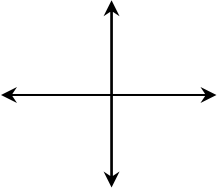
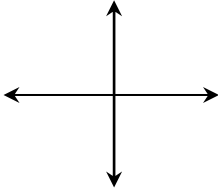
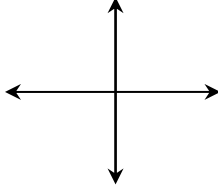
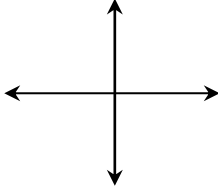
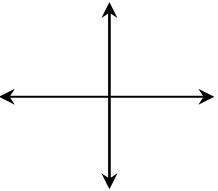
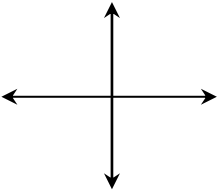
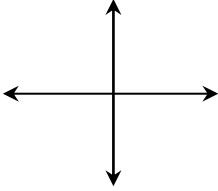
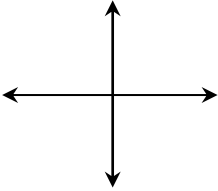


Name: \_\_\_\_\_

**PRE-CALCULUS REVIEW: Packet #1****Topic A: Basic Parent Functions and Transformations****Directions:** For each function family, give the parent function and sketch the shape of the graph.

<b>Linear</b> 	<b>Absolute Value</b> 	<b>Quadratic</b> 	<b>Cubic</b> 
<b>Square Root</b> 	<b>Cube Root</b> 	<b>Reciprocal</b> 	<b>Greatest Integer</b> 

**Recall the following transformations rules given a function  $f(x)$ :**

<b>Translations (Shifts)</b>	<b>Reflections</b>	<b>Dilations (compress/stretch)</b>
$f(x + h)$ shifts left	$-f(x)$ reflects over the $x$ -axis	$a \cdot f(x)$ is a vertical compression when $ a  < 1$ and a vertical stretch when $ a  > 1$
$f(x - h)$ shifts right		
$f(x) + k$ shifts up	$f(-x)$ reflects over the $y$ -axis	$f(b \cdot x)$ is a horizontal stretch when $ b  < 1$ and a horizontal compression when $ b  > 1$
$f(x) - k$ shifts down		

**Directions:** Describe the transformations from the parent function.

1. $f(x) = 3(x + 7)^3 + 4$	2. $f(x) =  -2(x - 1)  - 9$	3. $f(x) = -\frac{x + 8}{6} - 5$
4. The greatest integer parent function is reflected in the $y$ -axis, horizontally stretched by a factor of 4, and translated 3 units left. Write an equation to represent the <b>new function</b> .		5. The square root parent function is reflected in the $x$ -axis, vertically compressed by a factor of $\frac{1}{2}$ , then translated 5 units right and 1 unit down. Write an equation to represent the <b>new function</b> .



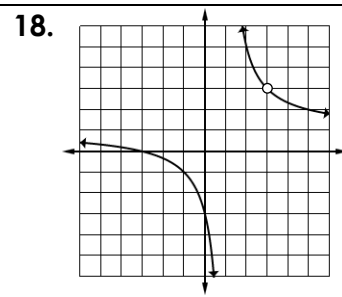
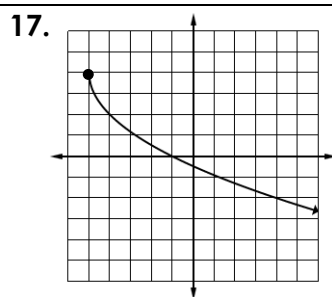
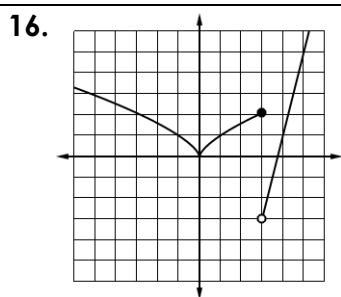
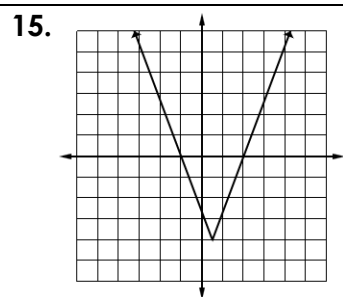
**Directions:** Determine algebraically if the function is even, odd, or neither. If even or odd, describe the symmetry.

13.  $f(x) = \sqrt[4]{2x^2 + 9}$

14.  $f(x) = \frac{-x^3 + 7}{4}$

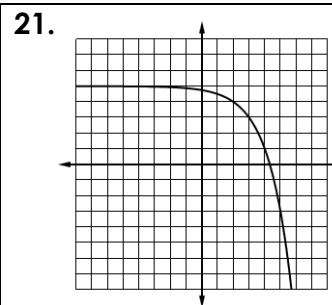
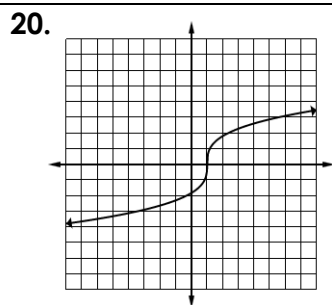
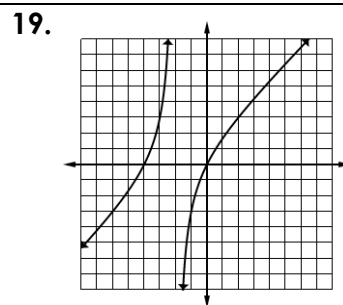
**Topic C: Continuity**

**Directions:** Determine if the functions below are continuous. If discontinuous, identify the type and location of discontinuity.



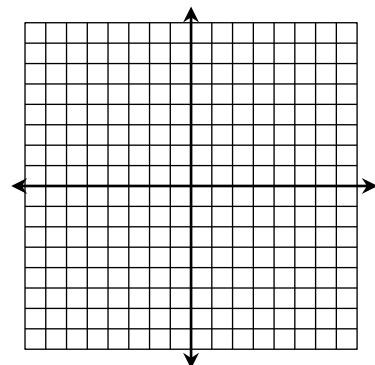
**Topic D: Inverse Functions**

**Directions:** Determine if the graph represents a one-to-one function.

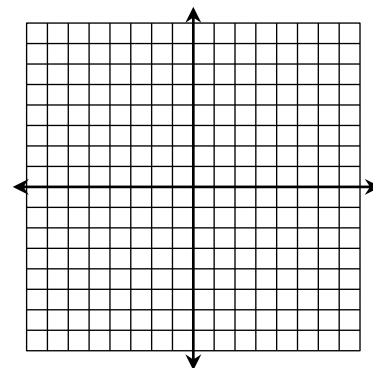


**Directions:** Find the inverse of each function. Then, graph both the function and its inverse.

22.  $f(x) = \frac{5}{x} + 2$



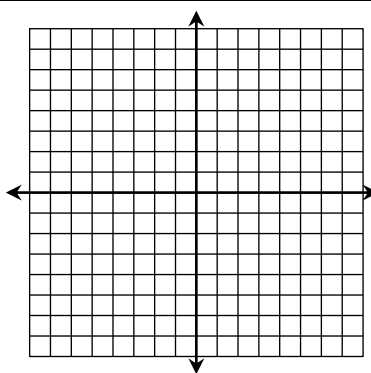
23.  $f(x) = 2\sqrt[3]{x-4}$



**Topic E: Piecewise Functions**

**Directions:** Graph the piecewise function. Identify the domain, range, and state the location and type of any discontinuities.

24.  $f(x) = \begin{cases} -1 & \text{if } x < -2 \\ x^2 - 6 & \text{if } -2 \leq x < 3 \\ -\frac{4}{3}x + 3 & \text{if } x \geq 3 \end{cases}$

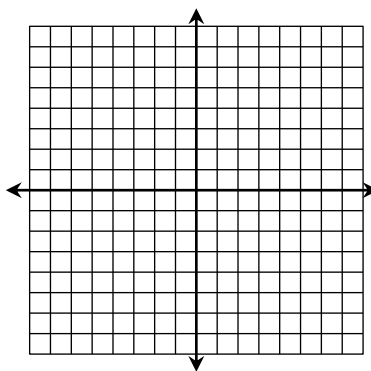


**Domain:**

**Range:**

**Discontinuities:**

25.  $f(x) = \begin{cases} \frac{4}{x+2} & \text{if } x \leq 1 \\ -3x + 8 & \text{if } x > 1 \end{cases}$

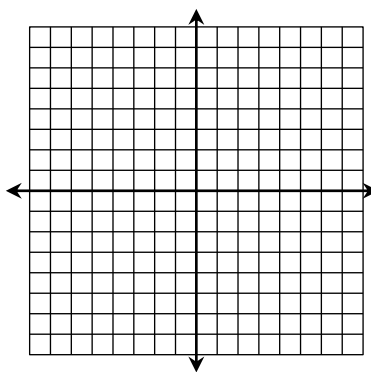


**Domain:**

**Range:**

**Discontinuities:**

26.  $f(x) = \begin{cases} \sqrt{8-2x} & \text{if } x < 2 \\ 4|x-3| & \text{if } 2 \leq x \leq 5 \\ 2x-7 & \text{if } x > 5 \end{cases}$



**Domain:**

**Range:**

**Discontinuities:**

# Pre-Calculus Review

## QUIZ 1

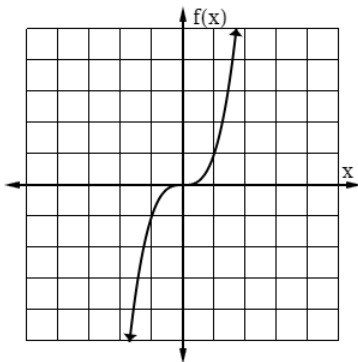
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Date: \_\_\_\_\_ Per: \_\_\_\_\_

1. The minimum point on the graph of the function  $f(x)$  is  $(-4, -1)$ . What is the minimum point on the graph of the function  $f(x + 2)$ ?

- A.  $(-4, -3)$
- B.  $(-4, 1)$
- C.  $(-2, -1)$
- D.  $(-6, -1)$

2. If the function below is reflected in the  $x$ -axis, then horizontally compressed by a factor of  $1/3$ , which equation represents the new function?



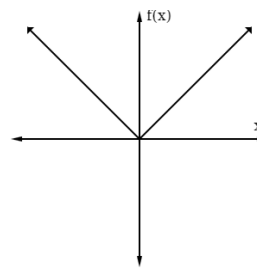
- A.  $f(x) = -3x^3$
- B.  $f(x) = \left(-\frac{1}{3}x\right)^3$
- C.  $f(x) = -(3x)^3$
- D.  $f(x) = -\left(\frac{1}{3}x\right)^3$

3. The function  $f(x) = \sqrt{x}$  is transformed to create function  $g$  below. Which describes a transformation that took place?

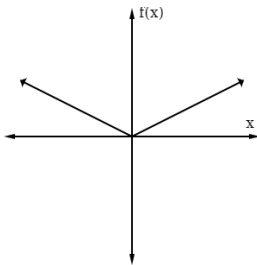
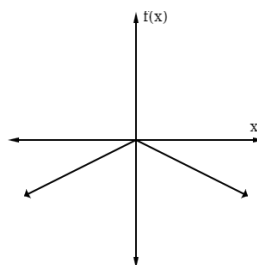
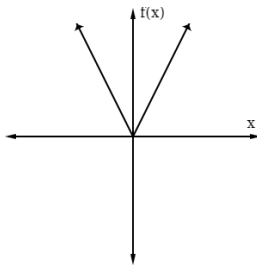
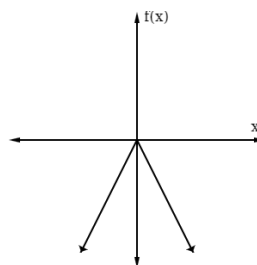
$$g(x) = \frac{1}{2}\sqrt{-(x-1)}$$

- A. a shift one unit to the left
- B. a shift one unit down
- C. a horizontal stretch of 2
- D. a reflection in the  $y$ -axis

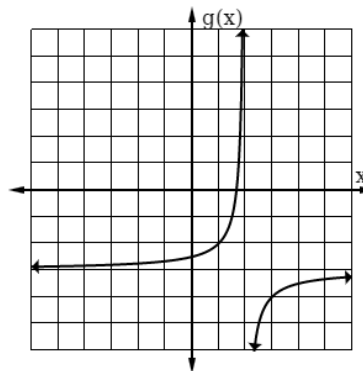
4. The graph of  $f(x)$  is shown below.



Which graph best represents  $a \cdot f(-x)$  if  $0 < a < 1$ ?

- A. 
- B. 
- C. 
- D. 

5. A function  $f$  was reflected in the  $x$ -axis, then translated three units left and two units down to create function  $g$  shown below. Which function represents  $f$ ?



- A.  $f(x) = \frac{1}{x+1} + 5$
- B.  $f(x) = \frac{1}{x-5} + 1$
- C.  $f(x) = \frac{1}{x+1} - 3$
- D.  $f(x) = \frac{1}{x-5} - 3$

6. If  $p$  and  $q$  are real numbers, which statement is true?

$$f(x) = (x - p)^2 + q$$

- A. The function decreases on the interval  $[-q, \infty)$ .
- B. The function increases on the interval  $[p, \infty)$ .
- C. The domain of the function is  $[p, \infty)$ .
- D. The range of the function is  $[-q, \infty)$ .

7. What is the domain of the function below?

$$f(x) = \sqrt{2x + 6}$$

- A.  $(-\infty, \infty)$                       C.  $[-3, \infty)$   
 B.  $[6, \infty)$                           D.  $[3, \infty)$

8. What set of transformations would cause  $f$  and  $g$  to have the same range?

$$f(x) = -(x + 5)^2 - 4; \quad g(x) = (x - 1)^2 - 2$$

- A. Reflect  $g$  in the  $x$ -axis; Translate  $f$  two units up.  
 B. Reflect  $f$  in the  $x$ -axis; Translate  $g$  four units down.  
 C. Reflect  $g$  in the  $x$ -axis; Translate  $f$  six units up.  
 D. Reflect  $f$  in the  $x$ -axis; Translate  $g$  two units down.

9. Which function is odd?

- A.  $f(x) = \sqrt{x^2 - 3}$                       C.  $f(x) = |x + 2|$   
 B.  $f(x) = \frac{4}{x}$                                   D.  $f(x) = 2x^4 - 8x^2 + 3$

10. Which equation, if true, will show that  $y^2 = x - 4$  is symmetric to the  $x$ -axis?

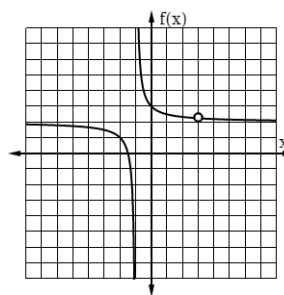
- A.  $(-y)^2 = x - 4$                       C.  $(-y)^2 = -x - 4$   
 B.  $y^2 = -x - 4$                           D.  $y^2 = -(x - 4)$

11. Given the function  $f(x)$  below, which statement is true?

$$f(x) = \begin{cases} |x + 1|, & x < 1 \\ x^3 - 2, & x \geq 1 \end{cases}$$

- A.  $f$  has a jump discontinuity at  $x = 1$ .  
 B.  $f$  has an infinite discontinuity at  $x = 1$ .  
 C.  $f$  has a removable discontinuity at  $x = 1$ .  
 D.  $f$  is continuous on the real numbers.

12. Given the graph of  $f(x)$  below, which statement is true?

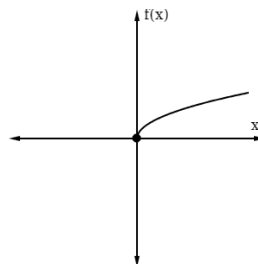


- A.  $f$  has a jump discontinuity at  $x = -1$ .  
 B.  $f$  has a removable discontinuity at  $x = -1$ .  
 C.  $f$  has a removable discontinuity at  $x = 3$ .  
 D.  $f$  has an infinite discontinuity at  $x = 3$ .

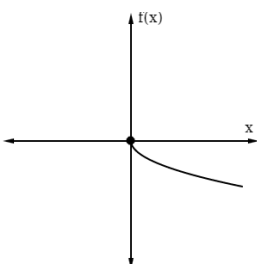
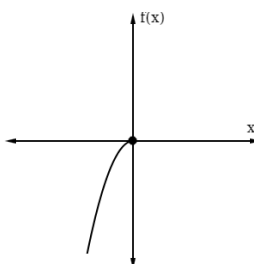
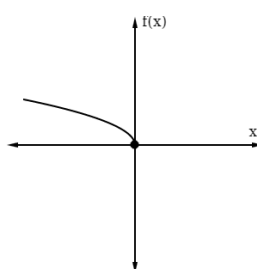
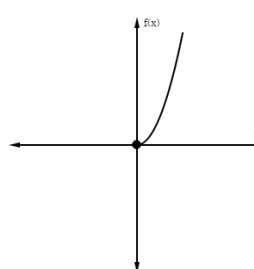
13. The graph of  $f(x) = x^3 + 5$  is reflected in the  $x$ -axis, then translated two units left. Which equation represents the inverse of the new function?

- A.  $f^{-1} = \sqrt[3]{-x + 5} - 4$                       C.  $f^{-1} = \sqrt[3]{x + 3}$   
 B.  $f^{-1} = \sqrt[3]{-x + 5} - 2$                       D.  $f^{-1} = \sqrt[3]{x + 7}$

14. The graph of  $f(x)$  is shown below.



Which graph best represents the inverse of  $f(x)$ ?

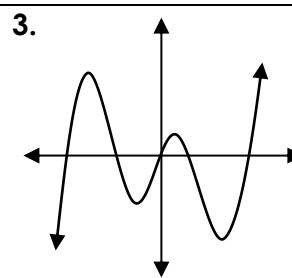
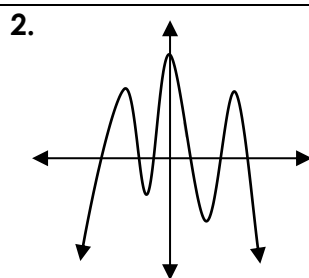
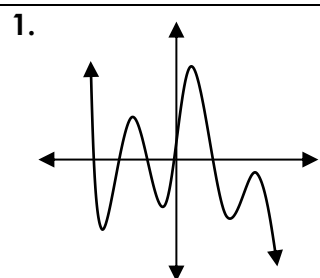
- A.                       C.   
 B.                       D. 

Name: \_\_\_\_\_

**PRE-CALCULUS REVIEW: Packet #2**

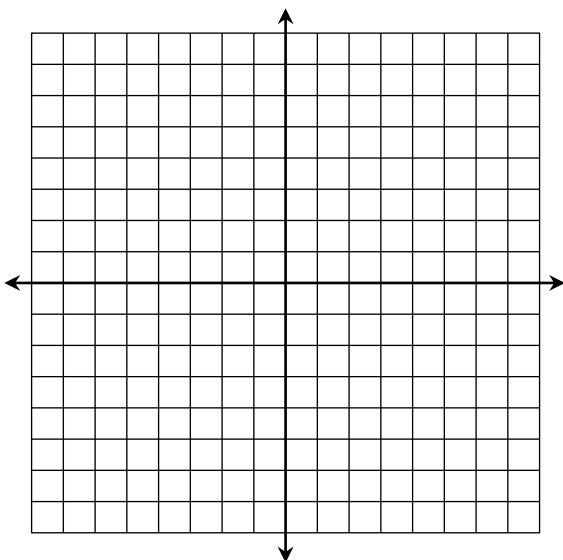
**Topic A: Graphs of Polynomial Functions**

**Directions:** Given the graph of each polynomial function below, determine the sign of the leading coefficient and whether the function has an even or odd degree.



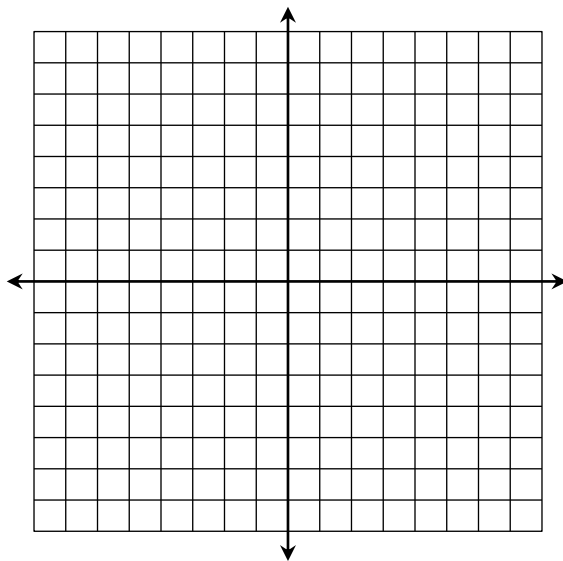
**Directions:** Graph each function, then identify its key characteristics.

4.  $f(x) = 2x^3 + x^2 - 6x + 3$



<b>Domain:</b>	<b>Range:</b>
<b>x-intercept(s):</b>	
<b>y-intercept:</b>	
<b>Rel. Minimum(s):</b>	
<b>Rel. Maximum(s):</b>	
<b>Inc. Interval(s):</b>	
<b>Dec. Interval(s):</b>	
<b>End Behavior:</b>	

5.  $f(x) = -x^4 + x^3 + 4x^2 - 6$



<b>Domain:</b>	<b>Range:</b>
<b>x-intercept(s):</b>	
<b>y-intercept:</b>	
<b>Rel. Minimum(s):</b>	
<b>Rel. Maximum(s):</b>	
<b>Inc. Interval(s):</b>	
<b>Dec. Interval(s):</b>	
<b>End Behavior:</b>	

### Topic B: Zeros of Polynomial Functions

**Directions:** Find all zeros for each function. Simplify all irrational zeros and complex solutions.

6.  $f(x) = 9x^4 + 68x^2 - 32$

7.  $f(x) = 2x^3 - 11x^2 + 2x + 15$

8.  $f(x) = x^3 - 6x^2 + 15x - 28$

9.  $f(x) = x^3 - 6x^2 - 12x + 8$

**Directions:** Identify the zeros, their multiplicities, and describe the effect on the graph.

10.  $f(x) = 2x^4 - 24x^3 + 72x^2$

Zero	Multiplicity	Effect

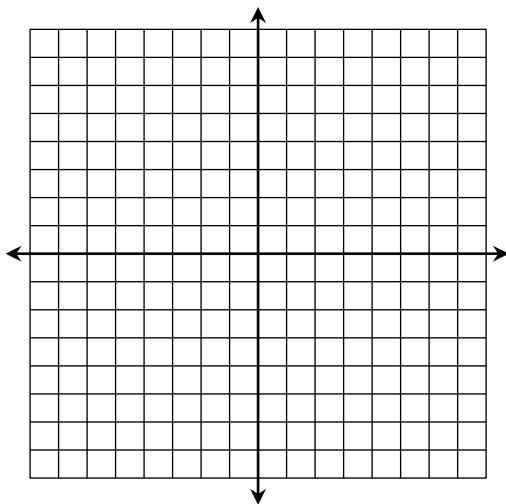
11.  $f(x) = x^5 + 3x^4 - 9x^3 - 27x^2$

Zero	Multiplicity	Effect

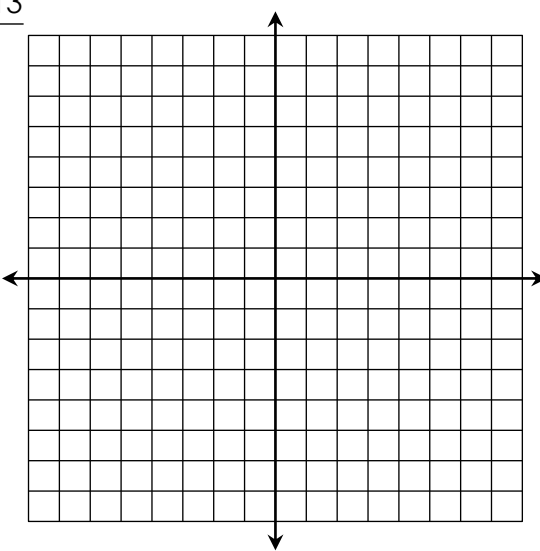
### Topic C: Graphs of Rational Functions

**Directions:** Graph each function, then identify its key characteristics.

12.  $f(x) = \frac{3x^2 + 7x + 2}{x^2 + x - 2}$

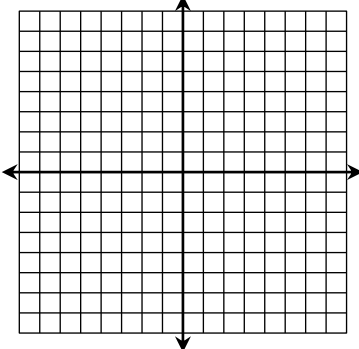
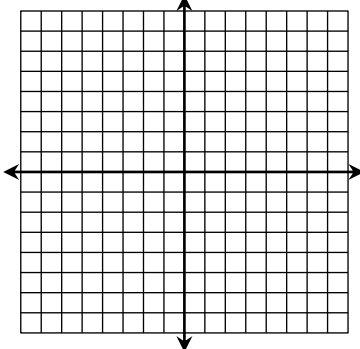
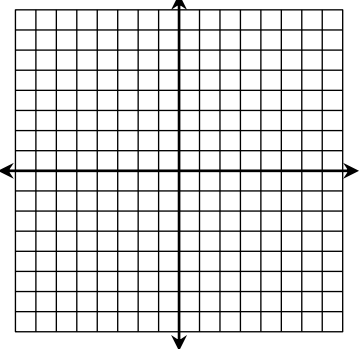
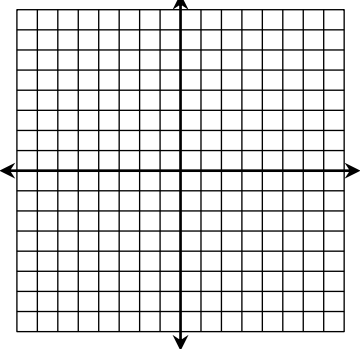


<b>Domain:</b>	<b>Range:</b>
<b>x-intercept(s):</b>	
<b>y-intercept:</b>	
<b>Vertical Asymptote:</b>	
<b>Horizontal Asymptote:</b>	
<b>Slant Asymptote:</b>	
<b>Hole(s):</b>	

<p>13. <math>f(x) = \frac{x^2 + 7x + 13}{x + 4}</math></p> 	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 2px;">Domain:</td> <td style="width: 50%; padding: 2px;">Range:</td> </tr> <tr> <td colspan="2" style="padding: 2px;">x-intercept(s):</td> </tr> <tr> <td colspan="2" style="padding: 2px;">y-intercept:</td> </tr> <tr> <td colspan="2" style="padding: 2px;">Vertical Asymptote:</td> </tr> <tr> <td colspan="2" style="padding: 2px;">Horizontal Asymptote:</td> </tr> <tr> <td colspan="2" style="padding: 2px;">Slant Asymptote:</td> </tr> <tr> <td colspan="2" style="padding: 2px;">Hole(s):</td> </tr> </table>	Domain:	Range:	x-intercept(s):		y-intercept:		Vertical Asymptote:		Horizontal Asymptote:		Slant Asymptote:		Hole(s):	
Domain:	Range:														
x-intercept(s):															
y-intercept:															
Vertical Asymptote:															
Horizontal Asymptote:															
Slant Asymptote:															
Hole(s):															

**Topic D: Graphs of Exponential & Logarithmic Functions**

**Directions:** Graph each function, then identify its key characteristics.

<p>14. <math>f(x) = \frac{3}{4} \cdot (2)^{x+1} - 5</math></p> 	<p>15. <math>f(x) = -e^{-x-3} + 1</math></p> 		
Domain:	Range:	Domain:	Range:
y-intercept:	Asymptote:	y-intercept:	Asymptote:
<p>16. <math>f(x) = \log_2(x + 6) - 3</math></p> 	<p>17. <math>f(x) = \log_{\frac{1}{3}}(x + 5) + 2</math></p> 		
Domain:	Range:	Domain:	Range:
x-intercept:	Asymptote:	x-intercept:	Asymptote:

**Topic E: Solving Logarithmic & Exponential Equations****Directions:** Solve each equation, rounding to the nearest ten-thousandths place when necessary.

**18.**  $2 \cdot \ln(2n) = \ln(n+3) + \ln(3n-2)$

**19.**  $\log_7 3 + \log_7(3y^2 + 8) = 2$

**20.**  $\left(\frac{1}{16}\right)^{2x-1} = 64^x \cdot \left(\frac{1}{4}\right)^{x+3}$

**21.**  $9 \cdot 5^{-4c-3} + 5 = 93$

**22.** The population of a certain town was 24,250. Fifteen years later, the population decreased to 10,625. If the population followed a continuous exponential decay model, find the rate at which the population decreased.**23.** Myles was gifted \$3,000 as a graduation present. He deposited half into an account that earns 3.25% interest compounded monthly and the other half into an account that earns 2.5% interest compounded continuously. If Myles does not make any additional deposits or withdrawals, how long will it take for each account to double in value?**24.** The number of radio commercials for a product affects the amount of people who purchase the product. Use the table below and a logistic function to determine how many commercials it takes for 17% of people to purchase the product.

Commercials	People (%)
0	1.7
20	4
40	5.2
60	9.1
80	16.4
100	20.7
120	24.3

# Pre-Calculus Review

## QUIZ 2

Name: \_\_\_\_\_

Date: \_\_\_\_\_ Per: \_\_\_\_\_

1. Which statement is true regarding the end behavior of the function below?

$$f(x) = -5(x+2)^7(x-3)^4(x-1)^3$$

- A.  $f$  is increasing at both its left and right ends
- B.  $f$  is decreasing at both its left and right ends
- C.  $f$  is increasing at its left end and decreasing at its right end
- D.  $f$  is decreasing at its left end and increasing at its right end

2. Which statement is true about the graph of the function  $f(x) = -3x^4 + 12x^2$ ?

- A.  $f$  crosses the  $x$ -axis at  $x = -2$ ,  $x = 0$ , and  $x = 2$ .
- B.  $f$  touches, but does not cross the  $x$ -axis at  $x = -2$  and  $x = 2$ .
- C.  $f$  crosses the  $x$ -axis at  $x = 0$  and touches but does not cross the  $x$ -axis at  $x = -2$  and  $x = 2$ .
- D.  $f$  crosses the  $x$ -axis at  $x = -2$  and  $x = 2$ , then touches but does not cross the  $x$ -axis at  $x = 0$ .

3. Which function has the following characteristics?

- It has a relative maximum at  $(-1, -8)$ .
- It has a  $y$ -intercept at  $(0, -9)$ .
- It has zeros at  $x = \{-3, 1\}$ .

- A.  $f(x) = x^4 - 4x^2 - 12x - 9$
- B.  $f(x) = x^4 - 4x^3 + 4x^2 - 9$
- C.  $f(x) = x^4 - 4x^3 - 2x^2 + 12x + 9$
- D.  $f(x) = x^4 + 4x^3 + 4x^2 - 9$

4. What are the zeros of the function below?

$$f(x) = x^3 + 3x^2 - 11x - 21$$

- A.  $\{3, -3 \pm \sqrt{2}\}$
- B.  $\{-3, 3 \pm \sqrt{2}\}$
- C.  $\{3, -2 \pm 3\sqrt{2}\}$
- D.  $\{-3, 2 \pm 3\sqrt{2}\}$

5. What are the zeros of the function below?

$$f(x) = x^4 + x^2 - 72$$

- A.  $\{\pm 2\sqrt{2}, \pm 3\}$
- B.  $\{\pm 2i\sqrt{2}, \pm 3i\}$
- C.  $\{\pm 2i\sqrt{2}, \pm 3\}$
- D.  $\{\pm 2\sqrt{2}, \pm 3i\}$

6. Which function has an  $x$ -intercept at  $(1, 0)$  and a horizontal asymptote of  $y = -4/3$ ?

- A.  $f(x) = \frac{8x^2 + 5x - 3}{-6x^2 + 6x}$
- B.  $f(x) = \frac{4x^2 - 3x - 1}{-3x^2 - 3x}$
- C.  $f(x) = \frac{3x^2 + 7x + 4}{-4x^2 + 4x}$
- D.  $f(x) = \frac{12x^2 - 13x - 4}{-9x^2 + 9x}$

7. If the function below has a  $y$ -intercept at  $(0, -4)$  and a vertical asymptote at  $x = 2$ , what are the values of  $a$  and  $b$ ?

$$f(x) = \frac{-16}{ax + b}$$

- A.  $a = 2$  and  $b = -4$
- B.  $a = -2$  and  $b = 4$
- C.  $a = -4$  and  $b = 2$
- D.  $a = -2$  and  $b = -4$

8. Which characteristics describes the graph of the function below?

$$f(x) = \frac{x^2 - 3x - 4}{x - 2}$$

- A. There are zeros at  $x = -4$  and  $x = 1$
- B. The  $y$ -intercept is  $(0, -2)$ .
- C. There is a vertical asymptote at  $x = -1$ .
- D. There is a slant asymptote at  $y = x - 1$ .

9. Which statement is true regarding the two functions below?

FUNCTION A	FUNCTION B
$f(x) = \log_3(x - 4) - 1$	$f(x) = 2^{x-1} - 4$

- A. Function A has a vertical asymptote at  $x = 4$  and Function B has a horizontal asymptote at  $y = -4$ .
- B. Function A has a vertical asymptote at  $x = -4$  and Function B has a horizontal asymptote at  $y = 4$ .
- C. Function A has a vertical asymptote at  $x = 1$  and Function B has a horizontal asymptote at  $y = -1$ .
- D. The functions are inverses of each other.

10. Which function has the characteristics below?

- $f(x)$  has an  $x$ -intercept at  $(2, 0)$ .
- As  $x$  approaches  $\infty$ ,  $f(x)$  approaches  $\infty$ .

- A.  $f(x) = \left(\frac{1}{2}\right)^{x-3} - 2$
- B.  $f(x) = \ln(x - 1)$
- C.  $f(x) = 2^{x+2} - 1$
- D.  $f(x) = \log_2(x + 3)$

11. Find the solution to the equation below.

$$2 \cdot \log(k - 1) = \frac{1}{3} \log 8 + \log(3k + 5)$$

- A.  $k = \{1, 9\}$
- B.  $k = \{-1, 9\}$
- C.  $k = 9$
- D.  $k = 1$

12. Find the solution to the equation below.

$$81^{5-3x} - 27^x = 0$$

- A. 3
- B.  $\frac{16}{3}$
- C.  $\frac{4}{3}$
- D.  $\frac{3}{5}$

13. Which value is the closest to the solution of the equation below?

$$13^{6-2p} + 5 = 85$$

- A. 1.8147
- B. 1.8695
- C. 1.9722
- D. 2.1458

14. Finn inherited \$8,000 from his grandparents. He placed the money in an account that earns 6.75% interest compounded semiannually. If he makes no other deposits or withdrawals, how much interest will he have earned after 5 years?

- A. \$2,731.27
- B. \$2,895.92
- C. \$3,015.78
- D. \$3,149.24

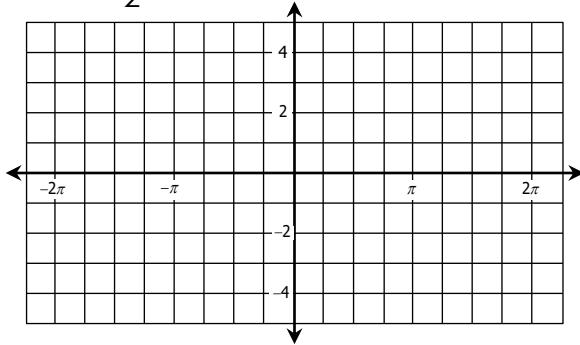
15. At the beginning of 2012, Jensyn deposited \$500 into an account that earns 4% interest compounded continuously. If she makes no other deposits or withdrawals, in what year can she expect to triple her investment?

- A. 2038
- B. 2039
- C. 2040
- D. 2041

**Topic A: Graphing Trigonometric Functions**

**Directions:** Graph each function and identify its key characteristics.

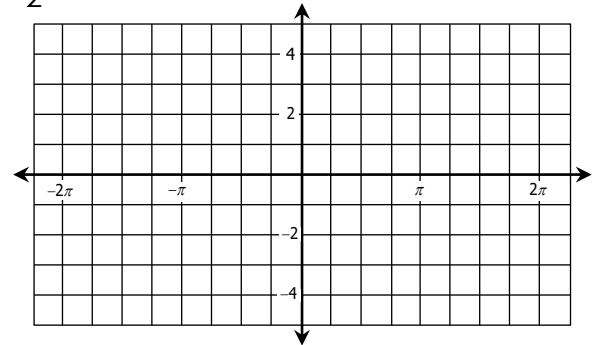
1.  $f(x) = -4 \cdot \cos \frac{1}{2}x$



**Amplitude:**

**Period:**

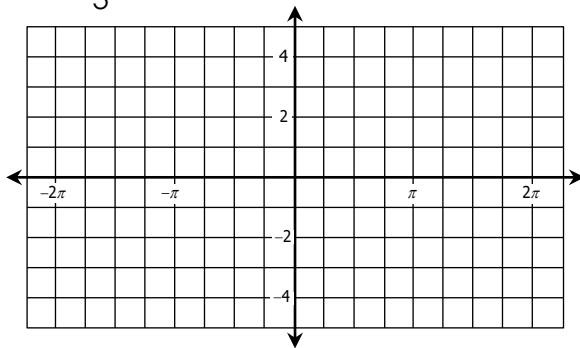
2.  $f(x) = \frac{1}{2} \cdot \cot x$



**Amplitude:**

**Period:**

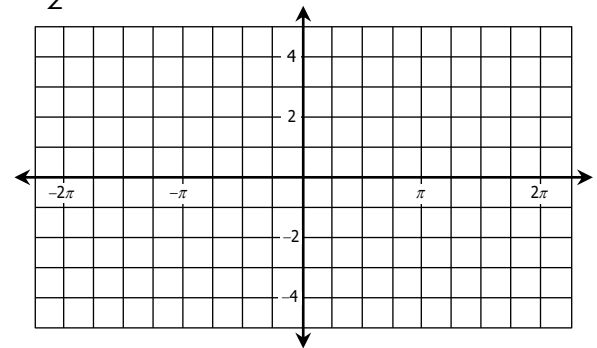
3.  $f(x) = 2 \cdot \sin \frac{8}{3}x$



**Amplitude:**

**Period:**

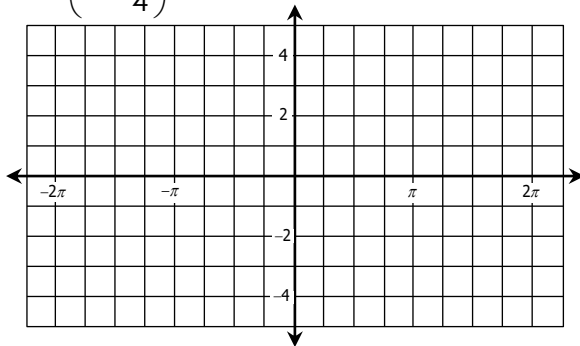
4.  $f(x) = -\frac{5}{2} \cdot \sec 2x$



**Amplitude:**

**Period:**

5.  $f(x) = \tan \left( x + \frac{\pi}{4} \right) + 1$



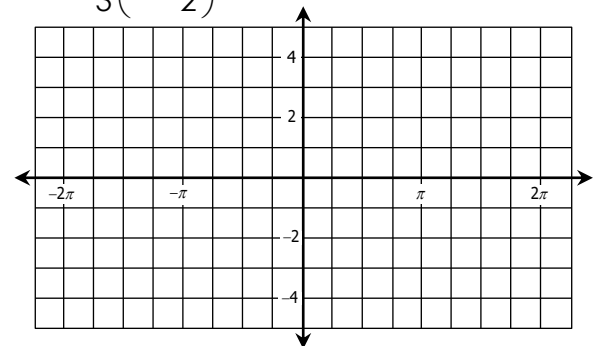
**Amplitude:**

**Period:**

**Phase Shift:**

**Vertical Shift:**

6.  $f(x) = -\csc \frac{4}{3} \left( x - \frac{\pi}{2} \right) - 3$



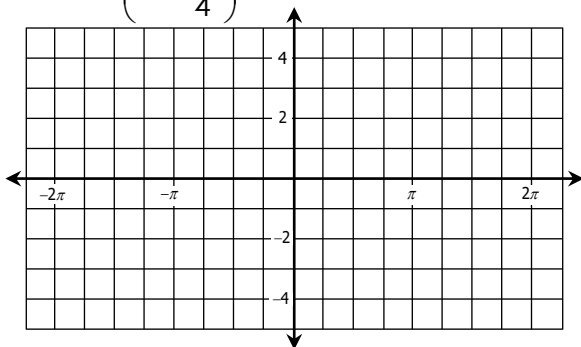
**Amplitude:**

**Period:**

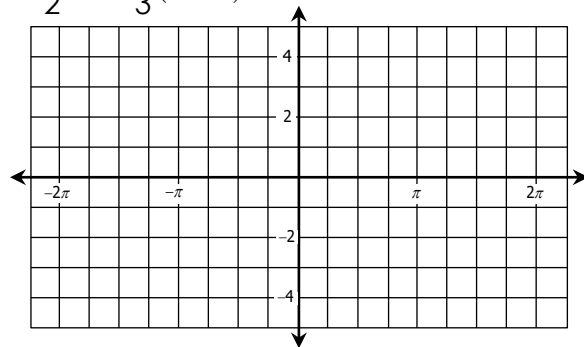
**Phase Shift:**

**Vertical Shift:**

7.  $f(x) = 2 \cdot \cos 4 \left( x + \frac{3\pi}{4} \right) - 1$



8.  $f(x) = -\frac{3}{2} \cdot \sec \frac{2}{3} (x - \pi) + 2$



Amplitude:

Period:

Phase Shift:

Vertical Shift:

Amplitude:

Period:

Phase Shift:

Vertical Shift:

### Topic B: Inverses & Compositions of Trigonometric Functions

**Directions:** Give the exact value, if it exists.

9.  $\arcsin(-1)$

10.  $\tan^{-1}(-\sqrt{3})$

11.  $\arccos(-2)$

12.  $\arctan\left(\tan \frac{\pi}{2}\right)$

13.  $\sin^{-1}\left(\cos \frac{7\pi}{6}\right)$

14.  $\cos(\arctan 0)$

### Topic C: Using Identities to Find Exact Values

**Directions:** Find each exact value.

15. If  $\cos \theta = \frac{1}{3}$  and  $0 < \theta < \frac{\pi}{2}$ , find  $\cos 2\theta$ .

16. If  $\tan \beta = -\frac{5\sqrt{7}}{7}$  and  $\frac{\pi}{2} < \beta < \pi$ , find  $\sin 2\beta$ .

17.  $\sin 202.5^\circ$

18.  $\tan \frac{5\pi}{12}$

19. If  $\sin y = \frac{2\sqrt{6}}{5}$  and  $\frac{\pi}{2} < y < \pi$ , find  $\cos \frac{y}{2}$ .

20. If  $\tan \alpha = -\frac{\sqrt{3}}{3}$  and  $\frac{3\pi}{2} < \alpha < 2\pi$ , find  $\tan \frac{\alpha}{2}$ .

21.  $\cos 75^\circ$

22.  $\tan \frac{11\pi}{12}$

23.  $3 \left( \sin \frac{31\pi}{12} + \sin \frac{13\pi}{12} \right)$

24.  $\cos \frac{13\pi}{12} - \cos \frac{7\pi}{12}$

25.  $\cos \frac{3\pi}{4} \cdot \sin \frac{\pi}{12}$

26.  $4\cos \frac{\pi}{12} \cdot \cos \frac{\pi}{12}$

**Topic D: Simplifying Trigonometric Expressions****Directions:** Simplify each trigonometric expression.

27.  $\frac{\cos^2 A - \cos^2 A \cdot \sec^2 A}{\sin^2 A}$

28.  $\frac{(\cot \beta + \csc \beta)(\cot \beta - \csc \beta)}{\csc \beta}$

29.  $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x}$

30.  $\frac{\sec \theta \cdot \sin \theta}{\cot \theta + \tan \theta}$

**Topic E: Solving Trigonometric Equations****Directions:** Solve each equation for the given interval.

31.  $10\sin \alpha \cdot \cos \alpha - 2\sqrt{3}\sin \alpha = -7\sqrt{3}\sin \alpha; [0, \pi]$

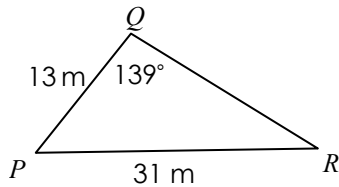
32.  $5\csc^2 \theta - 5 = 5\cot \theta; [\pi, 2\pi]$

33.  $\cos 2y + 3\cos^2 y - 1 = \cos^2 y + 1$ ;  $[0, 2\pi]$

34.  $\sin 3x + \sin x = \cos x$ ;  $\left[0, \frac{\pi}{2}\right]$

**Topic F: Law of Sines & Cosines**

35. Solve the triangle below. Round all answers to the nearest tenth.



$QR =$

$m\angle P =$

$m\angle R =$

36. In  $\triangle LMN$ , if  $m\angle M = 55^\circ$ ,  $MN = 19$  in, and  $LN = 16$  in, find  $m\angle N$ .

37. In  $\triangle XYZ$ , if  $m\angle X = 97^\circ$ ,  $XY = 23$  cm, and  $YZ = 22$  cm, find  $m\angle Z$ .

38. A fox leaves his den and travels N  $54^\circ$  W. After traveling 3.6 miles, he changes direction and travels 8 miles due east before stopping. How far is the fox from his den?

39. Ed is staining his triangular deck. If the sides of the deck measure 40 feet, 51 feet, and 50 feet, and each can of stain covers 175 square feet, determine how many cans Ed will need.

# Pre-Calculus Review

## QUIZ 3

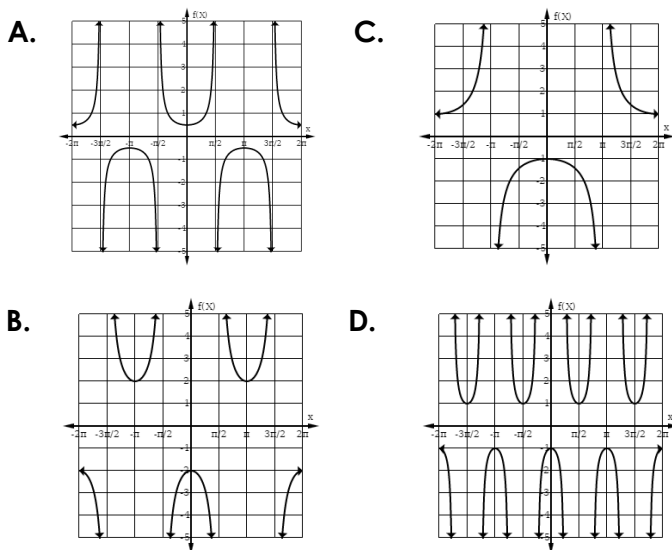
Name: \_\_\_\_\_

Date: \_\_\_\_\_ Per: \_\_\_\_\_

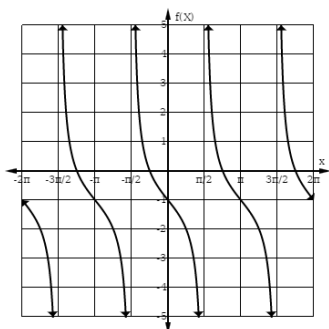
1. Which function has a period of  $2\pi$  and an amplitude of 4?

- A.  $f(x) = 4 \cdot \cos x$       C.  $f(x) = 2 \cdot \sin 4x$   
 B.  $f(x) = 4 \cdot \cos 2x$       D.  $f(x) = 4 \cdot \sin \frac{1}{2}x$

2. The graph of  $f(x) = \sec x$  undergoes a reflection about the  $x$ -axis and a horizontal stretch by a factor of 2. Which graph shows this transformed function?



3. Which function is shown on the graph below?



- A.  $f(x) = \cot\left(x + \frac{\pi}{4}\right) - 1$   
 B.  $f(x) = \cot\left(x + \frac{\pi}{2}\right) - 1$   
 C.  $f(x) = -\tan\left(x + \frac{\pi}{2}\right) - 1$   
 D.  $f(x) = -\tan\left(x + \frac{\pi}{4}\right) - 1$

4. The graph of the function  $f(x) = \sin x$  undergoes a vertical stretch by a factor of 3, a phase shift of  $\pi/4$  radians to the right, then a horizontal compression by a factor of  $1/2$ . Which statement is true regarding the transformed function?

- A. The  $y$ -intercept is  $(0, 3)$ .  
 B. The function intersects the  $x$ -axis at  $\left(\frac{\pi}{4} + \frac{k\pi}{2}, 0\right)$  where  $k$  is an integer.  
 C. The period is  $2\pi$ .  
 D. The amplitude is 2.

5. Find the exact value of the expression below.

$$\sin\left[\arctan\left(-\frac{\sqrt{3}}{3}\right)\right]$$

- A.  $\frac{\sqrt{3}}{2}$       C.  $\frac{1}{2}$   
 B.  $-\frac{\sqrt{3}}{2}$       D.  $-\frac{1}{2}$

6. If  $\sec x = -\frac{5}{2}$  and  $\csc x > 0$ , find the exact value of  $\tan x$ .

- A.  $\frac{\sqrt{21}}{2}$       C.  $\frac{5\sqrt{21}}{21}$   
 B.  $-\frac{\sqrt{21}}{2}$       D.  $-\frac{5\sqrt{21}}{21}$

7. If  $\cos \theta = -\frac{8}{17}$  and  $\pi \leq \theta < \frac{3\pi}{2}$ , find the exact value of  $\tan 2\theta$ .

- A.  $-\frac{289}{161}$       C.  $-\frac{240}{161}$   
 B.  $-\frac{161}{289}$       D.  $-\frac{161}{240}$

8. If  $\tan A = -2\sqrt{6}$  and  $\frac{3\pi}{2} \leq A < 2\pi$ , find the exact value of  $\sin \frac{A}{2}$ .

- A.  $\frac{2\sqrt{5}}{5}$                       C.  $\frac{\sqrt{10}}{5}$   
 B.  $-\frac{2\sqrt{5}}{5}$                       D.  $-\frac{\sqrt{10}}{5}$

12. Find all solutions to the equation below on the given interval.

$$3\cos^2\theta + 2 - \sin^2\theta = -4\cos\theta; [0, 2\pi)$$

- A.  $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$                       C.  $\left\{\frac{5\pi}{6}, \frac{7\pi}{6}\right\}$   
 B.  $\left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$                       D.  $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

9. Find the exact value of  $\tan \frac{19\pi}{12}$ .

- A.  $-2 - \sqrt{3}$   
 B.  $2 + \sqrt{3}$   
 C.  $\sqrt{3} - 2$   
 D.  $2 - \sqrt{3}$

13. Find all solutions to the equation below on the given interval.

$$3\sin u = -4\sin^2 u - \cos 2u; [0, 2\pi)$$

- A.  $\left\{\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$                       C.  $\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}\right\}$   
 B.  $\left\{\frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}\right\}$                       D.  $\left\{\frac{3\pi}{4}, \pi, \frac{5\pi}{4}\right\}$

10. Find the exact value of the expression below.

$$\sin \frac{3\pi}{4} \cdot \cos \frac{\pi}{12}$$

- A.  $\frac{-1 + \sqrt{3}}{4}$                       C.  $\frac{1 - \sqrt{3}}{4}$   
 B.  $\frac{-1 - \sqrt{3}}{4}$                       D.  $\frac{1 + \sqrt{3}}{4}$

14. In  $\triangle ABC$ ,  $m\angle A = 48^\circ$ ,  $AB = 27$  feet, and  $BC = 21$  feet. Which statement is true?

- A. No triangle exists with these measurements.  
 B.  $m\angle B$  can be  $24.8^\circ$  or  $107.2^\circ$ .  
 C.  $m\angle B$  can be  $59.2^\circ$  or  $24.8^\circ$ .  
 D.  $m\angle C$  can only be  $24.8^\circ$ .

11. Find all solutions to the equation below on the given interval.

$$\cos x \cdot \sec x = \sqrt{2} \cos x; [\pi, 2\pi)$$

- A.  $\left\{\frac{5\pi}{4}\right\}$                       C.  $\left\{\frac{5\pi}{4}, \frac{7\pi}{4}\right\}$   
 B.  $\left\{\frac{7\pi}{4}\right\}$                       D.  $\left\{\frac{4\pi}{3}, \frac{7\pi}{3}\right\}$

15. A triangular greenspace is formed by three roads in a neighborhood: Derby Lane, Harvest Drive, and Cary Drive. The length of the greenspace along Derby Lane and Harvest Drive is 15 meters and 21 meters, respectively. If the angle formed by Derby Lane and Harvest Drive is  $58^\circ$ , find the angle formed by Derby Lane and Cary Drive.

- A.  $44.3^\circ$   
 B.  $49.5^\circ$   
 C.  $77.7^\circ$   
 D.  $65.3^\circ$

Name: \_\_\_\_\_

**PRE-CALCULUS REVIEW: Packet #4****Topic A: Vectors & Vector Operations****Directions:** Use the initial and terminal points of the vectors below to give the component form, linear combination using standard vectors  $\mathbf{i}$  and  $\mathbf{j}$ , magnitude, and direction angle for each one.

1.  $\overline{CD}$  with  $C(-7, 5)$  and  $D(-5, 4)$

2.  $\overline{PQ}$  with  $P(-1, -1)$  and  $Q(5, 3)$

Component Form:

Linear Combination:

Component Form:

Linear Combination:

Magnitude:

Direction Angle:

Magnitude:

Direction Angle:

**Directions:** Find each of the following for  $\mathbf{a} = \langle 8, 0 \rangle$ ,  $\mathbf{b} = \langle -3, 7 \rangle$ , and  $\mathbf{c} = \langle 6, -9 \rangle$ .

3.  $\mathbf{b} - \mathbf{a}$

4.  $2\mathbf{a} + 3\mathbf{c}$

5.  $-\frac{1}{3}\mathbf{c} - \frac{5}{2}\mathbf{a}$

**Directions:** Find the component form of each vector with the given magnitude and direction angle.

6.  $\|\mathbf{k}\| = 14$ ,  $\theta = 225^\circ$

7.  $\|\mathbf{q}\| = 5$ ,  $\theta = 110^\circ$

**Directions:** Convert each vector to trigonometric form.

8.  $\mathbf{z} = 5\mathbf{i} - 5\mathbf{j}$

9.  $\overline{EF}$  with  $E(3, 1)$  and  $F(2, -2)$

**Topic B: Dot Products & Angle Between Vectors****Directions:** Find the dot product of the given vectors, then determine whether the vectors are orthogonal.

10.  $\mathbf{c} = \langle 6, 4 \rangle$  and  $\mathbf{d} = \langle -2, 3 \rangle$

11.  $\mathbf{p} = 3\mathbf{i} - 4\mathbf{j}$  and  $\mathbf{q} = -12\mathbf{i} + 9\mathbf{j}$

**Directions:** Find the angle between each vector pair.

12.  $\mathbf{x} = \langle -7, 2 \rangle$  and  $\mathbf{y} = \langle 5, 6 \rangle$

13.  $\mathbf{m} = -\mathbf{i}$  and  $\mathbf{n} = 3\mathbf{i} + 10\mathbf{j}$

### Topic C: Vector Applications

14. Leon kicks a soccer ball off the ground so that it travels with a velocity of 71 kilometers per hour at an angle of  $41^\circ$  with the ground. Find the magnitude of the horizontal and vertical components.

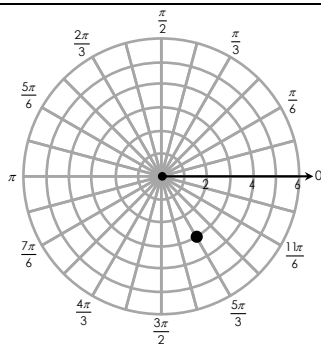
15. A blimp is traveling at a speed of 44 miles per hour in the direction of  $N 62^\circ E$ . While in transit, the blimp encounters wind traveling at a velocity of 29 miles per hour traveling in the direction of  $N 4^\circ W$ . Find the resultant speed and direction of the blimp.

16. Two tugboats are pulling a freighter into a port. If Tugboat A is traveling in the direction  $N 37^\circ W$  with a force of 31,000 kilograms, Tugboat B is traveling in the direction  $S 11^\circ W$  with a force of 26,250 kilograms, find the magnitude and direction angle of the resultant force on the freighter.

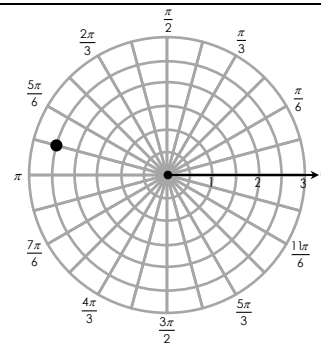
### Topic D: Polar vs. Rectangular Coordinates

**Directions:** Given each point on the polar grids below, name two different pairs of polar coordinates if  $0 \leq \theta \leq 2\pi$ .

17.



18.



**Directions:** Convert each pair of polar coordinates to rectangular coordinates. Round to the nearest hundredth if necessary.

19.  $\left(-7, \frac{\pi}{3}\right)$

20.  $\left(6, \frac{7\pi}{4}\right)$

21.  $\left(3, -\frac{2\pi}{15}\right)$

**Directions:** Convert each pair of rectangular coordinates to polar coordinates. Round to the nearest hundredth if necessary. If  $0 \leq \theta < 2\pi$ , give two possible solutions.

22. (5, 5)

23.  $(-3\sqrt{3}, -3)$

24. (-2, 6)

### Topic E: Polar Graphs

**Directions:** Classify the curve of each polar equation.

25.  $r = 2 + 4\cos\theta$

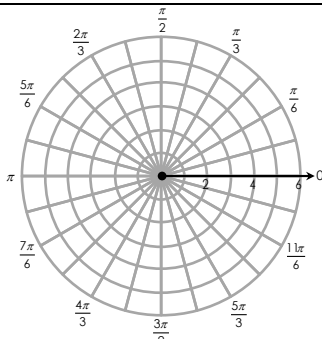
26.  $r^2 = 4\cos(2\theta)$

27.  $r = 4 + 2\sin\theta$

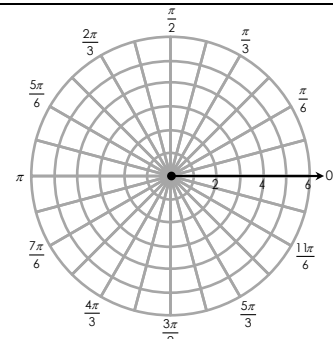
28.  $r = 7\cos(5\theta)$

**Directions:** Classify each of the following, then sketch the graph of each equation.

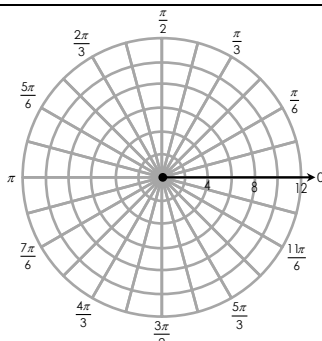
29.  $r = 5\sin(2\theta)$



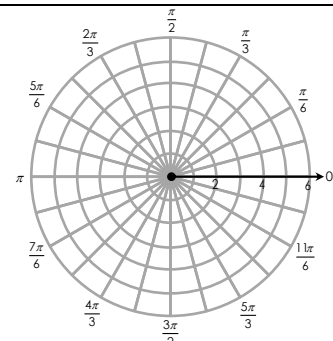
30.  $r = -4\cos\theta$



31.  $r^2 = 81\sin(2\theta)$



32.  $r = 3 + 3\sin\theta$



### Topic F: Polar vs. Rectangular Equations

**Directions:** Convert the equations from polar to rectangular form.

33.  $y = \frac{x^2}{3}$

34.  $(x - 4)^2 + (y + 1)^2 = 17$

**Directions:** Convert the equations from rectangular to polar form.

35.  $r = -8\csc\theta$

36.  $r = -4\cos\theta + 10\sin\theta$

### Topic G: Polar Forms of Complex Numbers

**Directions:** Convert each complex number to polar form.

37.  $8 - 8i$

38.  $-\sqrt{15} + 5i$

**Directions:** Convert the polar form of each complex number to rectangular form.

39.  $2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

40.  $\sqrt{2}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

**Directions:** Find each product or quotient. Express your answer in rectangular form.

41.  $5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) \cdot 3\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

42.  $4\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) \div 6\sqrt{2}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

**Directions:** Find each power. Express your answer in rectangular form.

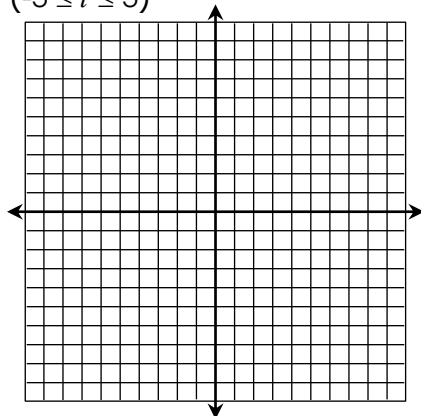
43.  $\left[3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^6$

44.  $(\sqrt{3} - i)^4$

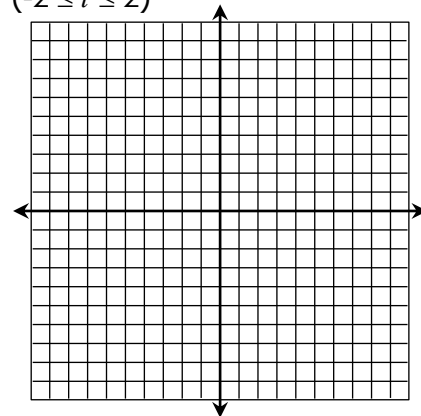
### Topic H: Graphing Parametric Equations

**Directions:** Sketch the curve given by each set of parametric equations over the given interval. Use arrows to indicate the direction of the curve as  $t$  increases.

45.  $x = 2t - 1, y = t^2 \quad (-3 \leq t \leq 3)$



46.  $x = t^2 - 1, y = 2t \quad (-2 \leq t \leq 2)$



### Topic I: Parametric vs. Rectangular Equations

**Directions:** Write each set of parametric equations in rectangular form. Note any restrictions on the domain.

47.  $x = 2t - 3, y = -t + 6$

48.  $x = t^2 - 4, y = 3t$

49.  $x = 3\cos\theta$ ,  $y = 5\sin\theta$

50.  $x = \cos\theta + 4$ ,  $y = \sin\theta - 1$

**Topic J: Applications of Parametric Equations**

**Use for questions 51 and 52:** A ball was launched at an angle of elevation of  $82^\circ$  at an initial velocity of 36 meters per second from a height of 14 meters above ground.

51. Find the horizontal distance the ball has traveled at 2.5 seconds.

52. Find the vertical distance the ball has traveled at 2.5 seconds.

**Use for questions 53 and 54:** A soccer ball is kicked down the field. The ball travels at an angle of elevation of  $53^\circ$  at an initial velocity of 16 meters per second.

53. What was the maximum height of the ball?

54. What was the horizontal distance the ball traveled at its maximum height?

**Use for questions 55 and 56:** A frog leaped from the ground at an angle of elevation of  $53^\circ$ , leaping at an initial velocity of 8 feet per second.

55. How long was the frog in the air?

56. What was the horizontal distance traveled by the frog?

# Pre-Calculus Review

## QUIZ 4

Name: \_\_\_\_\_

Date: \_\_\_\_\_ Per: \_\_\_\_\_

1. Given  $R(4, -1)$  and  $S(-2, 7)$ , give the magnitude and approximate direction angle for  $\overline{RS}$ .

- A.  $2\sqrt{10}$ ;  $71.57^\circ$
- B.  $2\sqrt{10}$ ;  $108.43^\circ$
- C. 10;  $126.87^\circ$
- D. 10;  $143.13^\circ$

2. Vector  $a$  has a magnitude of 4 units and a direction of  $-30^\circ$ . Vector  $b$  has a magnitude of 2 units and a direction of  $90^\circ$ . Which is the component form of vector  $c$  if  $c = a + 2b$ ?

- A.  $\langle 2, -2\sqrt{3} \rangle$
- B.  $\langle -2, 2\sqrt{3} \rangle$
- C.  $\langle -2\sqrt{3}, 2 \rangle$
- D.  $\langle 2\sqrt{3}, 2 \rangle$

3. Find the approximate measure of the angle formed between the vectors given below.

$$u = -i + 7j; v = 2i + 3j$$

- A.  $37.78^\circ$
- B.  $38.19^\circ$
- C.  $41.82^\circ$
- D.  $48.34^\circ$

4. Two soccer players kick a ball at the same time. One player kicks the ball with a force of 148 newtons in the direction of  $N 40^\circ E$ . The other player kicks with a force of 115 newtons in the direction of  $N 70^\circ W$ . What is the resultant magnitude and direction of the resultant force on the ball?

- A. 153.26 newtons;  $N 4.84^\circ W$
- B. 161.54 newtons;  $N 4.84^\circ W$
- C. 153.26 newtons;  $N 4.16^\circ E$
- D. 161.54 newtons;  $N 4.16^\circ E$

5. A helicopter flying with a velocity of 125 miles per hour makes a  $36^\circ$  angle with the horizontal. What is the approximate vertical component of the helicopters velocity?

- A. 73.47 mph
- B. 83.92 mph
- C. 95.28 mph
- D. 101.13 mph

6. Which point maps to the same position on a polar coordinate as point  $P$  below?

$$P\left(-1, \frac{13\pi}{18}\right)$$

- A.  $\left(1, \frac{5\pi}{18}\right)$
- B.  $\left(1, -\frac{5\pi}{18}\right)$
- C.  $\left(-1, -\frac{5\pi}{18}\right)$
- D.  $\left(-1, \frac{13\pi}{9}\right)$

7. What are the rectangular coordinates of the polar coordinates given below?

$$\left(-10, -\frac{4\pi}{3}\right)$$

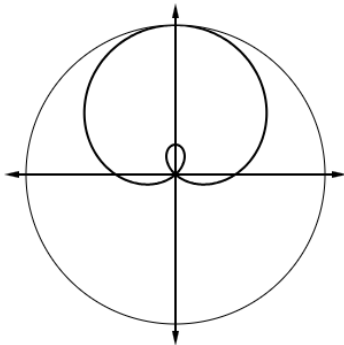
- A.  $(5\sqrt{3}, -5)$
- B.  $(-5\sqrt{3}, 5)$
- C.  $(5, -5\sqrt{3})$
- D.  $(-5, 5\sqrt{3})$

8. Which statement best describes the graph of the polar equation below?

$$r = \cos 3\theta$$

- A. A rose with 6 petals
- B. A rose with 3 petals.
- C. A cardioid.
- D. A lemniscate.

9. The graph of the equation  $r = a + b \sin \theta$  is shown below. Which gives possible values for  $a$  and  $b$ ?



- A.  $a = -2; b = -3$
- B.  $a = 2; b = -3$
- C.  $a = 3; b = 2$
- D.  $a = 2; b = 3$

10. A circle has a radius of 4 and a center at  $(0, 4)$  in the rectangular plane. What is the polar equation for this circle?

- A.  $r = 8\cos \theta$
- B.  $r = 4\cos \theta$
- C.  $r = 8\sin \theta$
- D.  $r = 4\sin \theta$

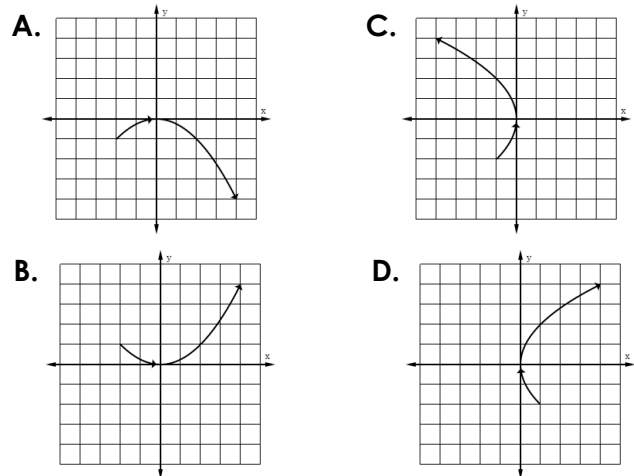
11. Which expression is the equivalent polar form of the expression below?

$$(1 - i\sqrt{3})^5$$

- A.  $16 \left( \cos \frac{10\pi}{3} + i \sin \frac{10\pi}{3} \right)$
- B.  $32 \left( \cos \frac{25\pi}{3} + i \sin \frac{25\pi}{3} \right)$
- C.  $243 \left( \cos \frac{10\pi}{3} + i \sin \frac{10\pi}{3} \right)$
- D.  $243 \left( \cos \frac{25\pi}{3} + i \sin \frac{25\pi}{3} \right)$

12. Which graph represents the pair of parametric equations below on the given interval?

$$x = -\frac{t^2}{4}, y = t; \quad -2 \leq t \leq 4$$



13. Give the rectangular form of the pair of parametric equations below.

$$x = t^2 - 2, y = 5t$$

- A.  $y = \pm 5\sqrt{x+2}$
- B.  $y = \pm 5\sqrt{x-2}$
- C.  $y = \pm 2\sqrt{x-5}$
- D.  $y = \pm 2\sqrt{x+5}$

14. Which set of parametric equations represent the rectangular equation shown below?

$$\frac{x^2}{81} + \frac{y^2}{16} = 1$$

- A.  $x = 4 \sin \theta, y = 9 \cos \theta$
- B.  $x = 9 \sin \theta, y = 4 \cos \theta$
- C.  $x = 4 \cos \theta, y = 9 \sin \theta$
- D.  $x = 9 \cos \theta, y = 4 \sin \theta$

15. A referee threw a flag at a speed of 30 feet per second at an angle of elevation of  $55^\circ$ . If the flag left his hand at a height of 4 feet, how far from the point where the referee is standing does the flag land?

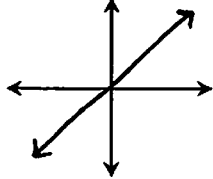
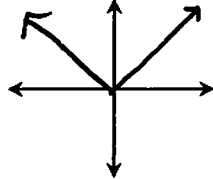
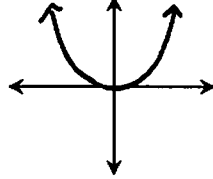
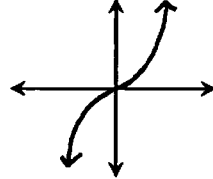
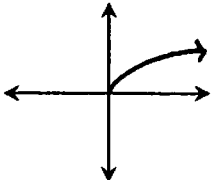
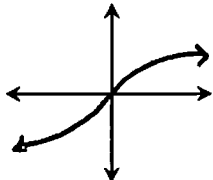
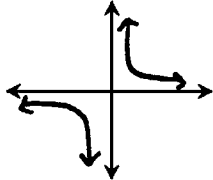
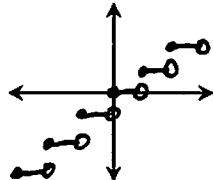
- A. 23.55 ft
- B. 24.73 ft
- C. 26.14 ft
- D. 28.91 ft

Name: \_\_\_\_\_

**PRE-CALCULUS REVIEW: Packet #1**

**Topic A: Basic Parent Functions and Transformations**

**Directions:** For each function family, give the parent function and sketch the shape of the graph.

<p><b>Linear</b></p> 	<p><b>Absolute Value</b></p> 	<p><b>Quadratic</b></p> 	<p><b>Cubic</b></p> 
<p><b>Square Root</b></p> 	<p><b>Cube Root</b></p> 	<p><b>Reciprocal</b></p> 	<p><b>Greatest Integer</b></p> 

**Recall the following transformations rules given a function  $f(x)$ :**

Translations (Shifts)	Reflections	Dilations (compress/stretch)
$f(x+h)$ shifts left	$-f(x)$ reflects over the $x$ -axis	$a \cdot f(x)$ is a vertical compression when $ a  < 1$ and a vertical stretch when $ a  > 1$
$f(x-h)$ shifts right		
$f(x)+k$ shifts up	$f(-x)$ reflects over the $y$ -axis	$f(b \cdot x)$ is a horizontal stretch when $ b  < 1$ and a horizontal compression when $ b  > 1$
$f(x)-k$ shifts down		

**Directions:** Describe the transformations from the parent function.

<p>1. <math>f(x) = 3(x+7)^3 + 4</math></p> <p>- Vert. stretch by 3</p> <p>- translate left 7 and up 4</p>	<p>2. <math>f(x) =  -2(x-1)  - 9</math></p> <p>- Reflect in <math>y</math>-axis</p> <p>- Horiz. Compression by <math>1/2</math></p> <p>- Translate right 1 and down 9</p>	<p>3. <math>f(x) = -\frac{x+8}{6} - 5</math></p> <p>- Reflect in <math>x</math>-axis</p> <p>- Vert. compression by <math>1/6</math></p> <p>- Translate left 8, down 2</p>
<p>4. The greatest integer parent function is reflected in the <math>y</math>-axis, horizontally stretched by a factor of 4, and translated 3 units left. Write an equation to represent the <b>new function</b>.</p> <p><math>f(x) = \left\lfloor -\frac{1}{4}(x+3) \right\rfloor</math></p>	<p>5. The square root parent function is reflected in the <math>x</math>-axis, vertically compressed by a factor of <math>1/2</math>, then translated 5 unit right and 1 unit down. Write an equation to represent the <b>new function</b>.</p> <p><math>f(x) = -\frac{1}{2}\sqrt{x-5} - 1</math></p>	

6. Transformations were applied to the quadratic parent function such that it creates an absolute maximum at  $(-7, -2)$ . Write an equation that could represent this new function.

$$f(x) = -(x+7)^2 - 2$$

7. A certain function is reflected in the  $x$ -axis, vertically compressed by a factor of  $\frac{3}{4}$ , translated right 3 and up 7. If the new function is represented by  $f(x) = 9|5(x-3)| + 2$ , write an equation that could represent the original function.

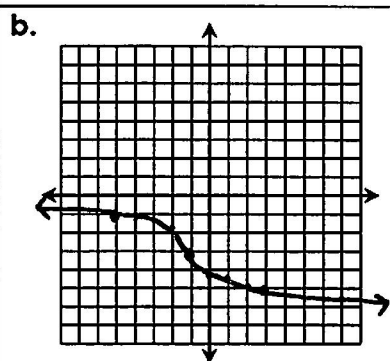
$$f(x) = -12|5x| - 5$$

**Directions:** Using the description of transformations from the parent function, (a) write a function, then (b) graph the function and state its domain and range.

8. The cube root function is reflected in the  $y$ -axis, horizontally compressed by a factor of  $\frac{1}{2}$ , and translated left 1 and down 3.

a.

$$f(x) = \sqrt[3]{-2(x+1)} - 3$$



Domain:

$$\mathbb{R}$$

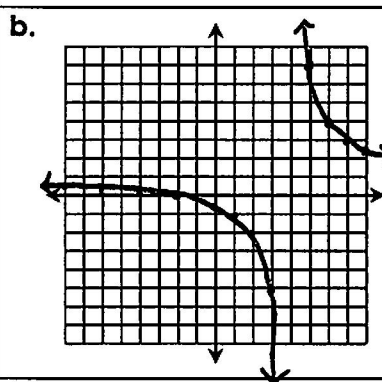
Range:

$$\mathbb{R}$$

9. The reciprocal function is vertically stretched by a factor of 3, horizontally stretched by a factor of 2, then translated right 4 and up 1.

a.

$$f(x) = \frac{3}{\frac{1}{2}(x-4)} + 1$$



Domain:

$$\{x | x \neq 4\}$$

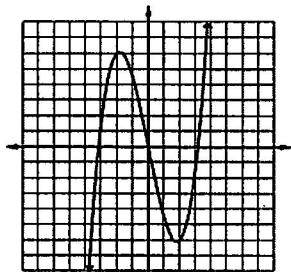
Range:

$$\{y | y \neq 1\}$$

### Topic B: Symmetry; Even and Odd Functions

**Directions:** Use the graph to determine if the relations given below are symmetric to the  $x$ -axis,  $y$ -axis, or origin. Confirm your answer algebraically.

10.  $y = \frac{1}{2}x^3 - 5x$

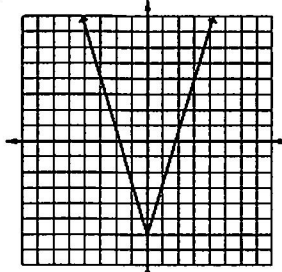


$$-y = \frac{1}{2}(-x)^3 - 5(-x)$$

$$-y = -\frac{1}{2}x^3 + 5x$$

Symmetric to origin

11.  $y = \frac{2}{3}|5x| - 6$



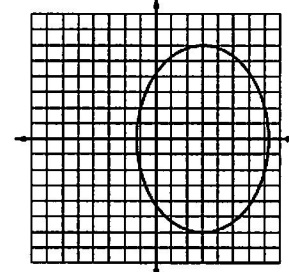
$$y = \frac{2}{3}|5(-x)| - 6$$

$$y = \frac{2}{3}|-5x| - 6$$

$$y = \frac{2}{3}|5x| - 6$$

Symmetric to  $y$ -axis

12.  $2(x-3)^2 + y^2 = 36$



$$2(x-3)^2 + (-y)^2 = 36$$

$$2(x-3)^2 + y^2 = 36$$

Symmetric to  $x$ -axis

**Directions:** Determine algebraically if the function is even, odd, or neither. If even or odd, describe the symmetry.

13.  $f(x) = \sqrt[4]{2x^2 + 9}$   
 $f(-x) = \sqrt[4]{2(-x)^2 + 9}$   
 $f(-x) = \sqrt[4]{2x^2 + 9}$

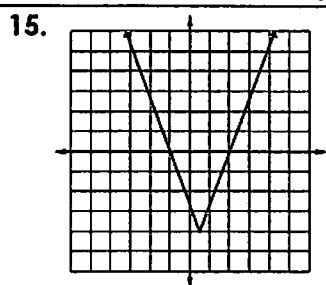
Even; Symmetric to y-axis

14.  $f(x) = \frac{-x^3 + 7}{4}$   
 $f(-x) = \frac{-(-x)^3 + 7}{4}$   
 $f(-x) = \frac{x^3 + 7}{4}$

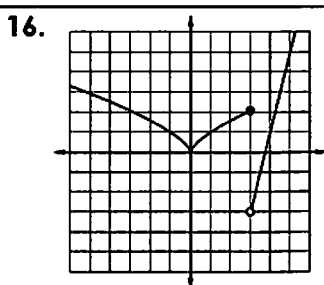
Neither

**Topic C: Continuity**

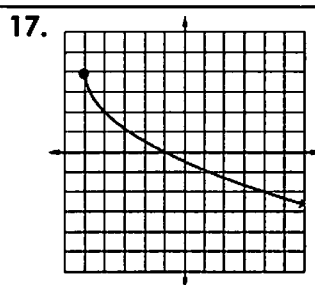
**Directions:** Determine if the functions below are continuous. If discontinuous, identify the type and location of discontinuity.



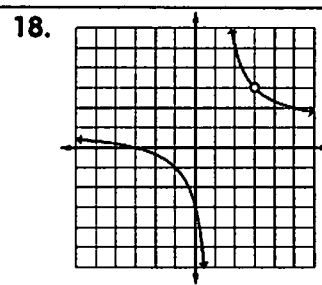
Continuous



Discontinuous;  
 $x=3$ ; jump



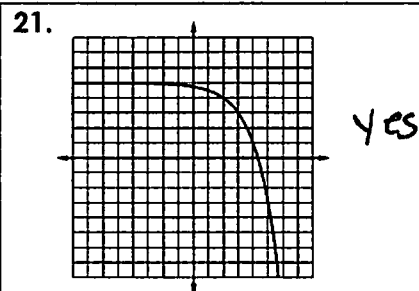
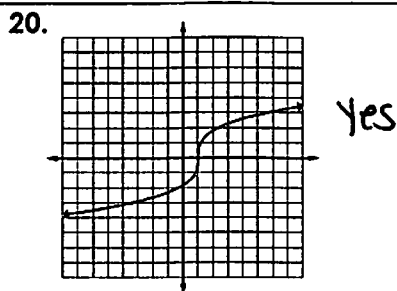
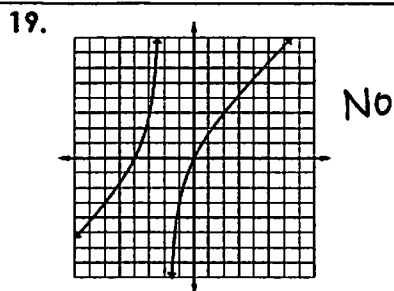
Continuous



Discontinuous  
 $x=1$ ; infinite  
 $x=3$ ; removable

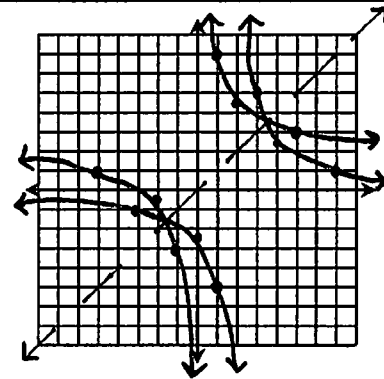
**Topic D: Inverse Functions**

**Directions:** Determine if the graph represents a one-to-one function.



**Directions:** Find the inverse of each function. Then, graph both the function and its inverse.

22.  $f(x) = \frac{5}{x} + 2$   
 $x = \frac{5}{y} + 2$   
 $x - 2 = \frac{5}{y}$   
 $y(x - 2) = 5$   
 $y = \frac{5}{x - 2}$



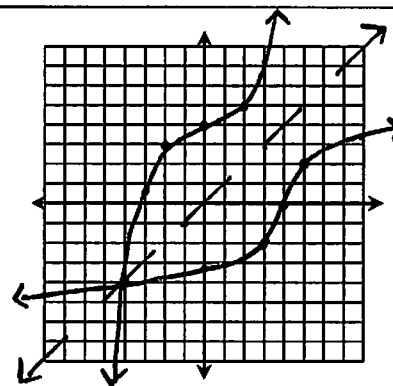
$$23. f(x) = 2\sqrt[3]{x-4}$$

$$x = 2\sqrt[3]{y-4}$$

$$\frac{1}{2}x = \sqrt[3]{y-4}$$

$$\frac{1}{8}x^3 = y-4$$

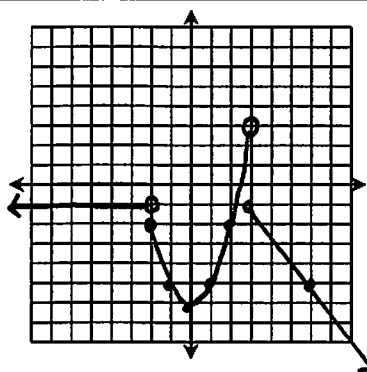
$$\frac{1}{8}x^3 + 4 = y$$



### Topic E: Piecewise Functions

**Directions:** Graph the piecewise function. Identify the domain, range, and state the location and type of any discontinuities.

$$24. f(x) = \begin{cases} -1 & \text{if } x < -2 \\ x^2 - 6 & \text{if } -2 \leq x < 3 \\ -\frac{4}{3}x + 3 & \text{if } x \geq 3 \end{cases}$$



Domain:

$$\mathbb{R}$$

Range:

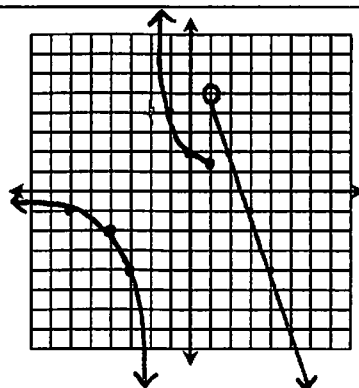
$$\{y \mid y < 3\}$$

Discontinuities:

$x = -2$ ; jump

$x = 3$ ; jump

$$25. f(x) = \begin{cases} \frac{4}{x+2} & \text{if } x \leq 1 \\ -3x + 8 & \text{if } x > 1 \end{cases}$$



Domain:

$$\{x \mid x \neq -2\}$$

Range:

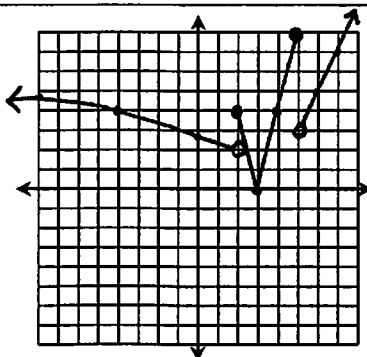
$$\mathbb{R}$$

Discontinuities:

$x = -2$ ; infinite

$x = 1$ ; jump

$$26. f(x) = \begin{cases} \sqrt{8-2x} & \text{if } x < 2 \\ 4|x-3| & \text{if } 2 \leq x \leq 5 \\ 2x-7 & \text{if } x > 5 \end{cases}$$



Domain:

$$\mathbb{R}$$

Range:

$$\{y \mid y \geq 0\}$$

Discontinuities:

$x = 2$ ; jump

$x = 5$ ; jump

# Pre-Calculus Review

## QUIZ 1

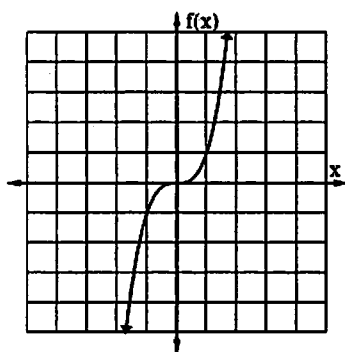
Name: \_\_\_\_\_

Date: \_\_\_\_\_ Per: \_\_\_\_\_

1. The minimum point on the graph of the function  $f(x)$  is  $(-4, -1)$ . What is the minimum point on the graph of the function  $f(x + 2)$ ?

- A.  $(-4, -3)$
- B.  $(-4, 1)$
- C.  $(-2, -1)$
- D.  $(-6, -1)$

2. If the function below is reflected in the  $x$ -axis, then horizontally compressed by a factor of  $1/3$ , which equation represents the new function?



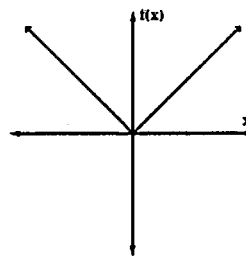
- A.  $f(x) = -3x^3$
- B.  $f(x) = \left(-\frac{1}{3}x\right)^3$
- C.  $f(x) = -(3x)^3$
- D.  $f(x) = -\left(\frac{1}{3}x\right)^3$

3. The function  $f(x) = \sqrt{x}$  is transformed to create function  $g$  below. Which describes a transformation that took place?

$$g(x) = \frac{1}{2}\sqrt{-(x-1)}$$

- A. a shift one unit to the left
- B. a shift one unit down
- C. a horizontal stretch of 2
- D. a reflection in the  $y$ -axis

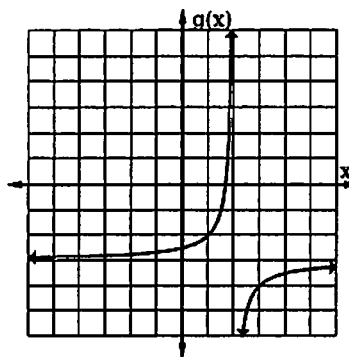
4. The graph of  $f(x)$  is shown below.



Which graph best represents  $a \cdot f(-x)$  if  $0 < a < 1$ ?

- A.
- B.
- C.
- D.

5. A function  $f$  was reflected in the  $x$ -axis, then translated three units left and two units down to create function  $g$  shown below. Which function represents  $f$ ?



- A.  $f(x) = \frac{1}{x+1} + 5$
- B.  $f(x) = \frac{1}{x-5} + 1$
- C.  $f(x) = \frac{1}{x+1} - 3$
- D.  $f(x) = \frac{1}{x-5} - 3$

6. If  $p$  and  $q$  are real numbers, which statement is true?

$$f(x) = (x - p)^2 + q$$

- A. The function decreases on the interval  $[-q, \infty)$ .
- B. The function increases on the interval  $[p, \infty)$ .
- C. The domain of the function is  $[p, \infty)$ .
- D. The range of the function is  $[-q, \infty)$ .

7. What is the domain of the function below?

$$f(x) = \sqrt{2x+6}$$

- A.  $(-\infty, \infty)$        C.  $[-3, \infty)$   
 B.  $[6, \infty)$       D.  $[3, \infty)$

8. What set of transformations would cause  $f$  and  $g$  to have the same range?

$$f(x) = -(x+5)^2 - 4; \quad g(x) = (x-1)^2 - 2$$

- A. Reflect  $g$  in the  $x$ -axis; Translate  $f$  two units up.  
 B. Reflect  $f$  in the  $x$ -axis; Translate  $g$  four units down.  
 C. Reflect  $g$  in the  $x$ -axis; Translate  $f$  six units up.  
 D. Reflect  $f$  in the  $x$ -axis; Translate  $g$  two units down.

9. Which function is odd?

- A.  $f(x) = \sqrt{x^2-3}$       C.  $f(x) = |x+2|$   
 B.  $f(x) = \frac{4}{x}$       D.  $f(x) = 2x^4 - 8x^2 + 3$

10. Which equation, if true, will show that  $y^2 = x - 4$  is symmetric to the  $x$ -axis?

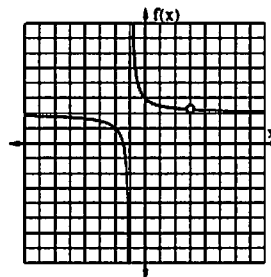
- A.  $(-y)^2 = x - 4$       C.  $(-y)^2 = -x - 4$   
 B.  $y^2 = -x - 4$       D.  $y^2 = -(x - 4)$

11. Given the function  $f(x)$  below, which statement is true?

$$f(x) = \begin{cases} |x+1|, & x < 1 \\ x^3 - 2, & x \geq 1 \end{cases}$$

- A.  $f$  has a jump discontinuity at  $x = 1$ .  
 B.  $f$  has an infinite discontinuity at  $x = 1$ .  
 C.  $f$  has a removable discontinuity at  $x = 1$ .  
 D.  $f$  is continuous on the real numbers.

12. Given the graph of  $f(x)$  below, which statement is true?

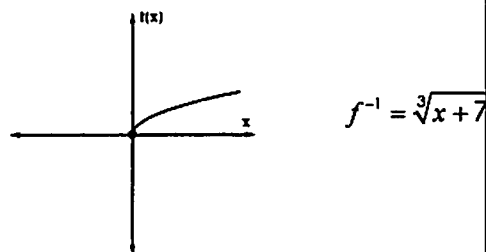


- A.  $f$  has a jump discontinuity at  $x = -1$ .  
 B.  $f$  has a removable discontinuity at  $x = -1$ .  
 C.  $f$  has a removable discontinuity at  $x = 3$ .  
 D.  $f$  has an infinite discontinuity at  $x = 3$ .

13. The graph of  $f(x) = x^3 + 5$  is reflected in the  $x$ -axis, then translated two units left. Which equation represents the inverse of the new function?

- A.  $f^{-1} = \sqrt[3]{-x+5} - 4$       C.  $f^{-1} = \sqrt[3]{x+3}$   
 B.  $f^{-1} = \sqrt[3]{-x+5} - 2$       D.  $f^{-1} = \sqrt[3]{x+7}$

14. The graph of  $f(x)$  is shown below.



Which graph best represents the inverse of  $f(x)$ ?

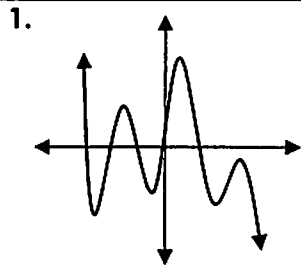
- A.      C.   
 B.       D.

Name: \_\_\_\_\_

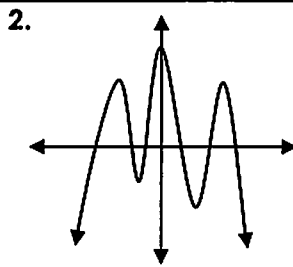
**PRE-CALCULUS REVIEW: Packet #2**

**Topic A: Graphs of Polynomial Functions**

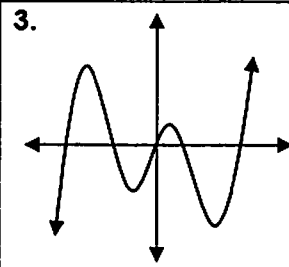
**Directions:** Given the graph of each polynomial function below, determine the sign of the leading coefficient and whether the function has an even or odd degree.



Negative, odd



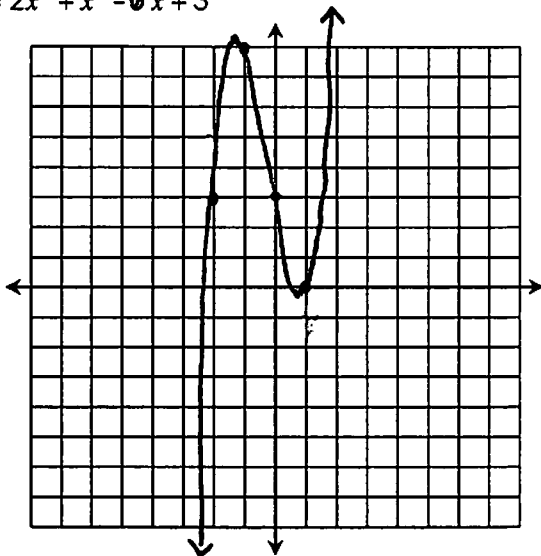
Negative, Even



Positive, odd

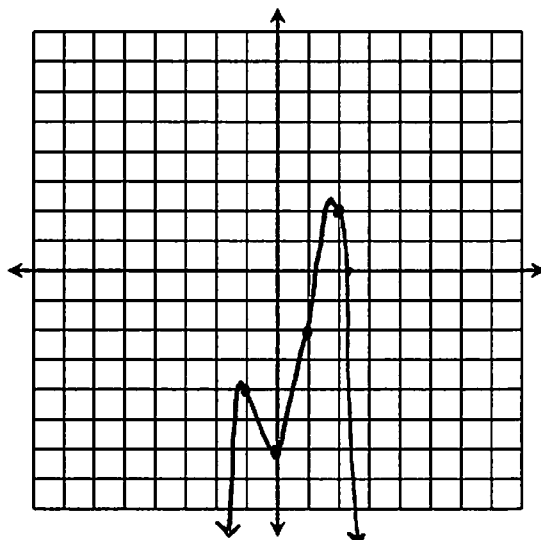
**Directions:** Graph each function, then identify its key characteristics.

4.  $f(x) = 2x^3 + x^2 - 6x + 3$



Domain:	$\mathbb{R}$	Range:	$\mathbb{R}$
x-intercept(s):	$(-2.19, 0), (0.69, 0), (1, 0)$		
y-intercept:	$(0, 3)$		
Rel. Minimum(s):	$(0.85, -0.15)$		
Rel. Maximum(s):	$(-1.18, 8.19)$		
Inc. Interval(s):	$(-\infty, -1.18), (0.85, \infty)$		
Dec. Interval(s):	$(-1.18, 0.85)$		
End Behavior:	<p>As <math>x \rightarrow \infty, f(x) \rightarrow \infty</math>                  As <math>x \rightarrow -\infty, f(x) \rightarrow -\infty</math></p>		

5.  $f(x) = -x^4 + x^3 + 4x^2 - 6$



Domain:	$\mathbb{R}$	Range:	$\{y \mid y \leq 2.31\}$
x-intercept(s):	$(1.28, 0), (2.25, 0)$		
y-intercept:	$(0, -6)$		
Rel. Minimum(s):	$(0, -6)$		
Rel. Maximum(s):	$(-1.09, -3.95), (1.84, 2.31)$		
Inc. Interval(s):	$(-\infty, -1.09), (0, 1.84)$		
Dec. Interval(s):	$(-1.09, 0), (1.84, \infty)$		
End Behavior:	<p>As <math>x \rightarrow \infty, f(x) \rightarrow -\infty</math>                  As <math>x \rightarrow -\infty, f(x) \rightarrow -\infty</math></p>		

### Topic B: Zeros of Polynomial Functions

**Directions:** Find all zeros for each function. Simplify all irrational zeros and complex solutions.

6.  $f(x) = 9x^4 + 68x^2 - 32$

$$f(x) = (9x^2 - 4)(x^2 + 8)$$

$x^2 = \frac{4}{9}$	$x^2 = -8$
$x = \pm \frac{2}{3}$	$x = \pm 2i\sqrt{2}$

$x = \left\{ \pm \frac{2}{3}, \pm 2i\sqrt{2} \right\}$

7.  $f(x) = 2x^3 - 11x^2 + 2x + 15$   $\pm 1, \pm 3, \pm 5, \pm 15,$

$$\begin{array}{r} -1 \overline{) 2 \ -11 \ 2 \ 15} \\ \underline{-2 \ 13 \ -15} \\ 2 \ -13 \ 15 \ 0 \end{array}$$

$\pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$

$f(x) = (x+1)(2x^2 - 13x + 15)$   
 $f(x) = (x+1)(2x-3)(x-5)$

$x = \left\{ -1, \frac{3}{2}, 5 \right\}$

8.  $f(x) = x^3 - 6x^2 + 15x - 28$   $\pm 1, \pm 2, \pm 4, \pm 7,$

$$\begin{array}{r} 4 \overline{) 1 \ -6 \ 15 \ -28} \\ \underline{4 \ -8 \ 28} \\ 1 \ -2 \ 7 \ 0 \end{array}$$

$f(x) = (x-4)(x^2 - 2x + 7)$

$x = 4$

$x = 2 \pm \frac{\sqrt{(-2)^2 - 4(1)(7)}}{2(1)}$

$x = \frac{2 \pm \sqrt{-24}}{2}$

$x = \frac{2 \pm 2i\sqrt{6}}{2}$

$x = 1 \pm i\sqrt{6}$

$x = \{4, 1 \pm i\sqrt{6}\}$

9.  $f(x) = x^3 - 6x^2 - 12x + 8$   $\pm 1, \pm 2, \pm 4, \pm 8$

$$\begin{array}{r} -2 \overline{) 1 \ -6 \ -12 \ 8} \\ \underline{-2 \ 16 \ -8} \\ 1 \ -8 \ 4 \ 0 \end{array}$$

$f(x) = (x+2)(x^2 - 8x + 4)$

$x = -2$

$x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(4)}}{2(1)}$

$x = \frac{8 \pm \sqrt{48}}{2}$

$x = \frac{8 \pm 4\sqrt{3}}{2}$

$x = 4 \pm 2\sqrt{3}$

$x = \{-2, 4 \pm 2\sqrt{3}\}$

**Directions:** Identify the zeros, their multiplicities, and describe the effect on the graph.

10.  $f(x) = 2x^4 - 24x^3 + 72x^2$

$f(x) = 2x^2(x-6)^2$

Zero	Multiplicity	Effect
0	2	tangent
6	2	tangent

11.  $f(x) = x^5 + 3x^4 - 9x^3 - 27x^2$

$f(x) = x^2(x+3)^2(x-3)$

Zero	Multiplicity	Effect
0	2	tangent
-3	2	tangent
3	1	intersect

### Topic C: Graphs of Rational Functions

**Directions:** Graph each function, then identify its key characteristics.

12.  $f(x) = \frac{3x^2 + 7x + 2}{x^2 + x - 2}$

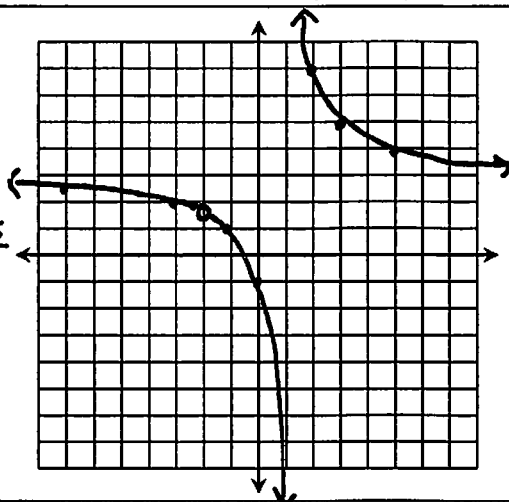
$$= \frac{(3x+1)(x+2)}{(x+2)(x-1)}$$

$$= \frac{3x+1}{x-1}$$

Hole  $(x=-2): \frac{3(-2)+1}{-2-1} = \frac{5}{3}$

VA:  $x-1=0$ ;  $x=1$

HA:  $y = \frac{3x}{x}$ ;  $y=3$



Domain: $\{x \mid x \neq -2, 1\}$	Range: $\{y \mid y \neq 3, 5/3\}$
x-intercept(s): $(-1/3, 0)$	
y-intercept: $(0, -1)$	
Vertical Asymptote: $x = 1$	
Horizontal Asymptote: $y = 3$	
Slant Asymptote: —	
Hole(s): $(-2, 5/3)$	

13.  $f(x) = \frac{x^2 + 7x + 13}{x + 4}$

SA:  

$$\begin{array}{r|rrr} -4 & 1 & 7 & 13 \\ & \downarrow & -4 & -12 \\ \hline & 1 & 3 & 1 \end{array}$$

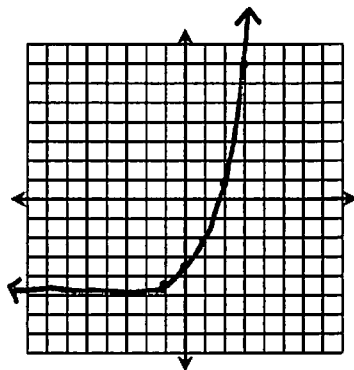
VA:  $x + 4 = 0$ ;  
 $x = -4$

Domain:	$\{x \mid x \neq -4\}$	Range:	$\{y \mid y \leq -3 \text{ or } y \geq 1\}$
x-intercept(s):	—		
y-intercept:	$(0, 3.25)$		
Vertical Asymptote:	$x = -4$		
Horizontal Asymptote:	—		
Slant Asymptote:	$y = x + 3$		
Hole(s):	—		

### Topic D: Graphs of Exponential & Logarithmic Functions

**Directions:** Graph each function, then identify its key characteristics.

14.  $f(x) = \frac{3}{4} \cdot (2)^{x+1} - 5$



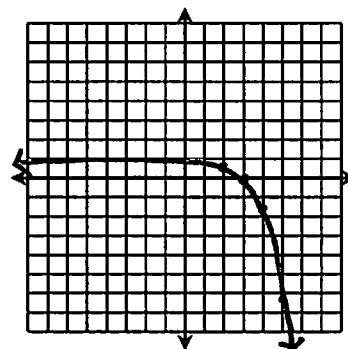
Domain:  $\mathbb{R}$

Range:  $\{y \mid y > -5\}$

y-intercept:  $(0, -3.5)$

Asymptote:  $y = -5$

15.  $f(x) = -e^{x-3} + 1$



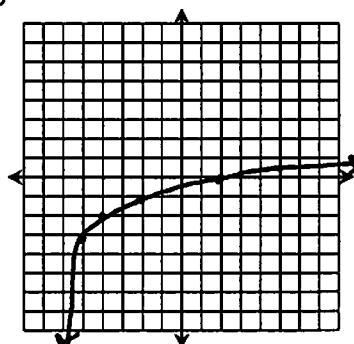
Domain:  $\mathbb{R}$

Range:  $\{y \mid y < 1\}$

y-intercept:  $(0, 0.95)$

Asymptote:  $y = 1$

16.  $f(x) = \log_2(x + 6) - 3$



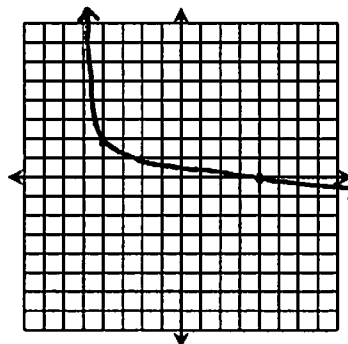
Domain:  $\{x \mid x > -6\}$

Range:  $\mathbb{R}$

x-intercept:  $(2, 0)$

Asymptote:  $x = -6$

17.  $f(x) = \log_3(x + 5) + 2$



Domain:  $\{x \mid x > -5\}$

Range:  $\mathbb{R}$

x-intercept:  $(4, 0)$

Asymptote:  $x = -5$

## Topic E: Solving Logarithmic & Exponential Equations

**Directions:** Solve each equation, rounding to the nearest ten-thousandths place when necessary.

18.  $2 \cdot \ln(2n) = \ln(n+3) + \ln(3n-2)$

$$\ln(2n)^2 = \ln(n+3)(3n-2)$$

$$4n^2 = 3n^2 + 7n - 6$$

$$n^2 - 7n + 6 = 0$$

$$(n-6)(n-1) = 0$$

$$n=6, n=1$$

$$n = \{1, 6\}$$

19.  $\log_7 3 + \log_7(3y^2 + 8) = 2$

$$\log_7 3(3y^2 + 8) = 2$$

$$9y^2 + 24 = 49$$

$$9y^2 = 25$$

$$y^2 = 25/9$$

$$y = \pm 5/3$$

$$y = \left\{ \pm \frac{5}{3} \right\}$$

20.  $\left(\frac{1}{16}\right)^{2x-1} = 64^x \cdot \left(\frac{1}{4}\right)^{x+3}$

$$4^{-2(2x-1)} = 4^{3x} \cdot 4^{-1(x+3)}$$

$$-4x + 2 = 3x - x - 3$$

$$-4x + 2 = 2x - 3$$

$$5 = 6x$$

$$x = 5/6$$

$$x = \frac{5}{6}$$

21.  $9 \cdot 5^{-4c-3} + 5 = 93$

$$9 \cdot 5^{-4c-3} = 88$$

$$5^{-4c-3} = 88/9$$

$$\frac{(-4c-3) \cdot \log 5}{\log 5} = \frac{\log 88/9}{\log 5}$$

$$-4c-3 = 1.4167$$

$$-4c = 4.4167$$

$$c = -1.1042$$

$$c = -1.1042$$

22. The population of a certain town was 24,250. Fifteen years later, the population decreased to 10,625. If the population followed a continuous exponential decay model, find the rate at which the population decreased.

$$10625 = 24250 e^{r \cdot 15}$$

$$.4381 = e^{15r}$$

$$\ln(.4381) = 15r$$

$$-.8252 = 15r$$

$$-.055 = r$$

5.5% decrease  
per year

23. Myles was gifted \$3,000 as a graduation present. He deposited half into an account that earns 3.25% interest compounded monthly and the other half into an account that earns 2.5% interest compounded continuously. If Myles does not make any additional deposits or withdrawals, how long will it take for each account to double in value?

$$3000 = 1500 \left(1 + \frac{0.0325}{12}\right)^{12t}$$

$$2 = 1.0027^{12t}$$

$$\frac{\log 2}{\log 1.0027} = \frac{12t \cdot \log 1.0027}{\log 1.0027}$$

$$256.2777 = 12t$$

$$t = 21.36$$

$$3000 = 1500 e^{.025t}$$

$$2 = e^{.025t}$$

$$\frac{\ln 2}{.025} = \frac{.025t}{.025}$$

$$t = 27.73$$

3.25% account = 22 years  
2.5% account = 28 years

24. The number of radio commercials for a product affects the amount of people who purchase the product. Use the table below and a logistic function to determine how many commercials it takes for 17% of people to purchase the product.

Commercials	People (%)
0	1.7
20	4
40	5.2
60	9.1
80	16.4
100	20.7
120	24.3

$$f(x) = \frac{28.89}{1 + 19.39e^{-.04x}}$$

$$17 = \frac{28.89}{1 + 19.39e^{-.04x}}$$

$$1 + 19.39e^{-.04x} = 1.70$$

$$e^{-.04x} = 0.04$$

$$-.04x = \ln .04$$

$$x = 83.06$$

84 Commercials

# Pre-Calculus Review

## QUIZ 2

Name: \_\_\_\_\_

Date: \_\_\_\_\_ Per: \_\_\_\_\_

1. Which statement is true about the graph of the function below?

$$f(x) = -5(x+2)^7(x-3)^4(x-1)^3$$

- A.  $f$  is increasing at both its left and right ends
- B.  $f$  is decreasing at both its left and right ends
- C.  $f$  is increasing at its left end and decreasing at its right end
- D.  $f$  is decreasing at its left end and increasing at its right end

2. Which statement is true about the graph of the function  $f(x) = -3x^4 + 12x^2$ ?

- A.  $f$  crosses the  $x$ -axis at  $x = -2$ ,  $x = 0$ , and  $x = 2$ .
- B.  $f$  touches, but does not cross the  $x$ -axis at  $x = -2$  and  $x = 2$ .
- C.  $f$  crosses the  $x$ -axis at  $x = 0$  and touches but does not cross the  $x$ -axis at  $x = -2$  and  $x = 2$ .
- D.  $f$  crosses the  $x$ -axis at  $x = -2$  and  $x = 2$ , then touches but does not cross the  $x$ -axis at  $x = 0$ .

3. Which function has the following characteristics?

- It has a relative maximum at  $(-1, -8)$ .
- It has a  $y$ -intercept at  $(0, -9)$ .
- It has zeros at  $x = \{-3, 1\}$ .

- A.  $f(x) = x^4 - 4x^2 - 12x - 9$
- B.  $f(x) = x^4 - 4x^3 + 4x^2 - 9$
- C.  $f(x) = x^4 - 4x^3 - 2x^2 + 12x + 9$
- D.  $f(x) = x^4 + 4x^3 + 4x^2 - 9$

4. What are the zeros of the function below?

$$f(x) = x^3 + 3x^2 - 11x - 21$$

$$\begin{array}{r|rrrr} 3 & 1 & 3 & -11 & -21 \\ & \downarrow & 3 & 18 & 21 \\ \hline & 1 & 6 & 7 & 0 \end{array}$$

$$f(x) = (x^2 + 6x + 7)(x - 3)$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(7)}}{2(1)} \quad | \quad x = 3$$

$$= \frac{-6 \pm \sqrt{8}}{2}$$

$$= \frac{-6 \pm 2\sqrt{2}}{2}$$

$$= -3 \pm \sqrt{2}$$

- A.  $\{3, -3 \pm \sqrt{2}\}$
- B.  $\{-3, 3 \pm \sqrt{2}\}$
- C.  $\{3, -2 \pm 3\sqrt{2}\}$
- D.  $\{-3, 2 \pm 3\sqrt{2}\}$

5. What are the zeros of the function below?

$$f(x) = x^4 + x^2 - 72$$

$$= (x^2 + 9)(x^2 - 8)$$

$$\begin{array}{l|l} x^2 = -9 & x^2 = 8 \\ \hline x = \pm 3i & x = \pm 2\sqrt{2} \end{array}$$

- A.  $\{\pm 2\sqrt{2}, \pm 3\}$
- B.  $\{\pm 2i\sqrt{2}, \pm 3i\}$
- C.  $\{\pm 2i\sqrt{2}, \pm 3\}$
- D.  $\{\pm 2\sqrt{2}, \pm 3i\}$

6. Which function has an  $x$ -intercept at  $(1, 0)$  and a horizontal asymptote of  $y = -4/3$ ?

$$HA: y = \frac{4x^2}{-3x^2} = -\frac{4}{3}$$

$$f(1) = \frac{4(1)^2 - 3(1) - 1}{-3(1)^2 - 3(1)} = \frac{0}{-6} = 0$$

- A.  $f(x) = \frac{8x^2 + 5x - 3}{-6x^2 + 6x}$
- C.  $f(x) = \frac{3x^2 + 7x + 4}{-4x^2 + 4x}$
- B.  $f(x) = \frac{4x^2 - 3x - 1}{-3x^2 - 3x}$
- D.  $f(x) = \frac{12x^2 - 13x - 4}{-9x^2 + 9x}$

7. If the function below has a  $y$ -intercept at  $(0, -4)$  and a vertical asymptote at  $x = 2$ , what are the values of  $a$  and  $b$ ?

$$f(x) = \frac{-16}{ax + b}$$

- A.  $a = 2$  and  $b = -4$
  - B.  $a = -2$  and  $b = 4$
  - C.  $a = -4$  and  $b = 2$
  - D.  $a = -2$  and  $b = -4$
- $$f(x) = \frac{-16}{-2x + 4}$$
- $$VA: -2x + 4 = 0$$
- $$x = 2$$

8. Which characteristics describes the graph of the function below?

$$f(x) = \frac{x^2 - 3x - 4}{x - 2}$$

$$\begin{array}{r} 2 \overline{) 1 \ -3 \ -4} \\ \underline{2 \phantom{-} \phantom{-}} \\ 1 \ -1 \ \cancel{-6} \end{array}$$

- A. There are zeros at  $x = -4$  and  $x = 1$
- B. The  $y$ -intercept is  $(0, -2)$ .
- C. There is a vertical asymptote at  $x = -1$ .
- D.** There is a slant asymptote at  $y = x - 1$ .

9. Which statement is true regarding the two functions below?

VA:  $x = 4$

HA:  $y = -4$

FUNCTION A	FUNCTION B
$f(x) = \log_3(x - 4) - 1$	$f(x) = 2^{x-1} - 4$

- A.** Function A has a vertical asymptote at  $x = 4$  and Function B has a horizontal asymptote at  $y = -4$ .
- B. Function A has a vertical asymptote at  $x = -4$  and Function B has a horizontal asymptote at  $y = 4$ .
- C. Function A has a vertical asymptote at  $x = 1$  and Function B has a horizontal asymptote at  $y = -1$ .
- D. The functions are inverses of each other.

10. Which function has the characteristics below?

- $f(x)$  has an  $x$ -intercept at  $(2, 0)$ .
- As  $x$  approaches  $\infty$ ,  $f(x)$  approaches  $\infty$ .

- A.  $f(x) = \left(\frac{1}{2}\right)^{x-3} - 2$
- B.  **$f(x) = \ln(x - 1)$**
- C.  $f(x) = 2^{x+2} - 1$
- D.  $f(x) = \log_2(x + 3)$

11. Find the solution to the equation below.

$$2 \cdot \log(k - 1) = \frac{1}{3} \log 8 + \log(3k + 5)$$

$$(k-1)^2 = 2(3k+5)$$

- A.  $k = \{1, 9\}$
  - B.  $k = \{-1, 9\}$
  - C.**  $k = 9$
  - D.  $k = 1$
- $k^2 - 2k + 1 = 6k + 10$   
 $k^2 - 8k - 9 = 0$   
 $(k-9)(k+1) = 0$   
 $k = 9, -1$

12. Find the solution to the equation below.

$$81^{5-3x} - 27^x = 0$$

$$3^4(5-3x) = 3^{3x}$$

$$20 - 12x = 3x$$

$$20 = 15x$$

- A. 3
- B.  $\frac{16}{3}$
- C.  **$\frac{4}{3} = x$**
- D.  $\frac{3}{5}$

13. Which value is the closest to the solution of the equation below?

$$13^{6-2p} + 5 = 85$$

$$13^{6-2p} = 80$$

$$(6-2p) \cdot \log 13 = \log 80$$

$$6-2p = 1.7084$$

$$-2p = -4.2916$$

$$p = 2.1458$$

- A. 1.8147
- B. 1.8695
- C. 1.9722
- D.** 2.1458

14. Finn inherited \$8,000 from his grandparents. He placed the money in an account that earns 6.75% interest compounded semiannually. If he makes no other deposits or withdrawals, how much interest will he have earned after 5 years?

$$= 8000 \left(1 + \frac{.0675}{2}\right)^{2 \cdot 5}$$

$$= 11,149.24$$

- A. \$2,731.27
- B. \$2,895.92
- C. \$3,015.78
- D.** \$3,149.24

15. In 2012, Jensyn deposited \$500 into an account that earns 4% interest compounded continuously. If she makes no other deposits or withdrawals, in what year can she expect to triple her investment?

$$1500 = 500 e^{.04t}$$

$$3 = e^{.04t}$$

$$\ln 3 = .04t$$

$$t = 27.47 \rightarrow 2039$$

- A. 2038
- B.** 2039
- C. 2040
- D. 2041

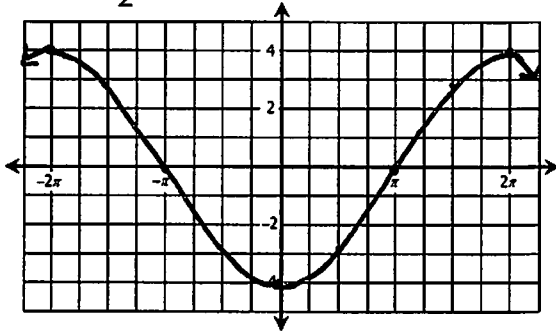
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**PRE-CALCULUS REVIEW: Packet #3**

**Topic A: Graphing Trigonometric Functions**

**Directions:** Graph each function and identify its key characteristics.

1.  $f(x) = -4 \cdot \cos \frac{1}{2}x$



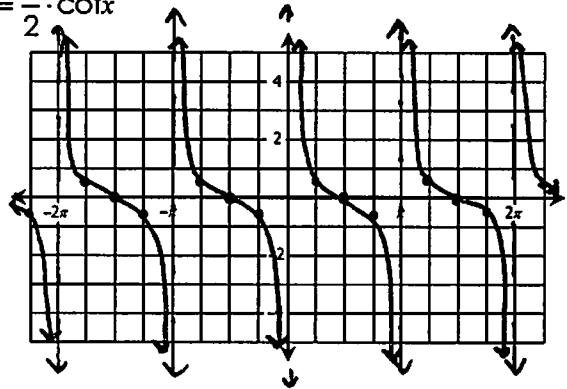
Amplitude:

4

Period:

$\frac{2\pi}{1/2} = 4\pi$

2.  $f(x) = \frac{1}{2} \cdot \cot x$



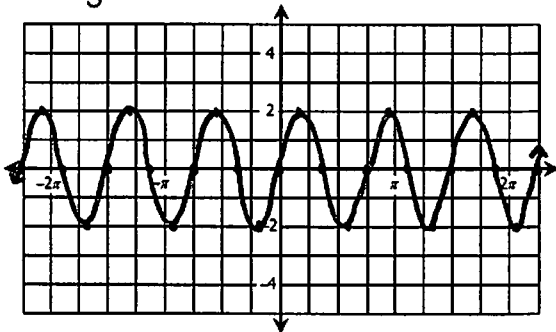
Amplitude:

Undefined

Period:

$\pi$

3.  $f(x) = 2 \cdot \sin \frac{8}{3}x$



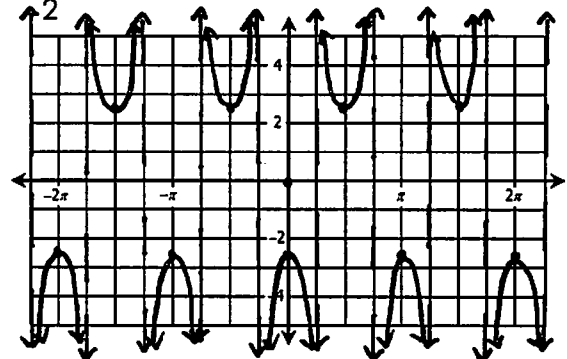
Amplitude:

2

Period:

$\frac{2\pi}{8/3} = \frac{3\pi}{4}$

4.  $f(x) = -\frac{5}{2} \cdot \sec 2x$



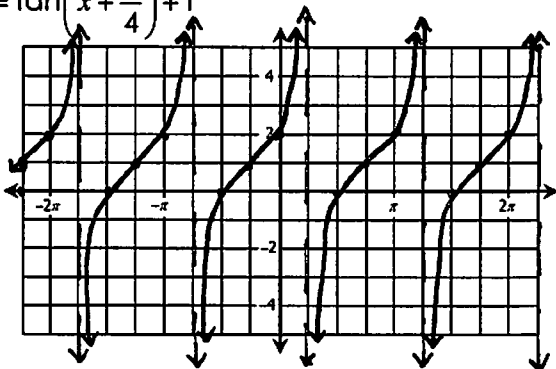
Amplitude:

Undefined

Period:

$\frac{2\pi}{2} = \pi$

5.  $f(x) = \tan \left( x + \frac{\pi}{4} \right) + 1$



Amplitude:

Undefined

Period:

$\pi$

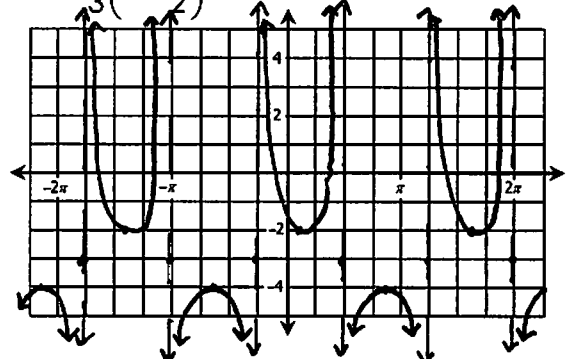
Phase Shift:

Left  $\pi/4$

Vertical Shift:

Up 1

6.  $f(x) = -\csc \frac{4}{3} \left( x - \frac{\pi}{2} \right) - 3$



Amplitude:

Undefined

Period:

$\frac{2\pi}{4/3} = \frac{3\pi}{2}$

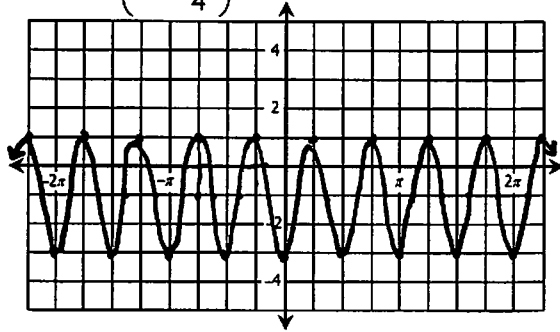
Phase Shift:

Right  $\pi/2$

Vertical Shift:

Down 3

$$7. f(x) = 2 \cdot \cos 4 \left( x + \frac{3\pi}{4} \right) - 1$$



Amplitude:

2

Period:

$$\frac{2\pi}{4} = \frac{\pi}{2}$$

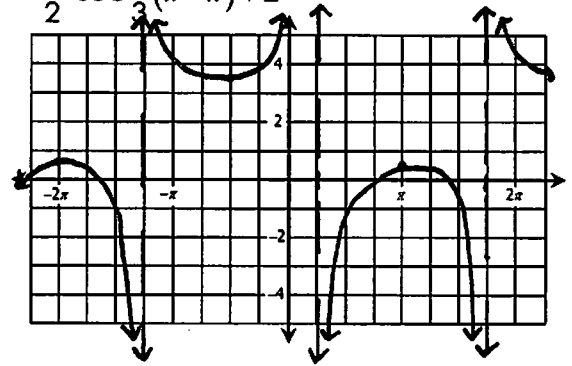
Phase Shift:

Left  $\frac{3\pi}{4}$

Vertical Shift:

Down 1

$$8. f(x) = -\frac{3}{2} \cdot \sec \frac{2}{3}(x - \pi) + 2$$



Amplitude:

Undefined

Phase Shift:

Right  $\pi$

Period:

$$\frac{2\pi}{2/3} = 3\pi$$

Vertical Shift:

Up 2

### Topic B: Inverses & Compositions of Trigonometric Functions

Directions: Give the exact value, if it exists.

9.  $\arcsin(-1)$

$$-\frac{\pi}{2}$$

10.  $\tan^{-1}(-\sqrt{3})$

$$-\frac{\pi}{3}$$

11.  $\arccos(-2)$

DNE

12.  $\arctan\left(\tan\frac{\pi}{2}\right)$

DNE

13.  $\sin^{-1}\left(\cos\frac{7\pi}{6}\right)$

$$-\frac{\pi}{3}$$

14.  $\cos(\arctan 0)$

1

### Topic C: Using Identities to Find Exact Values

Directions: Find each exact value.

15. If  $\cos\theta = \frac{1}{3}$  and  $0 < \theta < \frac{\pi}{2}$ , find  $\cos 2\theta$ .

$$\begin{aligned} \cos 2\theta &= 2\left(\frac{1}{3}\right)^2 - 1 \\ &= 2\left(\frac{1}{9}\right) - 1 \\ &= \frac{2}{9} - 1 = \boxed{-\frac{7}{9}} \end{aligned}$$

16. If  $\tan\beta = -\frac{5\sqrt{7}}{7}$  and  $\frac{\pi}{2} < \beta < \pi$ , find  $\sin 2\beta$ .

$$\begin{aligned} (5\sqrt{7})^2 + 7^2 &= x^2 \\ 175 + 49 &= x^2 \\ 224 &= x^2 \\ x &= 4\sqrt{14} \\ \sin\theta &= \frac{5\sqrt{2}}{8} \\ \cos\theta &= -\frac{\sqrt{14}}{8} \end{aligned} \quad \begin{aligned} \sin 2\theta &= 2\left(\frac{5\sqrt{2}}{8}\right)\left(-\frac{\sqrt{14}}{8}\right) \\ &= \frac{-10\sqrt{28}}{64} \\ &= \frac{-20\sqrt{7}}{16} \\ &= \boxed{-\frac{5\sqrt{7}}{4}} \end{aligned}$$

$$17. \sin 202.5^\circ \rightarrow \sin\left(\frac{405}{2}\right)$$

$$= -\sqrt{\frac{1 - \cos 405}{2}} = -\sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$$

$$= -\sqrt{\frac{2 - \sqrt{2}}{4}}$$

$$= \boxed{-\frac{\sqrt{2 - \sqrt{2}}}{2}}$$

$$18. \tan \frac{5\pi}{12} \rightarrow \tan\left(\frac{5\pi/6}{2}\right)$$

$$= \frac{\sin \frac{5\pi}{6}}{1 + \cos \frac{5\pi}{6}} = \frac{\frac{1}{2}}{1 + \left(-\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{\frac{1}{2}}{2 - \sqrt{3}}$$

$$= \frac{1^2(2 + \sqrt{3})}{2 - \sqrt{3}(2 + \sqrt{3})} = \frac{2 + \sqrt{3}}{1} = \boxed{2 + \sqrt{3}}$$

$$19. \text{ If } \sin y = \frac{2\sqrt{6}}{5} \text{ and } \frac{\pi}{2} < y < \pi, \text{ find } \cos \frac{y}{2}.$$

$$(2\sqrt{6})^2 + x^2 = 5^2 \quad \hookrightarrow \frac{\pi}{4} < \frac{y}{2} < \frac{\pi}{2} \quad (Q1)$$

$$24 + x^2 = 25$$

$$x^2 = 1$$

$$x = 1$$

$$\cos x = \frac{1}{5}$$

$$\cos \frac{y}{2} = \sqrt{\frac{1 + \frac{1}{5}}{2}}$$

$$= \sqrt{\frac{6/5}{2}}$$

$$= \sqrt{\frac{3}{5}} = \boxed{\frac{\sqrt{15}}{5}}$$

$$20. \text{ If } \tan \alpha = -\frac{\sqrt{3}}{3} \text{ and } \frac{3\pi}{2} < \alpha < 2\pi, \text{ find } \tan \frac{\alpha}{2}.$$

$$(\sqrt{3})^2 + 3^2 = x^2 \quad \hookrightarrow \frac{3\pi}{4} < \frac{\alpha}{2} < \pi$$

$$3 + 9 = x^2$$

$$12 = x^2$$

$$2\sqrt{3} = x$$

$$\tan \frac{\alpha}{2} = \frac{1 - \frac{\sqrt{3}}{2}}{-\frac{1}{2}}$$

$$\sin \alpha = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$$

$$\cos \alpha = -\frac{3}{2\sqrt{3}} = -\frac{\sqrt{3}}{2}$$

$$= \frac{2 - \sqrt{3}}{2}$$

$$= \frac{-\frac{1}{2}}{-\frac{1}{2}}$$

$$= \boxed{-2 + \sqrt{3}}$$

$$21. \cos 75^\circ \rightarrow \cos(45 + 30)$$

$$= \cos 45 \cdot \cos 30 - \sin 45 \cdot \sin 30$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

$$22. \tan \frac{11\pi}{12} \rightarrow \tan(225 - 60)$$

$$= \frac{\tan 225 - \tan 60}{1 + \tan 225 \cdot \tan 60} = \frac{1 - \sqrt{3}}{1 + 1 \cdot \sqrt{3}}$$

$$= \frac{1 - \sqrt{3}}{1 + \sqrt{3}} \cdot \frac{(1 - \sqrt{3})}{(1 - \sqrt{3})}$$

$$= \frac{4 - 2\sqrt{3}}{-2}$$

$$= \boxed{-2 + \sqrt{3}}$$

$$23. 3\left(\sin \frac{31\pi}{12} + \sin \frac{13\pi}{12}\right)$$

$$= 3\left[2 \cdot \sin\left(\frac{\frac{31\pi}{12} + \frac{13\pi}{12}}{2}\right) \cdot \cos\left(\frac{\frac{31\pi}{12} - \frac{13\pi}{12}}{2}\right)\right]$$

$$= 6 \sin \frac{11\pi}{6} \cdot \cos \frac{3\pi}{4}$$

$$= 6\left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{6\sqrt{2}}{4} = \boxed{\frac{3\sqrt{2}}{2}}$$

$$24. \cos \frac{13\pi}{12} - \cos \frac{7\pi}{12}$$

$$= -2 \cdot \sin\left(\frac{\frac{13\pi}{12} + \frac{7\pi}{12}}{2}\right) \cdot \sin\left(\frac{\frac{13\pi}{12} - \frac{7\pi}{12}}{2}\right)$$

$$= -2 \sin \frac{5\pi}{6} \cdot \sin \frac{\pi}{4}$$

$$= -2\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{-\frac{\sqrt{2}}{2}}$$

$$\begin{aligned}
 25. \cos \frac{3\pi}{4} \cdot \sin \frac{\pi}{12} &= \frac{1}{2} \left[ \sin \left( \frac{3\pi}{4} + \frac{\pi}{12} \right) - \sin \left( \frac{3\pi}{4} - \frac{\pi}{12} \right) \right] \\
 &= \frac{1}{2} \left[ \sin \frac{5\pi}{6} - \sin \frac{2\pi}{3} \right] \\
 &= \frac{1}{2} \left[ \frac{1}{2} - \frac{\sqrt{3}}{2} \right] = \frac{1}{2} \left[ \frac{1-\sqrt{3}}{2} \right] = \boxed{\frac{1-\sqrt{3}}{4}}
 \end{aligned}$$

$$\begin{aligned}
 26. 4\cos \frac{\pi}{12} \cdot \cos \frac{\pi}{12} &= 4 \cdot \frac{1}{2} \left[ \cos \left( \frac{\pi}{12} - \frac{\pi}{12} \right) + \cos \left( \frac{\pi}{12} + \frac{\pi}{12} \right) \right] \\
 &= 2 \left[ \cos 0 + \cos \frac{\pi}{6} \right] \\
 &= 2 \left[ 1 + \frac{\sqrt{3}}{2} \right] = 2 \left[ \frac{2+\sqrt{3}}{2} \right] = \boxed{2+\sqrt{3}}
 \end{aligned}$$

### Topic D: Simplifying Trigonometric Expressions

Directions: Simplify each trigonometric expression.

$$\begin{aligned}
 27. \frac{\cos^2 A - \cos^2 A \cdot \sec^2 A}{\sin^2 A} &= \frac{\cos^2 A (1 - \sec^2 A)}{\sin^2 A} \\
 &= \frac{\cos^2 A}{\sin^2 A} \cdot -\tan^2 A \\
 &= \cot^2 A \cdot -\tan^2 A = \boxed{-1}
 \end{aligned}$$

$$\begin{aligned}
 28. \frac{(\cot \beta + \csc \beta)(\cot \beta - \csc \beta)}{\csc \beta} &= \frac{\cot^2 \beta - \csc^2 \beta}{\csc \beta} \\
 &= \frac{-1}{\csc \beta} = \boxed{-\sin \beta}
 \end{aligned}$$

$$\begin{aligned}
 29. \frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} &= \frac{(1 + \sin x)^2}{\cos x (1 + \sin x)} + \frac{\cos^2 x}{\cos x (1 + \sin x)} \\
 &= \frac{1 + 2\sin x + \sin^2 x + \cos^2 x}{\cos x (1 + \sin x)} \\
 &= \frac{2 + 2\sin x}{\cos x (1 + \sin x)} = \frac{2(1 + \sin x)}{\cos x (1 + \sin x)} = \frac{2}{\cos x} = \boxed{2\sec x}
 \end{aligned}$$

$$\begin{aligned}
 30. \frac{\sec \theta \cdot \sin \theta}{\cot \theta + \tan \theta} &= \frac{\frac{1}{\cos \theta} \cdot \sin \theta}{\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta}} \\
 &= \frac{\tan \theta}{\frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta}} \\
 &= \frac{\tan \theta}{\frac{1}{\sin \theta \cos \theta}} \\
 &= \frac{\sin \theta}{\cos \theta} \cdot \sin \theta \cos \theta = \boxed{\sin^2 \theta}
 \end{aligned}$$

### Topic E: Solving Trigonometric Equations

Directions: Solve each equation for the given interval.

$$\begin{aligned}
 31. 10\sin \alpha \cdot \cos \alpha - 2\sqrt{3}\sin \alpha &= -7\sqrt{3}\sin \alpha; [0, \pi] \\
 10\sin \alpha \cos \alpha + 5\sqrt{3}\sin \alpha &= 0 \\
 5\sin \alpha (2\cos \alpha + \sqrt{3}) &= 0 \\
 \begin{array}{l|l}
 5\sin \alpha = 0 & 2\cos \alpha = -\sqrt{3} \\
 \sin \alpha = 0 & \cos \alpha = -\frac{\sqrt{3}}{2} \\
 \alpha = 0, \pi & \alpha = \frac{5\pi}{6}
 \end{array} & \boxed{\alpha = 0, \frac{5\pi}{6}, \pi}
 \end{aligned}$$

$$\begin{aligned}
 32. 5\csc^2 \theta - 5 &= 5\cot \theta; [\pi, 2\pi] \\
 5(\cot^2 \theta + 1) - 5 &= 5\cot \theta \\
 5\cot^2 \theta + 5 - 5 &= 5\cot \theta \\
 5\cot^2 \theta - 5\cot \theta &= 0 \\
 5\cot \theta (\cot \theta - 1) &= 0 \\
 \begin{array}{l|l}
 \cot \theta = 0 & \cot \theta = 1 \\
 \theta = \frac{3\pi}{2} & \theta = \frac{5\pi}{4}
 \end{array} & \boxed{\theta = \frac{5\pi}{4}, \frac{3\pi}{2}}
 \end{aligned}$$

33.  $\cos 2y + 3\cos^2 y - 1 = \cos^2 y + 1; [0, 2\pi]$

$$2\cos^2 y - 1 + 3\cos^2 y - 1 = \cos^2 y + 1$$

$$5\cos^2 y - 2 = \cos^2 y + 1$$

$$4\cos^2 y = 3$$

$$\cos^2 y = 3/4$$

$$\cos y = \pm \frac{\sqrt{3}}{2}$$

$$y = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

34.  $\sin 3x + \sin x = \cos x; \left[0, \frac{\pi}{2}\right]$

$$2 \cdot \sin\left(\frac{3x+x}{2}\right) \cdot \cos\left(\frac{3x-x}{2}\right) = \cos x$$

$$2 \cdot \sin 2x \cdot \cos x = \cos x$$

$$2\sin 2x \cos x - \cos x = 0$$

$$\cos x (2\sin 2x - 1) = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}$$

$$\sin 2x = \frac{1}{2}$$

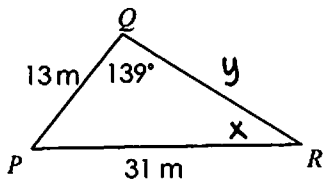
$$2x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{\pi}{2}$$

### Topic F: Law of Sines & Cosines

35. Solve the triangle below. Round all answers to the nearest tenth.



$$\frac{\sin 139}{31} = \frac{\sin x}{13}$$

$$\frac{31 \sin x}{31} = \frac{13 \sin 139}{31}$$

$$\sin x = .2751$$

$$x = 16$$

$$\frac{\sin 139}{31} = \frac{\sin 25}{y}$$

$$\frac{y \sin 139}{\sin 139} = \frac{31 \sin 25}{\sin 139}$$

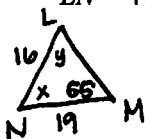
$$y = 20$$

$$QR = 20$$

$$m\angle P = 25^\circ$$

$$m\angle R = 16^\circ$$

36. In  $\triangle LMN$ , if  $m\angle M = 55^\circ$ ,  $MN = 19$  in, and  $LN = 16$  in, find  $m\angle N$ .



$$\frac{\sin 55}{16} = \frac{\sin y}{19}$$

$$\frac{16 \sin y}{16} = \frac{19 \sin 55}{16}$$

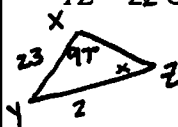
$$\sin y = .9727$$

$$y = 76.6^\circ, 103.4^\circ$$

$$m\angle N = 48.4^\circ$$

$$\text{or } 21.6^\circ$$

37. In  $\triangle XYZ$ , if  $m\angle X = 97^\circ$ ,  $XY = 23$  cm, and  $YZ = 22$  cm, find  $m\angle Z$ .



$$\frac{\sin 97}{22} = \frac{\sin x}{23}$$

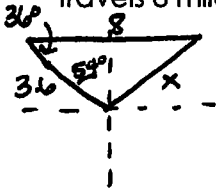
$$\frac{23 \sin 97}{22} = \frac{22 \sin x}{22}$$

$$1.0377 = \sin x$$

$$x = 0$$

No Solution

38. A fox leaves his den and travels N  $54^\circ$  W. After traveling 3.6 miles, he changes direction and travels 8 miles due east before stopping. How far is the fox from his den?



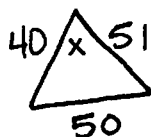
$$x^2 = 3.6^2 + 8^2 - 2(3.6)(8) \cos 36$$

$$x^2 = 76.96 - 57.6 \cos 36$$

$$x^2 = 30.3606$$

$$x = 5.5 \text{ mi}$$

39. Ed is staining his triangular deck. If the sides of the deck measure 40 feet, 51 feet, and 50 feet, and each can of stain covers 175 square feet, determine how many cans Ed will need.



$$50^2 = 40^2 + 51^2 - 2(40)(51) \cos x$$

$$2500 = 4201 - 4080 \cos x$$

$$-1701 = -4080 \cos x$$

$$.4169 = \cos x$$

$$x = 65.4^\circ$$

$$A = \frac{1}{2}(40)(51)(\sin 65.4)$$

$$= 927.42 \text{ ft}^2$$

$$\frac{927.42}{175} = 5.3$$

6 cans of Stain

# Pre-Calculus Review

## QUIZ 3

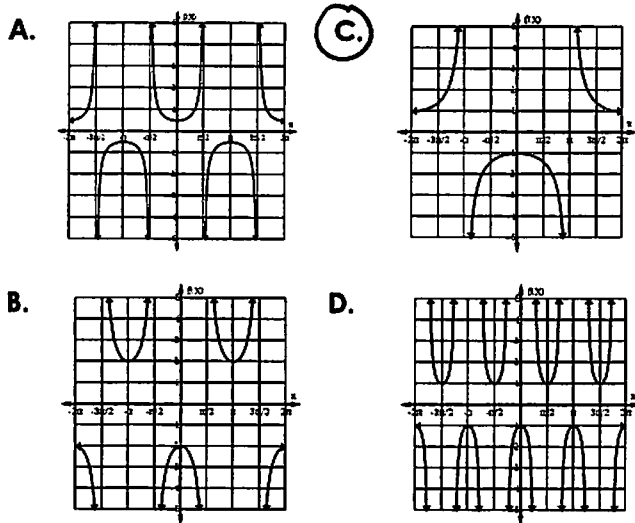
Name: \_\_\_\_\_

Date: \_\_\_\_\_ Per: \_\_\_\_\_

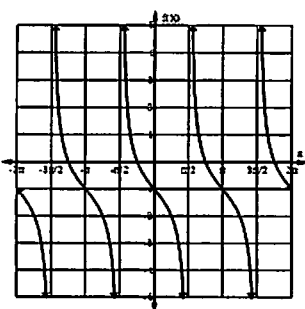
1. Which function has a period of  $2\pi$  and an amplitude of 4?

- A.  $f(x) = 4 \cdot \cos x$       C.  $f(x) = 2 \cdot \sin 4x$   
 B.  $f(x) = 4 \cdot \cos 2x$       D.  $f(x) = 4 \cdot \sin \frac{1}{2}x$

2. The graph of  $f(x) = \sec x$  undergoes a reflection about the  $x$ -axis and a horizontal stretch by a factor of 2. Which graph shows this transformed function?



3. Which function is shown on the graph below?



- A.  $f(x) = \cot\left(x + \frac{\pi}{4}\right) - 1$   
 B.  $f(x) = \cot\left(x + \frac{\pi}{2}\right) - 1$   
 C.  $f(x) = -\tan\left(x + \frac{\pi}{2}\right) - 1$   
 D.  $f(x) = -\tan\left(x + \frac{\pi}{4}\right) - 1$

4. The graph of the function  $f(x) = \sin x$  undergoes a vertical stretch by a factor of 3, a phase shift of  $\pi/4$  radians to the right, then a horizontal compression by a factor of  $1/2$ . Which statement is true regarding the transformed function?

- A. The  $y$ -intercept is  $(0, 3)$ .  
 B. The function intersects the  $x$ -axis at  $\left(\frac{\pi}{4} + \frac{k\pi}{2}, 0\right)$  where  $k$  is an integer.  
 C. The period is  $2\pi$ .  
 D. The amplitude is 2.

5. Find the exact value of the expression below.

$$\sin\left[\arctan\left(-\frac{\sqrt{3}}{3}\right)\right]$$

- A.  $\frac{\sqrt{3}}{2}$       C.  $\frac{1}{2}$   
 B.  $-\frac{\sqrt{3}}{2}$        D.  $-\frac{1}{2}$

6. If  $\sec x = -\frac{5}{2}$  and  $\csc x > 0$ , find the exact value of  $\tan x$ .

$$\cos x = -\frac{2}{5}$$

$$2^2 + x^2 = 5^2$$

$$x = \sqrt{21}$$

- A.  $\frac{\sqrt{21}}{2}$       C.  $\frac{5\sqrt{21}}{21}$   
 B.  $-\frac{\sqrt{21}}{2}$       D.  $-\frac{5\sqrt{21}}{21}$

7. If  $\cos \theta = -\frac{8}{17}$  and  $\pi \leq \theta < \frac{3\pi}{2}$ , find the exact value of  $\tan 2\theta$ .

$$8^2 + x^2 = 17^2$$

$$x = 15$$

$$\tan \theta = \frac{15}{8}$$

$$\frac{2\left(\frac{15}{8}\right)}{1 - \left(\frac{15}{8}\right)^2} = \frac{15/4}{-16/64} = \frac{-240}{16}$$

- A.  $-\frac{289}{161}$        C.  $-\frac{240}{161}$   
 B.  $-\frac{161}{289}$       D.  $-\frac{161}{240}$

8. If  $\tan A = -2\sqrt{6}$  and  $\frac{3\pi}{2} \leq A < 2\pi$ , find the exact value of  $\sin \frac{A}{2}$ .

$$= \sqrt{\frac{1 - 1/5}{2}} = \frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$$

- A.  $\frac{2\sqrt{5}}{5}$                       C.  $\frac{\sqrt{10}}{5}$   
 B.  $-\frac{2\sqrt{5}}{5}$                       D.  $-\frac{\sqrt{10}}{5}$

9. Find the exact value of  $\tan \frac{19\pi}{12} \rightarrow \tan\left(\frac{5\pi}{4} + \frac{\pi}{3}\right)$

$$= \frac{1 + \sqrt{3}}{1 - (1)(\sqrt{3})} = \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \cdot \frac{(1 + \sqrt{3})}{(1 + \sqrt{3})} = \frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$$

- A.  $-2 - \sqrt{3}$                       B.  $2 + \sqrt{3}$   
 C.  $\sqrt{3} - 2$                       D.  $2 - \sqrt{3}$

10. Find the exact value of the expression below.

$$\sin \frac{3\pi}{4} \cdot \cos \frac{\pi}{12}$$

$$= \frac{1}{2} \left[ \sin\left(\frac{3\pi}{4} + \frac{\pi}{12}\right) + \sin\left(\frac{3\pi}{4} - \frac{\pi}{12}\right) \right] = \frac{1}{2} \left[ \frac{1}{2} + \frac{\sqrt{3}}{2} \right]$$

- A.  $\frac{-1 + \sqrt{3}}{4}$                       C.  $\frac{1 - \sqrt{3}}{4}$   
 B.  $\frac{-1 - \sqrt{3}}{4}$                       D.  $\frac{1 + \sqrt{3}}{4}$

11. Find all solutions to the equation below on the given interval.

$$\cos x \cdot \sec x = \sqrt{2} \cos x; [\pi, 2\pi)$$

$$1 = \sqrt{2} \cos x$$

$$\frac{\sqrt{2}}{2} = \cos x$$

- A.  $\left\{\frac{5\pi}{4}\right\}$                       C.  $\left\{\frac{5\pi}{4}, \frac{7\pi}{4}\right\}$   
 B.  $\left\{\frac{7\pi}{4}\right\}$                       D.  $\left\{\frac{4\pi}{3}, \frac{7\pi}{3}\right\}$

12. Find all solutions to the equation below on the given interval.

$$3\cos^2\theta + 2 - \sin^2\theta = -4\cos\theta; [0, 2\pi)$$

$$3\cos^2\theta + 2 - (1 - \cos^2\theta) = -4\cos\theta$$

$$(2\cos\theta + 1)(2\cos\theta + 1) = 0$$

- A.  $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$                       C.  $\left\{\frac{5\pi}{6}, \frac{7\pi}{6}\right\}$   
 B.  $\left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$                       D.  $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

13. Find all solutions to the equation below on the given interval.

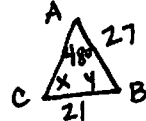
$$3\sin u = -4\sin^2 u - \cos 2u; [0, 2\pi)$$

$$3\sin u = -4\sin^2 u - (1 - 2\sin^2 u)$$

$$(2\sin u + 1)(\sin u + 1) = 0$$

- A.  $\left\{\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$                       C.  $\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}\right\}$   
 B.  $\left\{\frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}\right\}$                       D.  $\left\{\frac{3\pi}{4}, \pi, \frac{5\pi}{4}\right\}$

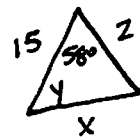
14. In  $\triangle ABC$ ,  $m\angle A = 48^\circ$ ,  $AB = 27$  feet, and  $BC = 21$  feet. Which statement is true?



$$\frac{\sin 48}{21} = \frac{\sin x}{27}$$

- A. No triangle exists with these measurements.  
 B.  $m\angle B$  can be  $24.8^\circ$  or  $107.2^\circ$ .  
 C.  $m\angle B$  can be  $59.2^\circ$  or  $24.8^\circ$ .  
 D.  $m\angle C$  can only be  $24.8^\circ$ .

15. A triangular greenspace is formed by three roads in a neighborhood: Derby Lane, Harvest Drive, and Cary Drive. The length of the greenspace along Derby Lane and Harvest Drive is 15 meters and 21 meters, respectively. If the angle formed by Derby Lane and Harvest Drive is  $58^\circ$ , find the angle formed by Derby Lane and Cary Drive.



- A.  $44.3^\circ$   
 B.  $49.5^\circ$   
 C.  $77.7^\circ$   
 D.  $65.3^\circ$

$$x^2 = 15^2 + 21^2 - 2(15)(21)\cos 58$$

$$x = 18.23$$

$$21^2 = 15^2 + 18.23^2 - 2(15)(18.23)\cos y$$

$$y = 77.7^\circ$$

Name: \_\_\_\_\_

## PRE-CALCULUS REVIEW: Packet #4

## Topic A: Vectors &amp; Vector Operations

**Directions:** Use the initial and terminal points of the vectors below to give the component form, linear combination using standard vectors  $i$  and  $j$ , magnitude, and direction angle for each angle.

1.  $\overline{CD}$  with  $C(-7, 5)$  and  $D(-5, 4)$ 

$$\|\overline{CD}\| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$$

$$\tan \theta = -\frac{1}{2}; \theta = 333.43^\circ$$

2.  $\overline{PQ}$  with  $P(-1, -1)$  and  $Q(5, 3)$ 

$$\|\overline{PQ}\| = \sqrt{6^2 + 4^2} = \sqrt{52} = 2\sqrt{13}$$

$$\tan \theta = \frac{4}{6}; \theta = 33.69^\circ$$

Component Form:

$$\langle 2, -1 \rangle$$

Linear Combination:

$$2i - j$$

Component Form:

$$\langle 6, 4 \rangle$$

Linear Combination:

$$6i + 4j$$

Magnitude:

$$\sqrt{5} \quad (2.24)$$

Direction Angle:

$$333.43^\circ$$

Magnitude:

$$2\sqrt{13} \quad (7.21)$$

Direction Angle:

$$33.69^\circ$$

**Directions:** Find each of the following for  $a = \langle 8, 0 \rangle$ ,  $b = \langle -3, 7 \rangle$ , and  $c = \langle 6, -9 \rangle$ .

3.  $b - a$ 

$$\langle -3, 7 \rangle - \langle 8, 0 \rangle$$

$$= \boxed{\langle -11, 7 \rangle}$$

4.  $2a + 3c$ 

$$2\langle 8, 0 \rangle + 3\langle 6, -9 \rangle$$

$$= \boxed{\langle 34, -27 \rangle}$$

5.  $-\frac{1}{3}c - \frac{5}{2}a$ 

$$-\frac{1}{3}\langle 6, -9 \rangle - \frac{5}{2}\langle 8, 0 \rangle$$

$$= \boxed{\langle -22, 37 \rangle}$$

**Directions:** Find the component form of each vector with the given magnitude and direction angle.

6.  $\|k\| = 14, \theta = 225^\circ$ 

$$k = \langle 14 \cos 225^\circ, 14 \sin 225^\circ \rangle$$

$$= \langle 14 \left(-\frac{\sqrt{2}}{2}\right), 14 \left(-\frac{\sqrt{2}}{2}\right) \rangle$$

$$= \boxed{\langle -7\sqrt{2}, -7\sqrt{2} \rangle}$$

7.  $\|q\| = 5, \theta = 110^\circ$ 

$$q = \langle 5 \cos 110^\circ, 5 \sin 110^\circ \rangle$$

$$= \boxed{\langle -1.71, 4.70 \rangle}$$

**Directions:** Convert each vector to trigonometric form.

8.  $z = 5i - 5j$ 

$$\|z\| = \sqrt{5^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}$$

$$\tan \theta = -5/5 = -1; \theta = 315^\circ$$

$$z = \langle 5\sqrt{2} \cos 315^\circ, 5\sqrt{2} \sin 315^\circ \rangle$$

$$\boxed{z = 5\sqrt{2} \langle \cos 315^\circ, \sin 315^\circ \rangle}$$

9.  $\overline{EF}$  with  $E(3, 1)$  and  $F(2, -2)$   $\langle -1, -3 \rangle$ 

$$\|\overline{EF}\| = \sqrt{(-1)^2 + (-3)^2} = \sqrt{10}$$

$$\tan \theta = -3/-1 = 3; \theta = 251.57^\circ$$

$$\boxed{\overline{EF} = \sqrt{10} \langle \cos 251.57^\circ, \sin 251.57^\circ \rangle}$$

## Topic B: Dot Products &amp; Angle Between Vectors

**Directions:** Find the dot product of the given vectors, then determine whether the vectors are orthogonal.

10.  $c = \langle 6, 4 \rangle$  and  $d = \langle -2, 3 \rangle$ 

$$c \cdot d = 6(-2) + 4(3)$$

$$= \boxed{0}$$

Orthogonal

11.  $p = 3i - 4j$  and  $q = -12i + 9j$ 

$$p \cdot q = 3(-12) - 4(9)$$

$$= \boxed{-72}$$

Not orthogonal

**Directions:** Find the angle between each vector pair.

12.  $x = \langle -7, 2 \rangle$  and  $y = \langle 5, 6 \rangle$

$$\cos \theta = \frac{-7(5) + 2(6)}{\sqrt{(-7)^2 + 2^2} \cdot \sqrt{5^2 + 6^2}} = \frac{-23}{\sqrt{3233}}$$

$$\theta = 113.86^\circ$$

13.  $m = -i$  and  $n = 3i + 10j$

$$\cos \theta = \frac{-1(3) + 0(10)}{\sqrt{(-1)^2 + 0^2} \cdot \sqrt{3^2 + 10^2}} = \frac{-3}{\sqrt{109}}$$

$$\theta = 106.7^\circ$$

### Topic C: Vector Applications

14. Leon kicks a soccer ball off the ground so that it travels with a velocity of 71 kilometers per hour at an angle of  $41^\circ$  with the ground. Find the magnitude of the horizontal and vertical components.

$$\cos 41^\circ = \frac{|x|}{71}$$

$$|x| = \cos 41^\circ \cdot 71$$

$$|x| = 53.58 \text{ km/hr}$$

$$\sin 41^\circ = \frac{|y|}{71}$$

$$|y| = \sin 41^\circ \cdot 71$$

$$|y| = 46.58 \text{ km/hr}$$

15. A blimp is traveling at a speed of 44 miles per hour in the direction of  $N 62^\circ E$ . While in transit, the blimp encounters wind traveling at a velocity of 29 miles per hour traveling in the direction of  $N 4^\circ W$ . Find the resultant speed and direction of the blimp.

$$V_1 = \langle 44 \cos 28^\circ, 44 \sin 28^\circ \rangle$$

$$V_2 = \langle 29 \cos 94^\circ, 29 \sin 94^\circ \rangle$$

$$V_1 + V_2 = \langle 36.83, 49.59 \rangle$$

$$\|V_1 + V_2\| = \sqrt{36.83^2 + 49.59^2} = 61.77 \text{ mph}$$

$$\tan \theta = \frac{49.59}{36.83} ; \theta = 53.4^\circ \rightarrow N 36.6^\circ E$$

16. Two tug boats are pulling a freighter into a port. If Tugboat A is traveling in the direction  $N 37^\circ W$  with a force of 31,000 kilograms, Tugboat B is traveling in the direction  $S 11^\circ W$  with a force of 26,250 kilograms, find the magnitude and direction angle of the resultant force on the freighter.

$$V_1 = \langle 31000 \cos 127^\circ, 31000 \sin 127^\circ \rangle$$

$$V_2 = \langle 26250 \cos 259^\circ, 26250 \sin 259^\circ \rangle$$

$$V_1 + V_2 = \langle -23665, -1010.01 \rangle$$

$$\|V_1 + V_2\| = \sqrt{(-23665)^2 + (-1010.01)^2} = 23686.54 \text{ kg}$$

$$\tan \theta = \frac{-1010.01}{-23665} ; \theta = 182.44^\circ \rightarrow S 87.56^\circ W$$

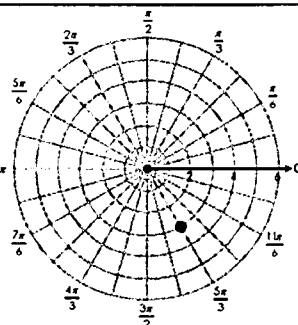
### Topic D: Polar vs. Rectangular Coordinates

**Directions:** Given each point on the polar grids below, name two different pairs of polar coordinates if  $0 \leq \theta < 2\pi$ .

17.

$$\left( 3, \frac{5\pi}{3} \right), \left( 3, -\frac{\pi}{3} \right),$$

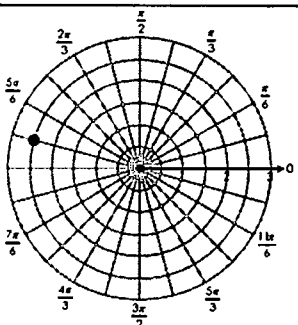
$$\left( -3, \frac{2\pi}{3} \right), \left( -3, -\frac{4\pi}{3} \right)$$



18.

$$\left( 2.5, \frac{11\pi}{12} \right), \left( 2.5, \frac{3\pi}{12} \right),$$

$$\left( -2.5, \frac{23\pi}{12} \right), \left( -2.5, -\frac{\pi}{12} \right)$$



**Directions:** Convert each pair of polar coordinates to rectangular coordinates. Round to the nearest hundredth if necessary.

19.  $(-7, \frac{\pi}{3})$   
 $x = -7 \cos \frac{\pi}{3} = -7(\frac{1}{2})$   
 $y = -7 \sin \frac{\pi}{3} = -7(\frac{\sqrt{3}}{2})$   
 $(-\frac{7}{2}, -\frac{7\sqrt{3}}{2})$

20.  $(6, \frac{7\pi}{4})$   
 $x = 6 \cos \frac{7\pi}{4} = 6(\frac{\sqrt{2}}{2})$   
 $y = 6 \sin \frac{7\pi}{4} = 6(-\frac{\sqrt{2}}{2})$   
 $(3\sqrt{2}, -3\sqrt{2})$

21.  $(3, -\frac{2\pi}{15})$   
 $x = 3 \cos(-\frac{2\pi}{15})$   
 $y = 3 \sin(-\frac{2\pi}{15})$   
 $(2.74, -1.22)$

**Directions:** Convert each pair of rectangular coordinates to polar coordinates. Round to the nearest hundredth if necessary. If  $0 \leq \theta < 2\pi$ , give two possible solutions.

22.  $(5, 5)$   
 $r = \sqrt{5^2 + 5^2} = 5\sqrt{2}$   
 $\theta = \tan^{-1}(5/5) = \frac{\pi}{4}$   
 $(5\sqrt{2}, \frac{\pi}{4}), (-5\sqrt{2}, \frac{5\pi}{4})$

23.  $(-3\sqrt{3}, -3)$   
 $r = \sqrt{(-3\sqrt{3})^2 + (-3)^2} = 6$   
 $\theta = \tan^{-1}(\frac{-3}{-3\sqrt{3}}) = \frac{7\pi}{6}$   
 $(6, \frac{7\pi}{6}), (-6, \frac{\pi}{6})$

24.  $(-2, 6)$   
 $r = \sqrt{(-2)^2 + 6^2} = 2\sqrt{10}$   
 $\theta = \tan^{-1}(\frac{6}{-2}) = 108.43^\circ$   
 $(2\sqrt{10}, 108.43^\circ), (-2\sqrt{10}, 288.43^\circ)$

### Topic E: Polar Graphs

**Directions:** Classify the curve of each polar equation.

25.  $r = 2 + 4 \cos \theta$

Looped Limacon

26.  $r^2 = 4 \cos(2\theta)$

Lemniscate

27.  $r = 4 + 2 \sin \theta$

Convex Limacon

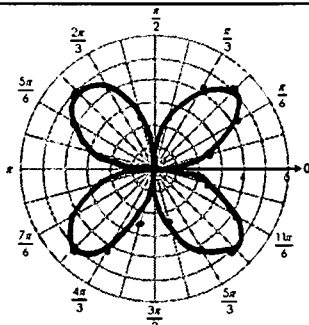
28.  $r = 7 \cos(5\theta)$

5-petal rose

**Directions:** Classify each of the following, then sketch the graph of each equation.

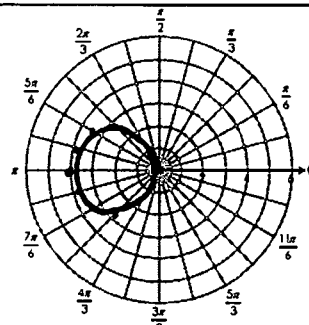
29.  $r = 5 \sin(2\theta)$

4-petal rose



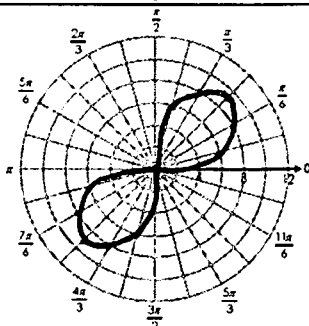
30.  $r = -4 \cos \theta$

Circle



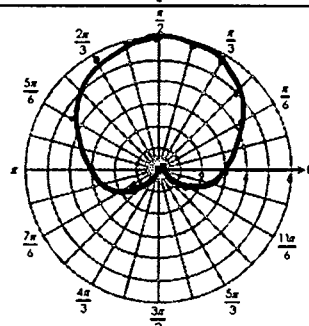
31.  $r^2 = 81 \sin(2\theta)$

Lemniscate



32.  $r = 3 + 3 \sin \theta$

Cardioid



### Topic F: Polar vs. Rectangular Equations

**Directions:** Convert the equations from polar to rectangular form.

33.  $y = \frac{x^2}{3}$        $r \sin \theta = \frac{(r \cos \theta)^2}{3}$   
 $3r \sin \theta = r^2 \cos^2 \theta$   
 $\frac{3 \sin \theta}{\cos^2 \theta} = r$   
 $3 \cdot \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} = r$   
 $r = 3 \tan \theta \sec \theta$

34.  $(x-4)^2 + (y+1)^2 = 17$   
 $x^2 - 8x + y^2 + 2y = 0$   
 $x^2 + y^2 - 8x + 2y = 0$   
 $(r \cos \theta)^2 + (r \sin \theta)^2 - 8r \cos \theta + 2r \sin \theta = 0$   
 $r^2 \cos^2 \theta + r^2 \sin^2 \theta = 8r \cos \theta - 2r \sin \theta$   
 $r^2 (\cos^2 \theta + \sin^2 \theta) = r(8 \cos \theta - 2 \sin \theta)$   
 $r^2 = r(8 \cos \theta - 2 \sin \theta)$   
 $r = 8 \cos \theta - 2 \sin \theta$

**Directions:** Convert the equations from rectangular to polar form.

35.  $r = -8 \csc \theta$   
 $r \sin \theta = -8 \csc \theta \sin \theta$   
 $r \sin \theta = -8$   
 $y = -8$

36.  $r = -4 \cos \theta + 10 \sin \theta$   
 $r^2 = -4r \cos \theta + 10r \sin \theta$   
 $x^2 + y^2 = -4x + 10y$   
 $x^2 + 4x + y^2 - 10y = 0$   
 $x^2 + 4x + 4 + y^2 - 10y + 25 = 4 + 25$   
 $(x+2)^2 + (y-5)^2 = 29$

### Topic G: Polar Forms of Complex Numbers

**Directions:** Convert each complex number to polar form.

37.  $8 - 8i$   
 $r = \sqrt{8^2 + (-8)^2} = \sqrt{128} = 8\sqrt{2}$   
 $\tan \theta = \frac{-8}{8} = -1$ ;  $\theta = \frac{7\pi}{4}$   
 $= 8\sqrt{2} \left( \cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$

38.  $-\sqrt{15} + 5i$   
 $r = \sqrt{(-\sqrt{15})^2 + 5^2} = \sqrt{40} = 2\sqrt{10}$   
 $\tan \theta = \frac{5}{-\sqrt{15}} = -\frac{\sqrt{15}}{3}$ ;  $\theta = 127.76^\circ$   
 $= 2\sqrt{10} \left( \cos 127.76^\circ + i \sin 127.76^\circ \right)$

**Directions:** Convert the polar form of each complex number to rectangular form.

39.  $2 \left( \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$   
 $= 2 \left( -\frac{1}{2} + \frac{\sqrt{3}}{2} i \right)$   
 $= -1 + i\sqrt{3}$

40.  $\sqrt{2} \left( \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$   
 $= \sqrt{2} (0 + 1i)$   
 $= i\sqrt{2}$

**Directions:** Find each product or quotient. Express your answer in rectangular form.

$$41. 5 \left( \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) \cdot 3 \left( \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$$

$$= 15 \left[ \cos \left( \frac{\pi}{2} + \frac{5\pi}{6} \right) + i \sin \left( \frac{\pi}{2} + \frac{5\pi}{6} \right) \right]$$

$$= 15 \left( -\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) = \boxed{-\frac{15}{2} - \frac{15\sqrt{3}}{2}i}$$

$$42. 4 \left( \cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right) \div 6\sqrt{2} \left( \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

$$= \frac{\sqrt{2}}{3} \left[ \cos \left( \frac{5\pi}{4} - \frac{\pi}{2} \right) + i \sin \left( \frac{5\pi}{4} - \frac{\pi}{2} \right) \right]$$

$$= \frac{\sqrt{2}}{3} \left( -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right)$$

$$= \boxed{-\frac{1}{3} + \frac{1}{3}i}$$

**Directions:** Find each power. Express your answer in rectangular form.

$$43. \left[ 3 \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \right]^6$$

$$= 3^6 \left( \cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right)$$

$$= 729 (0 - i) = \boxed{-729i}$$

$$44. (\sqrt{3} - i)^4 \quad r = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2$$

$$\tan \theta = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}; \theta = \frac{11\pi}{6}$$

$$z = 2 \left( \cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right)$$

$$z^4 = 2^4 \left( \cos \frac{44\pi}{6} + i \sin \frac{44\pi}{6} \right)$$

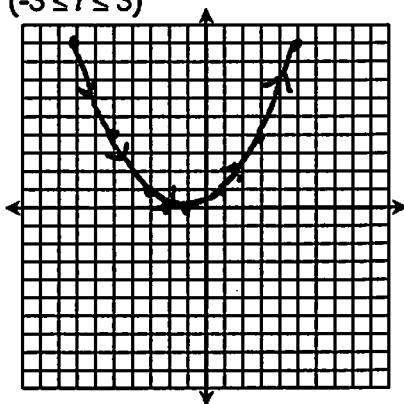
$$z^4 = 16 \left( -\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) = \boxed{-8 - 8i\sqrt{3}}$$

### Topic H: Graphing Parametric Equations

**Directions:** Sketch the curve given by each set of parametric equations over the given interval. Use arrows to indicate the direction of the curve as  $t$  increases.

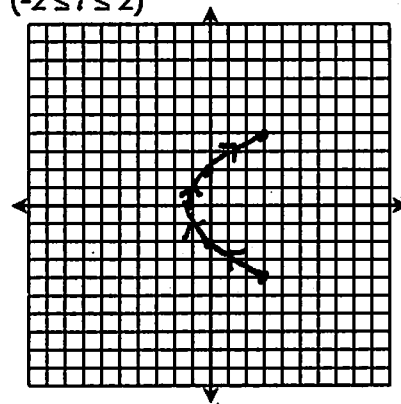
$$45. x = 2t - 1, y = t^2 \quad (-3 \leq t \leq 3)$$

t	x	y
-3	-7	9
-2	-5	4
-1	-3	1
0	-1	0
1	1	1
2	3	4
3	5	9



$$46. x = t^2 - 1, y = 2t \quad (-2 \leq t \leq 2)$$

t	x	y
-2	3	-4
-1	0	-2
0	-1	0
1	0	2
2	3	4



### Topic I: Parametric vs. Rectangular Equations

**Directions:** Write each set of parametric equations in rectangular form. Note any restrictions on the domain.

$$47. x = 2t - 3, y = -t + 6$$

$$x = 2(6 - y) - 3$$

$$x = 12 - 2y - 3$$

$$x = 9 - 2y$$

$$x - 9 = -2y$$

$$\boxed{-\frac{1}{2}x + \frac{9}{2} = y}$$

$$y - 6 = -t$$

$$-y + 6 = t$$

$$6 - y = t$$

$$48. x = t^2 - 4, y = 3t$$

$$t = \frac{1}{3}y$$

$$x = \left(\frac{1}{3}y\right)^2 - 4$$

$$x = \frac{1}{9}y^2 - 4$$

$$x + 4 = \frac{1}{9}y^2$$

$$9(x + 4) = y^2$$

$$\pm \sqrt{9(x + 4)} = y^2$$

$$\boxed{y = \pm 3\sqrt{x + 4}}$$

49.  $x = 3\cos\theta, y = 5\sin\theta$

$$\frac{x}{3} = \cos\theta ; \frac{y}{5} = \sin\theta$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$$

$$\boxed{\frac{x^2}{9} + \frac{y^2}{25} = 1}$$

50.  $x = \cos\theta + 4, y = \sin\theta - 1$

$$x - 4 = \cos\theta ; y + 1 = \sin\theta$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\boxed{(x-4)^2 + (y+1)^2 = 1}$$

**Topic J: Applications of Parametric Equations**

**Use for questions 51 and 52:** A projectile was launched at an angle of elevation of  $82^\circ$  at an initial velocity of 36 meters per second from a height of 14 meters above ground.

51. Find the horizontal distance the ball has traveled at 2.5 seconds.

$$x = t \cdot 36 \cos 82^\circ$$

$$x = 2.5 \cdot 36 \cos 82^\circ$$

$$\boxed{= 12.53 \text{ m}}$$

52. Find the vertical distance the ball has traveled at 2.5 seconds.

$$y = t \cdot 36 \sin 82^\circ - 4.9t^2 + 14$$

$$y = 2.5 \cdot 36 \sin 82^\circ - 4.9(2.5)^2 + 14$$

$$\boxed{= 72.5 \text{ m}}$$

**Use for questions 53 and 54:** A soccer ball is kicked down the field. The ball travels at an angle of elevation of  $53^\circ$  at an initial velocity of 16 meters per second.

53. What was the maximum height of the ball?

$$t = \frac{-16 \sin 53^\circ}{-9.8} = 1.3 \text{ sec}$$

$$y = t \cdot 16 \sin 53^\circ - 4.9t^2$$

$$y = 1.3 \cdot 16 \sin 53^\circ - 4.9(1.3)^2$$

$$\boxed{= 8.33 \text{ m}}$$

54. What was the horizontal distance the ball traveled at its maximum height?

$$x = t \cdot 16 \cos 53^\circ$$

$$x = 1.3 \cdot 16 \cos 53^\circ$$

$$\boxed{= 12.52 \text{ m}}$$

**Use for questions 55 and 56:** A frog leaped from the ground at an angle of elevation of  $39^\circ$ , leaping at an initial velocity of 8 feet per second.

55. How long was the frog in the air?

$$y = t \cdot 8 \sin 39^\circ - 16t^2$$

$$0 = t \cdot 8 \sin 39^\circ - 16t^2$$

$$t = \frac{-8 \sin 39^\circ \pm \sqrt{(8 \sin 39^\circ)^2 - 4(-16)(0)}}{2(-16)}$$

$$t = \frac{-8 \sin 39^\circ \pm \sqrt{(8 \sin 39^\circ)^2}}{-32}$$

$$t = \{0, 0.31\}$$

$$\boxed{0.31 \text{ sec}}$$

56. What was the horizontal distance traveled by the frog?

$$x = t \cdot 8 \cos 39^\circ$$

$$x = 0.31 \cdot 8 \cos 39^\circ$$

$$\boxed{= 1.93 \text{ ft}}$$

# Pre-Calculus Review

## QUIZ 4

Name: \_\_\_\_\_

Date: \_\_\_\_\_ Per: \_\_\_\_\_

1. Given  $R(4, -1)$  and  $S(-2, 7)$ , give the magnitude and approximate direction angle for  $\overrightarrow{RS}$ .

$$\|\overrightarrow{RS}\| = \sqrt{(-6)^2 + 8^2} = 10 \quad \langle -6, 8 \rangle$$

$$\tan \theta = \frac{8}{-6} = -\frac{4}{3}; \quad \theta = 126.87^\circ$$

- A.  $2\sqrt{10}; 71.57^\circ$   
 B.  $2\sqrt{10}; 108.43^\circ$   
 C. 10; 126.87°  
 D. 10; 143.13°

2. Vector  $a$  has a magnitude of 4 units and a direction of  $-30^\circ$ . Vector  $b$  has a magnitude of 2 units and a direction of  $90^\circ$ . Which is the component form of vector  $c$  if  $c = a + 2b$ ?

$$a = \langle 4 \cos(-30^\circ), 4 \sin(-30^\circ) \rangle$$

$$b = \langle 2 \cos 90^\circ, 2 \sin 90^\circ \rangle$$

- A.  $\langle 2, -2\sqrt{3} \rangle$   
 B.  $\langle -2, 2\sqrt{3} \rangle$   
 C.  $\langle -2\sqrt{3}, 2 \rangle$   
 D.  $\langle 2\sqrt{3}, 2 \rangle$

3. Find the approximate measure of the angle formed between the vectors given below.

$$u = -i + 7j; \quad v = 2i + 3j$$

$$\cos \theta = \frac{-1(2) + 7(3)}{\sqrt{(-1)^2 + 7^2} \cdot \sqrt{2^2 + 3^2}}$$

$$= \frac{19}{\sqrt{650}}$$

- A.  $37.78^\circ$   
 B.  $38.19^\circ$   
 C.  $41.82^\circ$   
 D.  $48.34^\circ$

4. Two soccer players kick a ball at the same time. One player kicks the ball with a force of 148 newtons in the direction of  $N 40^\circ E$ . The other player kicks with a force of 115 newtons in the direction of  $N 70^\circ W$ . What is the resultant magnitude and direction of the resultant force on the ball?

$$v_1 = \langle 148 \cos 50^\circ, 148 \sin 50^\circ \rangle$$

$$v_2 = \langle 115 \cos 160^\circ, 115 \sin 160^\circ \rangle$$

- A. 153.26 newtons;  $N 4.84^\circ W$   
 B. 161.54 newtons;  $N 4.84^\circ W$   
 C. 153.26 newtons;  $N 4.16^\circ E$   
 D. 161.54 newtons;  $N 4.16^\circ E$

5. A helicopter flying with a velocity of 125 miles per hour makes a  $36^\circ$  angle with the horizontal. What is the approximate vertical component of the helicopters velocity?

$$\sin 36^\circ = \frac{|y|}{125}$$

$$|y| = \sin 36 \cdot 125$$

- A. 73.47 mph  
 B. 83.92 mph  
 C. 95.28 mph  
 D. 101.13 mph

6. Which point maps to the same position on a polar coordinate as point  $P$  below?

$$P \left( -1, \frac{13\pi}{18} \right)$$

- A.  $\left( 1, \frac{5\pi}{18} \right)$       C.  $\left( -1, -\frac{5\pi}{18} \right)$   
 B.  $\left( 1, -\frac{5\pi}{18} \right)$       D.  $\left( -1, \frac{13\pi}{9} \right)$

7. What are the rectangular coordinates of the polar coordinates given below?

$$\left( -10, -\frac{4\pi}{3} \right)$$

$$x = -10 \cos \left( -\frac{4\pi}{3} \right)$$

$$= -10 \left( -\frac{1}{2} \right) = 5$$

$$y = -10 \sin \left( -\frac{4\pi}{3} \right)$$

$$= -10 \left( \frac{\sqrt{3}}{2} \right) = -5\sqrt{3}$$

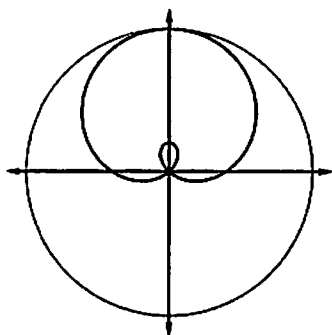
- A.  $(5\sqrt{3}, -5)$   
 B.  $(-5\sqrt{3}, 5)$   
 C.  $(5, -5\sqrt{3})$   
 D.  $(-5, 5\sqrt{3})$

8. Which statement best describes the graph of the polar equation below?

$$r = \cos 3\theta$$

- A. A rose with 6 petals
- B. A rose with 3 petals.
- C. A cardioid.
- D. A lemniscate.

9. The graph of the equation  $r = a + b \sin \theta$  is shown below. Which gives possible values for  $a$  and  $b$ ?



- A.  $a = -2; b = -3$
- B.  $a = 2; b = -3$
- C.  $a = 3; b = 2$
- D.  $a = 2; b = 3$

10. A circle has a radius of 4 and a center at  $(0, 4)$  in the rectangular plane. What is the polar equation for this circle?

- A.  $r = 8 \cos \theta$
- B.  $r = 4 \cos \theta$
- C.  $r = 8 \sin \theta$
- D.  $r = 4 \sin \theta$

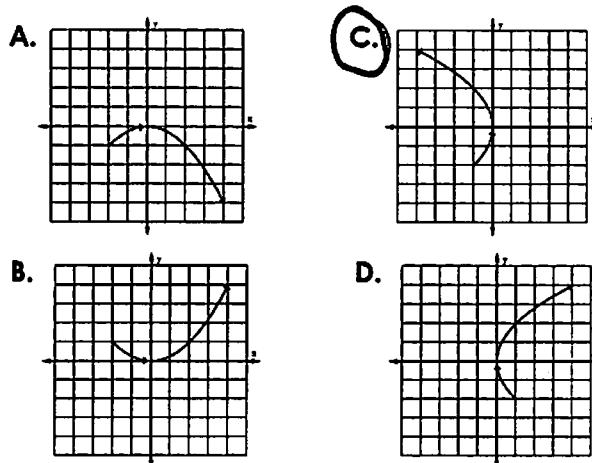
11. Which expression is the equivalent polar form of the expression below?

$$(1 - i\sqrt{3})^5 = 2^5 \left( \cos \frac{25\pi}{3} + i \sin \frac{25\pi}{3} \right)$$

- A.  $16 \left( \cos \frac{10\pi}{3} + i \sin \frac{10\pi}{3} \right)$
- B.  $32 \left( \cos \frac{25\pi}{3} + i \sin \frac{25\pi}{3} \right)$
- C.  $243 \left( \cos \frac{10\pi}{3} + i \sin \frac{10\pi}{3} \right)$
- D.  $243 \left( \cos \frac{25\pi}{3} + i \sin \frac{25\pi}{3} \right)$

12. Which graph represents the pair of parametric equations below on the given interval?

$$x = -\frac{t^2}{4}, y = t; -2 \leq t \leq 4$$



13. Give the rectangular form of the pair of parametric equations below.

$$x = t^2 - 2, y = 5t \rightarrow t = \frac{1}{5}y$$

- A.  $y = \pm 5\sqrt{x+2}$
  - B.  $y = \pm 5\sqrt{x-2}$
  - C.  $y = \pm 2\sqrt{x-5}$
  - D.  $y = \pm 2\sqrt{x+5}$
- $x = \left(\frac{1}{5}y\right)^2 - 2$   
 $x + 2 = \frac{y^2}{25}$   
 $25(x+2) = y^2$

14. Which set of parametric equations represent the rectangular equation shown below?

$$\frac{x^2}{81} + \frac{y^2}{16} = 1$$

- A.  $x = 4 \sin \theta, y = 9 \cos \theta$
  - B.  $x = 9 \sin \theta, y = 4 \cos \theta$
  - C.  $x = 4 \cos \theta, y = 9 \sin \theta$
  - D.  $x = 9 \cos \theta, y = 4 \sin \theta$
- $\frac{x}{9} = \cos \theta$   
 $\frac{y}{4} = \sin \theta$

15. A referee threw a flag at a speed of 30 feet per second at an angle of elevation of  $55^\circ$ . If the flag left his hand at a height of 4 feet, how far from the point where the referee is standing does the flag land?

- A. 23.55 ft
  - B. 24.73 ft
  - C. 26.14 ft
  - D. 28.91 ft
- $y = t \cdot 30 \sin 55^\circ - 16t^2 + 4$   
 $t = \frac{-30 \sin 55^\circ \pm \sqrt{(30 \sin 55^\circ)^2 - 4(-16)(4)}}{2(-16)}$   
 $t = \{-1.5, 1.68\}$   
 $x = 1.68 \cdot 30 \cos 55^\circ$

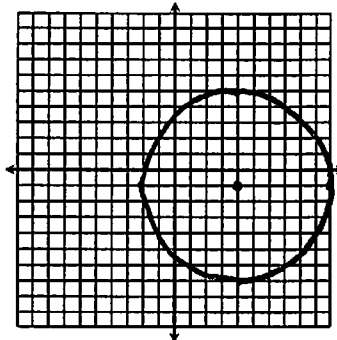
Name: \_\_\_\_\_

**PRE-CALCULUS REVIEW: Packet #5**

**Topic A: Graphing Conics**

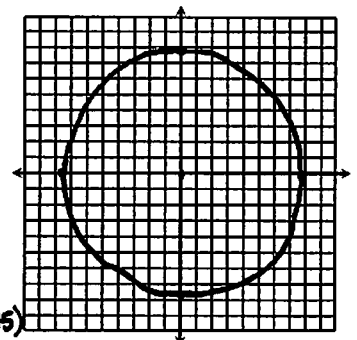
**Directions:** Graph each conic, then identify its key characteristics.

1.  $(x-4)^2 + (y+1)^2 = 36$



Center:  $(4, -1)$   
Radius:  $6$

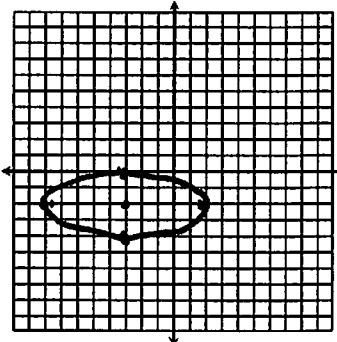
2.  $8x^2 + 8y^2 = 480$   
 $x^2 + y^2 = 60$



Center:  $(0, 0)$   
Radius:  $2\sqrt{15}$  (7.15)

3.  $\frac{(x+3)^2}{25} + \frac{(y+2)^2}{4} = 1$

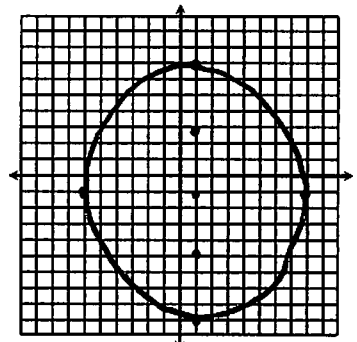
$c^2 = 25 - 4 = 21$   
 $c = \sqrt{21} \approx 4.6$   
 $e = \frac{4.6}{5} = 0.92$



Center:  $(-3, -2)$   
Vertices:  $(2, -2), (-8, -2)$   
Foci:  $(1.6, -2), (-7.6, -2)$   
Eccentricity:  $e = 0.92$

4.  $\frac{(x-1)^2}{49} + \frac{(y+1)^2}{64} = 1$

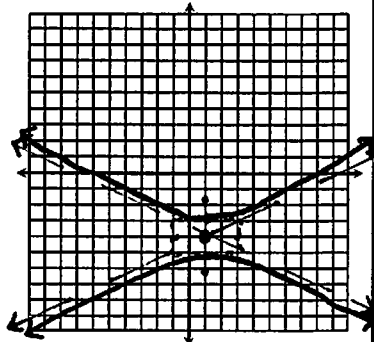
$c^2 = 64 - 49 = 15$   
 $c = \sqrt{15} \approx 3.9$   
 $e = \frac{3.9}{8} = 0.4875$



Center:  $(1, -1)$   
Vertices:  $(1, 7), (1, -9)$   
Foci:  $(1, -4.9), (1, 2.9)$   
Co-vertices:  $(-6, -1), (8, -1)$   
Eccentricity:  $e = 0.4875$

5.  $(y+4)^2 - \frac{(x-1)^2}{4} = 1$

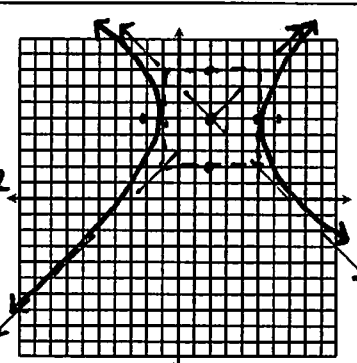
$c^2 = 1 + 4 = 5$   
 $c = \sqrt{5} \approx 2.2$   
 $y = \pm \frac{1}{2}(x-1) - 4$   
 $e = \frac{2.2}{1} = 2.2$



Center:  $(1, -4)$   
Vertices:  $(1, -5), (1, -3)$   
Foci:  $(1, -1.8), (1, -6.2)$   
Asymptotes:  $y = \frac{1}{2}x - \frac{9}{2}, y = -\frac{1}{2}x - \frac{7}{2}$   
Eccentricity:  $e = 2.2$

6.  $\frac{(x-2)^2}{9} - \frac{(y-5)^2}{9} = 1$

$c^2 = 9 + 9 = 18$   
 $c = \sqrt{18} = 3\sqrt{2} \approx 4.2$   
 $y = \pm (x-2) + 5$   
 $e = \frac{4.2}{3} = 1.4$

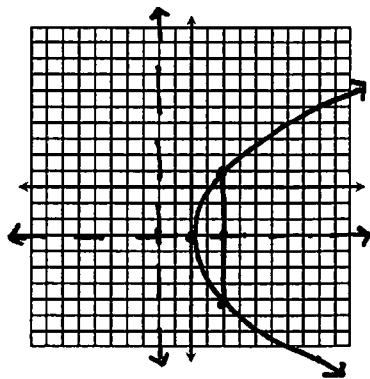


Center:  $(2, 5)$   
Vertices:  $(-1, 5), (5, 5)$   
Foci:  $(6.2, 5), (-2.2, 5)$   
Co-vertices:  $(2, 2), (2, 8)$   
Asymptotes:  $y = x + 3, y = -x + 7$   
Eccentricity:  $e = 1.4$

$$7. (y+3)^2 = 8x$$

$$8 = 4p$$

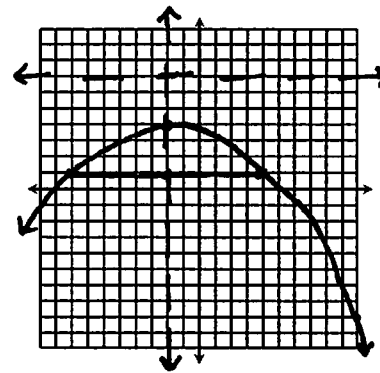
$$2 = p$$



$$8. (x+2)^2 = -12(y-4)$$

$$-12 = 4p$$

$$-3 = p$$



Vertex: $(0, -3)$	Directrix: $x = -2$	Vertex: $(-2, 4)$	Directrix: $y = 7$
Focus: $(2, -3)$	Axis of Symmetry: $y = -3$	Focus: $(-2, 1)$	Axis of Symmetry: $x = -2$
Endpoints of Latus Rectum: $(2, 1)$ and $(2, -7)$		Endpoints of Latus Rectum: $(-8, 1)$ and $(4, 1)$	

### Topic B: Writing Equations of Conics

**Directions:** Write an equation of a circle with the given characteristics.

9. center:  $(5, -7)$ , diameter:  $20\sqrt{2}$

$$(x-5)^2 + (y+7)^2 = (10\sqrt{2})^2$$

$$(x-5)^2 + (y+7)^2 = 200$$

10. endpoints of diameter:  $(11, 5)$  and  $(-5, 3)$

$$MP = \left( \frac{11-5}{2}, \frac{5+3}{2} \right) = (3, 4)$$

$$\text{radius} = \sqrt{(11-3)^2 + (5-4)^2} = \sqrt{64+1} = \sqrt{65}$$

$$(x-3)^2 + (y-4)^2 = 65$$

**Directions:** Write an equation of an ellipse with the given characteristics.

11. vertices:  $(18, -9)$  and  $(-2, -9)$

co-vertices:  $(8, -2)$  and  $(8, -16)$

$$MP = \left( \frac{18-2}{2}, \frac{-9-9}{2} \right) = (8, -9)$$

$$a = 10 \text{ (Horiz.)}$$

$$b = 7$$

$$\frac{(x-8)^2}{100} + \frac{(y+9)^2}{49} = 1$$

12. vertices:  $(-7, 13)$  and  $(-7, -11)$

co-vertices:  $(-1, 1)$  and  $(-13, 1)$

$$MP = \left( \frac{-7-7}{2}, \frac{13-11}{2} \right) = (-7, 1)$$

$$a = 12 \text{ (vert.)}$$

$$b = 6$$

$$\frac{(x+7)^2}{36} + \frac{(y-1)^2}{144} = 1$$

13. vertices:  $(-5, -7)$  and  $(-5, -13)$

foci:  $(-5, -10 \pm \sqrt{5})$

$$MP = \left( \frac{-5-5}{2}, \frac{-7-13}{2} \right) = (-5, -10)$$

$$a = 3 \text{ (vert.)}$$

$$c = \sqrt{5}$$

$$(\sqrt{5})^2 = a^2 - b^2$$

$$-4 = -b^2$$

$$b = 2$$

$$\frac{(x+5)^2}{4} + \frac{(y+10)^2}{9} = 1$$

14. co-vertices:  $(11, 8)$  and  $(9, 8)$

foci:  $(10, 8 \pm 3\sqrt{7})$

$$MP = \left( \frac{11+9}{2}, \frac{8+8}{2} \right) = (10, 8)$$

$$b = 1 \text{ (vert.)}$$

$$c = 3\sqrt{7}$$

$$(3\sqrt{7})^2 = a^2 - 1^2$$

$$63 = a^2 - 1$$

$$8 = a$$

$$(x-10)^2 + \frac{(y-8)^2}{64} = 1$$

**Directions:** Write an equation of a hyperbola with the given characteristics.

15. vertices:  $(-2, -5)$  and  $(-2, -13)$   
 co-vertices:  $(3, -9)$  and  $(-7, -9)$   
 $MP = \left( \frac{-2-2}{2}, \frac{-5-13}{2} \right) = (-2, -9)$

$a = 4$  (vert.)  
 $b = 5$

$$\frac{(y+9)^2}{16} - \frac{(x+2)^2}{25} = 1$$

16. co-vertices:  $(5, 6)$  and  $(5, 10)$   
 foci:  $(5 \pm 5\sqrt{5}, 8)$   
 $MP = \left( \frac{5+5}{2}, \frac{6+10}{2} \right) = (5, 8)$

$b = 2$  (horiz.)  
 $c = 5\sqrt{5}$

$$(5\sqrt{5})^2 = a^2 + 2^2$$

$$125 = a^2 + 4$$

$$a = 11$$

$$\frac{(x-5)^2}{121} - \frac{(y-8)^2}{4} = 1$$

17. vertices:  $(0, -1)$  and  $(20, -1)$   
 eccentricity:  $\frac{\sqrt{5}}{2}$   
 $MP = \left( \frac{0+20}{2}, \frac{-1-1}{2} \right) = (10, -1)$

$a = 10$  (horiz.)  
 $c = \frac{\sqrt{5}}{2}$ ;  $c = 5\sqrt{5}$   
 $(5\sqrt{5})^2 = 10^2 + b^2$   
 $b = 5$

$$\frac{(x-10)^2}{100} - \frac{(y+1)^2}{25} = 1$$

18. foci:  $(-7, -7 \pm 5\sqrt{2})$   $MP = (-7, -7)$   
 eccentricity:  $\sqrt{2}$   
 $c = 5\sqrt{2}$  (vert.)  
 $e = \frac{\sqrt{2}}{1}$ ;  $a = 5$

$$(5\sqrt{2})^2 = 5^2 + b^2$$

$$25 = b^2$$

$$b = 5$$

$$\frac{(y+7)^2}{25} - \frac{(x+7)^2}{25} = 1$$

**Directions:** Write an equation of a parabola with the given characteristics.

19. vertex:  $(-3, 7)$ , focus:  $(-3, \frac{11}{2})$   
 $p = -1.5$ ; down

$$(x+3)^2 = -6(y-7)$$

20. vertex:  $(11, 0)$ , directrix:  $x = 5$   
 $p = 6$ ; right

$$y^2 = 24(x-11)$$

21. focus:  $(\frac{13}{2}, 4)$ , directrix:  $x = \frac{23}{2}$   
 vertex:  $(9, 4)$   
 $p = 2.5$ ; right

$$(y-4)^2 = 10(x-9)$$

22. endpoints of latus rectum:  $(-1, -5)$  and  $(3, -5)$ ;  
 opens up  
 vertex:  $(1, -5)$   
 $4p = 4$

$$(x-1)^2 = 4(y+5)$$

### Topic C: Conics in General Form

**Directions:** Classify each conic, then write the equation of the conic in standard form.

23.  $-9x^2 + 4y^2 + 72x + 48y - 324 = 0$  **Hyperbola**  
 $-9x^2 + 72x + 4y^2 + 48y = 324$   
 $-9(x^2 - 8x + 16) + 4(y^2 + 12y + 36) = 324 - 144 + 144$   
 $\frac{-9(x-4)^2}{324} + \frac{4(y+6)^2}{324} = \frac{324}{324}$

$$\frac{(y+6)^2}{81} - \frac{(x-4)^2}{36} = 1$$

24.  $25x^2 + 16y^2 - 150x - 32y - 159 = 0$  **Ellipse**  
 $25x^2 - 150x + 16y^2 - 32y = 159$   
 $25(x^2 - 6x + 9) + 16(y^2 - 2y + 1) = 159 + 225 + 16$   
 $\frac{25(x-3)^2}{400} + \frac{16(y-1)^2}{400} = \frac{400}{400}$

$$\frac{(x-3)^2}{16} + \frac{(y-1)^2}{25} = 1$$

25.  $x^2 + y^2 + 14x + 8y - 68 = 0$  Circle

$$x^2 + 14x + y^2 + 8y = 68$$

$$(x^2 + 14x + 49) + (y^2 + 8y + 16) = 68 + 49 + 16$$

$$(x+7)^2 + (y+4)^2 = 133$$

26.  $y^2 - 16x - 10y - 7 = 0$  Parabola

$$y^2 - 10y = 16x + 7$$

$$y^2 - 10y + 25 = 16x + 7 + 25$$

$$(y-5)^2 = 16x + 32$$

$$(y-5)^2 = 16(x+2)$$

**Topic D: Applications of Conic Sections**

27. A taxi driver will agree to drive passengers to their destinations up to 20 miles from his house. Natalie lives 16.2 miles west and 11.3 miles south of the taxi driver's house. Write (a) an inequality to represent the region in which the taxi driver is willing to drive, then (b) determine whether Natalie's house is eligible for pick up.

a)  $x^2 + y^2 \leq 20^2$

$$x^2 + y^2 \leq 400$$

b)  $16.2^2 + 11.3^2 \leq 400$

$$262.44 + 127.69 \leq 400$$

$$390.13 \leq 400$$

Yes, Natalie can be picked up.

28. The cross section of a trophy revealed a hyperbolic shape with a height of 2 feet, a base of 6 inches across, and a width of 2 inches at its narrowest point. Find the width of the trophy 8 inches up from its base.

$$\frac{3^2}{12} - \frac{12^2}{b^2} = 1$$

$$9 - \frac{144}{b^2} = 1$$

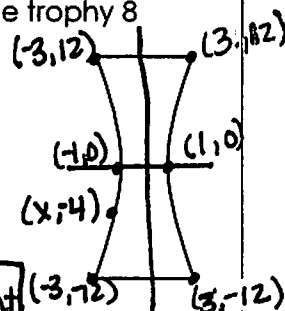
$$\frac{-144}{b^2} = -8 ; 18 = b^2$$

$$\frac{x^2}{1} - \frac{y^2}{18} = 1 \Rightarrow \left[ \frac{x^2}{1} - \frac{(-4)^2}{18} = 1 \right] \cdot 18$$

$$18x^2 - 16 = 18$$

$$18x^2 = 34$$

$$x^2 = \frac{17}{9} ; x = 1.37$$



Trophy is 2.74 in wide at that height

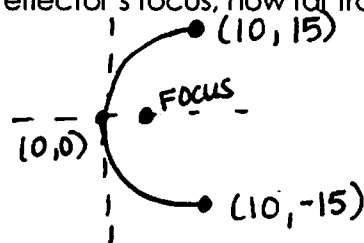
29. On the sideline of a televised football game, the crew uses a microphone reflector with a parabolic shape to collect the sounds from the players. The reflector spans 30 inches across and 10 inches deep. If the microphone needs to be placed at the reflector's focus, how far from the vertex should the microphone be placed?

$$y^2 = 4px$$

$$15^2 = 4p(10)$$

$$225 = 40p$$

$$p = 5.625 \text{ in}$$



30. An archeological dig site in the shape of an ellipse has an eccentricity of 0.6 and a major axis length of 20 feet. Write an equation to represent the outline of the dig site.

$$e = 0.6 = \frac{c}{a}$$

$$a = 10$$

$$c = 6$$

$$b^2 = a^2 - c^2$$

$$36 = 100 - b^2$$

$$-64 = -b^2$$

$$b = 8$$

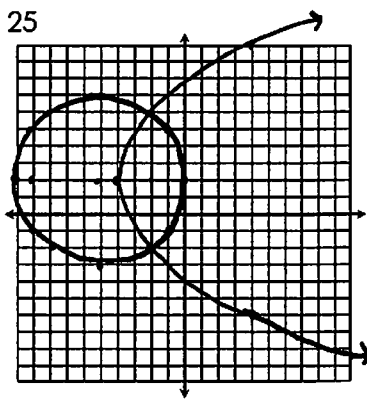
$$\frac{x^2}{100} + \frac{y^2}{64} = 1$$

### Topic E: Non-Linear Systems of Equations

**Directions:** Solve each system of equations by graphing.

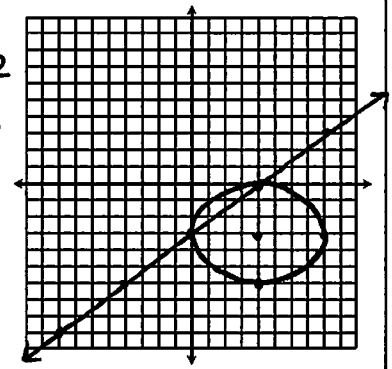
31. 
$$\begin{cases} (y-2)^2 = 8(x+4) \\ (x+5)^2 + (y-2)^2 = 25 \end{cases}$$

$(-2, 6)$  and  $(-2, -2)$



32. 
$$\begin{cases} \frac{(x-4)^2}{16} + \frac{(y+3)^2}{9} = 1 \\ 3x - 4y = 12 \end{cases}$$

$(0, -3)$  and  $(4, 0)$



**Directions:** Solve each system of equations by substitution or elimination.

33. 
$$\begin{cases} y = x^2 + 9x - 3 \\ y = -x^2 - 3x + 11 \end{cases}$$

$x^2 + 9x - 3 = -x^2 - 3x + 11$

$2x^2 + 12x - 14 = 0$

$2(x^2 + 6x - 7) = 0$

$2(x+7)(x-1) = 0$

$x = -7 \quad x = 1$

$y = (-7)^2 + 9(-7) - 3 = -17$   
 $y = 1^2 + 9(1) - 3 = 7$

$(-7, -17)$   
 $(1, 7)$

34. 
$$\begin{cases} y^2 - x^2 = 28 \rightarrow y^2 = 28 + x^2 \\ 4x^2 + 5y^2 = 212 \end{cases}$$

$4x^2 + 5(28 + x^2) = 212$

$4x^2 + 5x^2 + 140 = 212$

$9x^2 = 72$

$x^2 = 8$

$x = \pm 2\sqrt{2}$

$y^2 - (2\sqrt{2})^2 = 28$

$y^2 = 36$

$y = \pm 6$

$(2\sqrt{2}, 6)$   
 $(-2\sqrt{2}, 6)$   
 $(2\sqrt{2}, -6)$   
 $(-2\sqrt{2}, -6)$

### Topic F: Solving Three-Variable Systems with Matrices

**Directions:** Write the system of equations as an augmented matrix. Then, use row operations to transform the matrix to reduced row-echelon form and give the solution.

35. 
$$\begin{cases} -x - 2y + z = 10 \\ 4x + 3y + 3z = 6 \\ 3x + 4y = -11 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & -1 & -10 \\ 0 & -5 & 7 & 46 \\ 0 & -2 & 3 & 19 \end{bmatrix} \times R_3 - 3R_1 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 0 & 2 & 9 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{bmatrix} \times \frac{1}{5} R_2 \rightarrow R_2$$

$$\begin{bmatrix} -1 & -2 & 1 & 10 \\ 4 & 3 & 3 & 6 \\ 3 & 4 & 0 & -11 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & 9 \\ 0 & -5 & 7 & 46 \\ 0 & -2 & 3 & 19 \end{bmatrix} \times R_1 + R_3 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{bmatrix} \times R_1 - 2R_3 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 2 & -1 & -10 \\ 4 & 3 & 3 & 6 \\ 3 & 4 & 0 & -11 \end{bmatrix} \times -R_1 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 0 & 2 & 9 \\ 0 & -5 & 7 & 46 \\ 0 & 0 & 1 & 3 \end{bmatrix} \times 5R_3 - 2R_2 \rightarrow R_3$$

$[3, -5, 3]$

$$\begin{bmatrix} 1 & 2 & -1 & -10 \\ 0 & -5 & 7 & 46 \\ 3 & 4 & 0 & -11 \end{bmatrix} \times R_2 - 4R_1 \rightarrow R_2$$

$$\begin{bmatrix} 1 & 0 & 2 & 9 \\ 0 & -5 & 0 & 25 \\ 0 & 0 & 1 & 3 \end{bmatrix} \times R_2 - 7R_3 \rightarrow R_2$$

$$36. \begin{cases} 5x + y - z = 14 \\ -4x - 2y - 2z = -24 \\ -4x + 4y + z = -15 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & -3 & -10 \\ 0 & 3 & 7 & 32 \\ 0 & -6 & -3 & -9 \end{bmatrix} \xrightarrow[R_2]{R_2 - 2R_1} \begin{bmatrix} 1 & -1 & -3 & -10 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 5 \end{bmatrix} \xrightarrow[R_2]{\frac{1}{3}R_2} \begin{bmatrix} 1 & -1 & -3 & -10 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 & -1 & 14 \\ -4 & -2 & -2 & -24 \\ -4 & 4 & 1 & -15 \end{bmatrix} \xrightarrow[R_3]{2R_2 + R_3} \begin{bmatrix} 1 & -1 & -3 & -10 \\ 0 & 3 & 7 & 32 \\ 0 & 0 & 11 & 65 \end{bmatrix} \xrightarrow[R_1]{R_1 + R_2} \begin{bmatrix} 1 & 0 & -3 & -11 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -3 & -10 \\ -4 & -2 & -2 & -24 \\ -4 & 4 & 1 & -15 \end{bmatrix} \xrightarrow[R_1]{R_1 + R_2} \begin{bmatrix} 1 & -1 & -3 & -10 \\ 0 & 3 & 7 & 32 \\ 0 & 0 & 1 & 5 \end{bmatrix} \xrightarrow[R_3]{\frac{1}{11}R_3} \begin{bmatrix} 1 & -1 & -3 & -10 \\ 0 & 3 & 7 & 32 \\ 0 & 0 & 1 & 5 \end{bmatrix} \xrightarrow[R_1]{R_1 + 3R_3} \begin{bmatrix} 1 & -1 & 0 & 4 \\ 0 & 3 & 7 & 32 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -3 & -10 \\ -4 & -2 & -2 & -24 \\ 0 & -6 & -3 & -9 \end{bmatrix} \xrightarrow[R_3]{R_2 - R_3} \begin{bmatrix} 1 & -1 & -3 & -10 \\ 0 & 3 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{bmatrix} \xrightarrow[R_2]{R_2 - 7R_3} \begin{bmatrix} 1 & -1 & -3 & -10 \\ 0 & 3 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -3 & -10 \\ 2 & 1 & 1 & 12 \\ 0 & -6 & -3 & -9 \end{bmatrix} \xrightarrow[R_2]{-\frac{1}{2}R_2} \begin{bmatrix} 1 & -1 & -3 & -10 \\ 1 & 0.5 & 0.5 & 6 \\ 0 & -6 & -3 & -9 \end{bmatrix}$$

$$\boxed{[4, -1, 5]}$$

**Directions:** Using a graphing calculator, determine whether the system has one solution, no solution, or an infinite number of solutions. Identify the solution if there is one.

$$37. \begin{cases} -18x + 8y = 13 \\ 9x - 4y = 6 \end{cases}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -4/9 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**No Solution**

$$38. \begin{cases} 5x - 2y + 3z = 22 \\ -4x + y - z = -12 \\ 7x + 5y - 8z = -18 \end{cases}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

**(2, 0, 4)**

$$39. \begin{cases} 4x - 3y - z = 11 \\ -2y + 2z = 10 \\ 2x - 3y + z = 13 \end{cases}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

**Infinite Solutions**

**Directions:** Write a system of equations to solve each problem. Use a method of your choice.

40. Michael bought three ice cream cones, five ice cream sundaes, and two milkshakes and paid \$50.10. Andrea bought four ice cream cones, two ice cream sundaes, and three milkshakes and paid \$38.20. If the cost per sundae is \$1 less than the combined cost of one ice cream cone and one milkshake, find the cost of each item.

$x =$  ice cream cones  
 $y =$  ice cream sundaes  
 $z =$  milkshakes

$$\begin{cases} 3x + 5y + 2z = 50.10 \\ 4x + 2y + 3z = 38.20 \\ y = x + z - 1 \end{cases}$$

$$\begin{bmatrix} 3 & 5 & 2 & 50.10 \\ 4 & 2 & 3 & 38.20 \\ -1 & 1 & -1 & -1 \end{bmatrix} \xrightarrow{\text{rref}} (2.95, 6.45, 4.50)$$

**Ice cream cone = \$2.95, Ice Cream Sundae = \$6.45, Milkshake = \$4.50**

41. An investment totaling \$108,000 was placed into three different accounts that earn 3%, 4.5%, and 7.25% annual simple interest. The amount of money placed in the account that earns 3% is double the amount of money placed in the account that earns 7.25%. If the amount interest earned after one year totaled \$4,793.50, how much money was placed in each account?

$x = 3\%$   
 $y = 4.5\%$   
 $z = 7.25\%$

$$\begin{cases} x + y + z = 108000 \\ x = 2z \\ .03x + .045y + .0725z = 4793.50 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & 108000 \\ 1 & 0 & -2 & 0 \\ .03 & .045 & .0725 & 4793.50 \end{bmatrix} \xrightarrow{\text{rref}} (53200, 28200, 26600)$$

**\$53,200 at 3%, \$28,200 at 4.5%, \$26,600 at 7.25%**

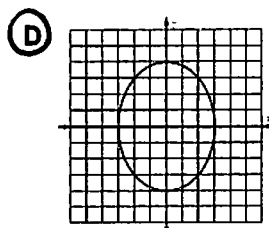
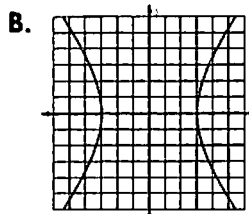
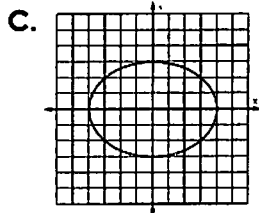
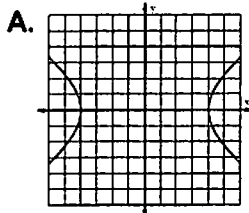
# Pre-Calculus Review

## QUIZ 5

Name: \_\_\_\_\_

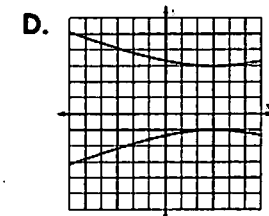
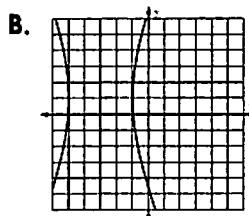
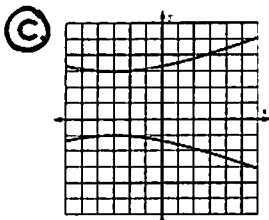
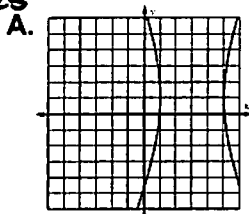
Date: \_\_\_\_\_ Per: \_\_\_\_\_

1. Which is the graph of  $16x^2 = 144 - 9y^2$ ?  $\frac{x^2}{9} + \frac{y^2}{16} = 1$



2. Which graph represents the equation below?

$\frac{(y-1)^2}{4} - \frac{(x+3)^2}{25} = 1$   $-4x^2 + 25y^2 - 24x - 50y - 111 = 0$



3. An ellipse has a center located at (2, 0), a horizontal major axis with a length of 40 units, and a focus located 16 units from its center. What is the equation of this ellipse?  $a=20$   
 $c=16$

A.  $\frac{(x-2)^2}{1600} + \frac{y^2}{576} = 1$

C.  $\frac{(x-2)^2}{400} + \frac{y^2}{256} = 1$

B.  $\frac{(x-2)^2}{400} + \frac{y^2}{144} = 1$

D.  $\frac{(x-2)^2}{1600} + \frac{y^2}{1296} = 1$

4. The equation  $y^2 = x$  is reflected over the y-axis, translated 3 units left, then vertically stretched by a factor of 8. Which correctly gives the focus and directrix of this new equation?  $4p=8; p=2$

A. focus: (1, 0); directrix:  $x = 5$

B. focus: (5, 0); directrix:  $x = 1$

C. focus: (-5, 0); directrix:  $x = -1$

D. focus: (-1, 0); directrix:  $x = -5$

5. Which is the equation of a hyperbola with a center at (1, 1), a vertex at (1, -3), and a focus at (1, -4)?  $c=5 \rightarrow b=3$   
 $a=4$

A.  $\frac{(y-1)^2}{16} - \frac{(x-1)^2}{9} = 1$

C.  $\frac{(y-1)^2}{4} - \frac{(x-1)^2}{9} = 1$

B.  $\frac{(x-1)^2}{16} - \frac{(y-1)^2}{9} = 1$

D.  $\frac{(x-1)^2}{4} - \frac{(y-1)^2}{9} = 1$

6. Which characteristic describes the graph of the equation below?

$$x^2 + 9y^2 - 2x - 90y + 190 = 0$$

$$\frac{(x-1)^2}{36} + \frac{(y-5)^2}{4} = 1$$

A. an ellipse with a major axis of 12 units

B. an ellipse with a center at (-1, 5)

C. a hyperbola with a transverse axis of 12 units

D. a hyperbola with a vertex at (6, 5)

7. A snowplow with a parabolic cross section is 3 feet wide at its opening. If the focus is located 4 inches from the vertex, find the depth of the snowplow at the vertex.

$$y^2 = 16x$$

$$18^2 = 16x$$

$$x = 20.25$$

A. 18.75 inches

B. 20.25 inches

C. 21.8 inches

D. 22.5 inches

8. A locket necklace in the shape of an ellipse is 25 millimeters long. If the eccentricity of the locket is 0.6, find the width of the locket

$$e = 0.6 \rightarrow c = 7.5$$

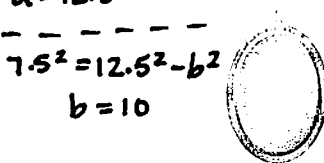
$$a = 12.5$$

A. 15 millimeters

B. 16 millimeters

C. 18 millimeters

D. 20 millimeters



9. Comets can travel on a hyperbolic path with the Sun at one of its foci and never return. If the hyperbolic path of a comet is modeled by the equation below, where  $x$  and  $y$  are measured in millions of kilometers, find the closest distance the comet can get to the Sun.

$$\frac{x^2}{9} - \frac{y^2}{40} = 1$$

$a = 3$   
 $b = 2\sqrt{10}$   
 $c^2 = 9 + 40 = 49$   
 $c = 7$

- A. 3 million kilometers
- B.** 4 million kilometers
- C. 7 million kilometers
- D. 9 million kilometers

12. Which sequence of row operations will transform matrix  $M$  to reduced row-echelon form?

$$M = \begin{bmatrix} 1 & 2 & -3 \\ -1 & 2 & 11 \end{bmatrix}$$

- |  |   |
|--|---|
| <p>A. <math>R_1 + R_2 \rightarrow R_1</math></p> <p><math>\frac{1}{4}R_1 \rightarrow R_1</math></p> <p><math>R_2 - 2R_1 \rightarrow R_2</math></p> | <p>C. <math>R_1 + R_2 \rightarrow R_2</math></p> <p><math>\frac{1}{2}R_1 \rightarrow R_1</math></p> <p><math>2R_1 - R_2 \rightarrow R_1</math></p>        |
| <p>B. <math>R_1 - R_2 \rightarrow R_2</math></p> <p><math>\frac{1}{2}R_2 \rightarrow R_2</math></p> <p><math>R_1 - 2R_2 \rightarrow R_1</math></p> | <p><b>D.</b> <math>R_1 + R_2 \rightarrow R_2</math></p> <p><math>\frac{1}{4}R_2 \rightarrow R_2</math></p> <p><math>R_1 - 2R_2 \rightarrow R_1</math></p> |

10. A parabola with an equation of  $y = 0.5x^2 - 8$  intersects a circle with a diameter of 10 units. If the center of the circle is located 5 units above the vertex of the parabola, at which points do the parabola and circle intersect?

$$y = 0.5x^2 - 8 \rightarrow 2y + 16 + y^2 + 6y + 9 = 25$$

$$x^2 + (y+3)^2 = 25 \rightarrow y^2 + 8y + 25 = 25$$

$$y(y+8) = 0$$

$$\frac{y=0}{(\pm 4, 0)} \quad \left| \quad \frac{y=-8}{(0, -8)} \right.$$

- A. (-5, 0), (0, -8), and (5, 0)
- B.** (-4, 0), (0, -8), and (4, 0)
- C. (-3, 0), (0, -8), and (3, 0)
- D. (-8, 0), (0, -4), and (8, 0)

13. The augmented matrix of a system of linear equations given is below. What is the solution, if any, for the system?

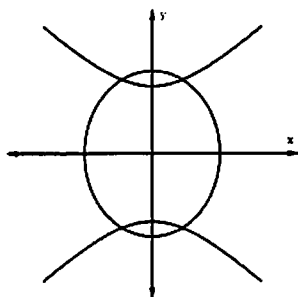
$$\left[ \begin{array}{ccc|c} 3 & -2 & -3 & 24 \\ 1 & 4 & -1 & -6 \\ 3 & 5 & -3 & 3 \end{array} \right]$$

↓

$$\left[ \begin{array}{ccc|c} 1 & 0 & -1 & 6 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

- A. (6, -3, 0)
- B. (-6, 3, 0)
- C.** infinitely many solutions
- D. no real solutions

11. A hyperbola intersects an ellipse as shown below. The equation of the ellipse is  $15x^2 + 10y^2 = 150$ . If the length of both axes of the hyperbola are the same as the length of the minor axis of the ellipse, what are the values of  $y$  at the points of intersection of the ellipse and the hyperbola?



$$15x^2 + 10y^2 = 150$$

$$-10x^2 + 10y^2 = 100$$

↓

$$x = \pm\sqrt{2}; y = \pm 2\sqrt{3}$$

- A.  $-\sqrt{2}$  and  $\sqrt{2}$
- B.  $-2\sqrt{2}$  and  $2\sqrt{2}$
- C.**  $-2\sqrt{3}$  and  $2\sqrt{3}$
- D.  $-3\sqrt{2}$  and  $3\sqrt{2}$

14. A jar of nickels, dimes, and quarters contains 47 coins. The number of nickels is three times the number of dimes. If the total value of the coins is \$5.75, how many quarters are in the jar?

$$\left[ \begin{array}{cccc} 1 & 1 & 1 & 47 \\ 1 & -3 & 0 & 0 \\ .05 & .10 & .25 & 5.75 \end{array} \right]$$

↓

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 24 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 15 \end{array} \right] \leftarrow \text{Quarters}$$

- A.** 15
- B. 16
- C. 18
- D. 24



10. Jamie purchased a car with 10,972 miles. After 12 weeks, her car has 12,259 miles. If Jamie continues to drive the same number of miles per week, determine the mileage on her car after one year.

$$a_1 = 10972$$

$$a_{12} = 12259$$

$$12259 = d(12-1) + 10972$$

$$1287 = 11d$$

$$d = 117$$

$$a_{52} = 117(52-1) + 10972$$

$$= 16,939 \text{ miles}$$

**Directions:** Use the given information about a geometric sequence to find the indicated value.

11. If  $a_1 = 343$  and  $r = \frac{2}{7}$ , find  $a_8$ .

$$a_8 = 343 \left(\frac{2}{7}\right)^{8-1}$$

$$= 343 \left(\frac{2}{7}\right)^7$$

$$= \frac{128}{2401}$$

12. If  $a_3 = -\frac{32}{45}$  and  $a_7 = -\frac{512}{3645}$ , find  $a_1$ .

$$a_7 = a_3 \cdot r^{7-3}$$

$$\frac{-512}{3645} = \frac{-32}{45} r^4$$

$$\frac{16}{81} = r^4 ; r = \frac{2}{3}$$

$$a_1 = \frac{-8}{5}$$

13. A camper was purchased for \$21,700. After 10 years, the camper's value decreased to \$10,500. Determine the rate at which the camper's value decreased per year.

$$a_1 = 21700$$

$$10500 = 21700 \cdot r^{10-1}$$

$$a_{10} = 10500$$

$$\frac{15}{31} = r^9$$

$$r = .9225$$

7.75% decrease per year

### Topic B: Arithmetic & Geometric Series

**Directions:** Find the indicated sum.

14.  $\left\{\frac{1}{2} + \frac{5}{6} + \frac{7}{6} + \frac{3}{2} + \dots\right\}; S_{20}$

$$S_{20} = 20 \left(\frac{\frac{1}{2} + \frac{41}{6}}{2}\right)$$

$$a_{20} = \frac{1}{3}(20-1) + \frac{1}{2}$$

$$= \frac{41}{6}$$

$$= \frac{220}{3}$$

15.  $\{256 - 64 + 16 - 4 + \dots\}; S_9$

$$S_{10} = \frac{256(1 - (-1/4)^9)}{1 - (-1/4)}$$

$$= \frac{52429}{256}$$

16.  $\sum_{p=5}^{17} (5p+2)$

$$a_1 = 5(5)+2 = 27$$

$$a_{13} = 5(17)+2 = 87$$

$$S_{13} = 13 \left(\frac{27+87}{2}\right)$$

$$= 741$$

17.  $\sum_{i=1}^{10} 0.4 \cdot (5)^{i-1}$

$$S_{10} = \frac{0.4(1-5^{10})}{1-5} = 976562.4$$

18.  $\sum_{n=1}^{\infty} 195 \cdot \left(\frac{-1}{5}\right)^{n-1}$

$$S = 195 \left(\frac{1}{1 - (-1/5)}\right)$$

$$= \frac{325}{2}$$

19.  $\sum_{j=1}^{\infty} \frac{1}{8} \cdot (2)^{j-1}$

No sum; divergent

20. A theatre has rows of seating such that the first row has six seats and each row thereafter has three more than the row in front of it. If there are 27 rows in the theatre, find the total number of seats.

$$a_1 = 6$$

$$d = 3$$

$$n = 27$$

$$a_{27} = 3(27-1) + 6 = 84$$

$$S_{27} = 27 \left( \frac{6+84}{2} \right)$$

$$= \boxed{1215 \text{ seats}}$$

21. Hermine deposited \$0.02 into his bank account. Each week, he made a deposit worth double the previous deposit. How long will it take before the account reaches a total of \$24,000?

$$24000 = \frac{.02(1-2^n)}{1-2}$$

$$-24000 = .02(1-2^n)$$

$$-1100000 = 1-2^n$$

$$-1100001 = -2^n$$

$$\frac{\log 1100001}{\log 2} = \frac{n \cdot \log 2}{\log 2}$$

$$n = 20.19$$

$\boxed{21 \text{ weeks}}$

### Topic C: Proof by Mathematical Induction

Directions: Use induction to prove each conjecture for all positive integers  $n$ .

22.  $1+4+7+\dots+(3n-2) = \frac{n(3n-1)}{2}$       $P_n: 1+4+7+\dots+(3n-2) = \frac{n(3n-1)}{2}$

①  $P_1: 1 = \frac{1(3 \cdot 1 - 1)}{2} \rightarrow 1 = 1$

②  $P_k: 1+4+7+\dots+(3k-2) = \frac{k(3k-1)}{2}$   
if  $n=k$  is true.

③ Prove  $P_{k+1}$  is true.

$$P_{k+1} = 1+4+7+\dots+(3k-2) + (3(k+1)-2) = \frac{(k+1)(3(k+1)-1)}{2}$$

$$\frac{k(3k-1)}{2} + (3(k+1)-2) = \frac{3k^2+5k+2}{2}$$

$$= \frac{(k+1)(3k+2)}{2}$$

$$\frac{3k^2-k}{2} + \frac{2(3k+1)}{2} = \frac{3k^2+5k+2}{2}$$

$$= \frac{3k^2+5k+2}{2}$$

$$\frac{3k^2+5k+2}{2} = \frac{3k^2+5k+2}{2} \quad \checkmark$$

23.  $3+8+15+\dots+(n^2+2n) = \frac{n(n+1)(2n+7)}{6}$       $P_n = 3+8+15+\dots+(n^2+n) = \frac{n(n+1)(2n+7)}{6}$

①  $P_1: 3 = \frac{1(1+1)(2 \cdot 1 + 7)}{6} \rightarrow 3 = 3$

②  $P_k: 3+8+15+\dots+(k^2+2k) = \frac{k(k+1)(2k+7)}{6}$

③ Prove  $P_{k+1}$  is true.

$$P_{k+1} = 3+8+15+\dots+(k^2+2k) + ((k+1)^2+2(k+1)) = \frac{(k+1)((k+1)+1)(2(k+1)+7)}{6}$$

$$\frac{k(k+1)(2k+7)}{6} + ((k+1)^2+2(k+1)) = \frac{2k^3+15k^2+31k+18}{6}$$

$$= \frac{(k+1)(k+2)(2k+9)}{6}$$

$$\frac{2k^3+9k^2+7k}{6} + \frac{6(k^2+4k+3)}{6} = \frac{2k^3+15k^2+31k+18}{6}$$

$$= \frac{2k^3+15k^2+31k+18}{6}$$

$$\frac{2k^3+15k^2+31k+18}{6} = \frac{2k^3+15k^2+31k+18}{6} \quad \checkmark$$

### Topic D: Pascal's Triangle & Binomial Theorem

**Directions:** Expand each expression using Pascal's triangle or the Binomial Theorem.

24.  $(c - 2d)^5$

$$= 1c^5(-2d)^0 + 5c^4(-2d)^1 + 10c^3(-2d)^2 + 10c^2(-2d)^3 + 5c^1(-2d)^4 + 1c^0(-2d)^5$$

$$= c^5 - 10c^4d + 40c^3d^2 - 80c^2d^3 + 80cd^4 - 32d^5$$

25.  $(5m + 4n)^4$

$$= 1(5m)^4(4n)^0 + 4(5m)^3(4n)^1 + 6(5m)^2(4n)^2 + 4(5m)^1(4n)^3 + 1(5m)^0(4n)^4$$

$$= 625m^4 + 2000m^3n + 2400m^2n^2 + 1280mn^3 + 256n^4$$

**Directions:** Find the indicated term of each binomial expansion.

26.  $(5y - 1)^8$ ; 7<sup>th</sup> term

$$\binom{8}{6} (5y)^2 (-1)^6$$

$$= 700y^2$$

27.  $(2a + 3b)^6$ ; 6<sup>th</sup> term

$$\binom{6}{5} (2a)^1 (3b)^5$$

$$= 2916ab^5$$

28.  $(h - 4k)^9$ ; 4<sup>th</sup> term

$$\binom{9}{3} (h)^6 (-4k)^3$$

$$= -5376h^6k^3$$

### Topic E: Limits

**Directions:** Evaluate each limit, if it exists.

29.  $\lim_{x \rightarrow 4} \frac{x-2}{2-\sqrt{x}}$

$$\infty$$

30.  $\lim_{x \rightarrow 6} -\frac{1}{x-8}$

$$\frac{1}{2}$$

31.  $\lim_{x \rightarrow -4} \frac{x^3 + 64}{16 - x^2}$

$$6$$

32.  $\lim_{x \rightarrow -2} \frac{x^2 + 5x + 6}{x^2 + 4x + 4}$

$$= \lim_{x \rightarrow -2} \frac{(x+2)(x+3)}{(x+2)(x+2)}$$

$$= \lim_{x \rightarrow -2} \frac{x+3}{x+2}$$

$$\text{DNE}$$

$$\begin{aligned}
 33. \lim_{x \rightarrow -4} \frac{2x^2 + 9x - 5}{4x^2 - 1} \\
 &= \lim_{x \rightarrow -4} \frac{(2x - 1)(x + 5)}{(2x + 1)(2x - 1)} \\
 &= \lim_{x \rightarrow -4} \frac{x + 5}{2x + 1} \Rightarrow \frac{6}{3} = \boxed{2}
 \end{aligned}$$

$$\begin{aligned}
 34. \lim_{x \rightarrow -\pi} \frac{2|\cos x|}{\cos x} \\
 &\Rightarrow \frac{2|\cos(-\pi)|}{\cos(-\pi)} = \frac{2(1)}{-1} \\
 &= \boxed{-2}
 \end{aligned}$$

$$\begin{aligned}
 35. \lim_{x \rightarrow -2} \frac{x + 2}{\sqrt{2 - 7x} - 4} \cdot \frac{(\sqrt{2 - 7x} + 4)}{(\sqrt{2 - 7x} + 4)} \\
 &= \lim_{x \rightarrow -2} \frac{(x + 2)(\sqrt{2 - 7x} + 4)}{(2 - 7x) - 16} \\
 &= \lim_{x \rightarrow -2} \frac{(x + 2)(\sqrt{2 - 7x} + 4)}{-7x - 14} \\
 &= \lim_{x \rightarrow -2} \frac{(x + 2)(\sqrt{2 - 7x} + 4)}{-7(x + 2)} \\
 &= \lim_{x \rightarrow -2} \frac{\sqrt{2 - 7x} + 4}{-7} \Rightarrow \frac{4 + 4}{-7} = \boxed{-\frac{8}{7}}
 \end{aligned}$$

$$\begin{aligned}
 36. \lim_{x \rightarrow 6} \frac{\sqrt{x + 3} - 3}{6 - x} \cdot \frac{(\sqrt{x + 3} + 3)}{(\sqrt{x + 3} + 3)} \\
 &= \lim_{x \rightarrow 6} \frac{(x + 3) - 9}{(6 - x)(\sqrt{x + 3} + 3)} \\
 &= \lim_{x \rightarrow 6} \frac{x - 6}{(6 - x)(\sqrt{x + 3} + 3)} \\
 &= \lim_{x \rightarrow 6} \frac{-1}{\sqrt{x + 3} + 3} \Rightarrow \frac{-1}{3 + 3} = \boxed{-\frac{1}{6}}
 \end{aligned}$$

$$\begin{aligned}
 37. \lim_{x \rightarrow \infty} \frac{3x^3 + 2}{x^2 + 4x - 7} \rightarrow \lim_{x \rightarrow \infty} \frac{3x + \frac{2}{x^2}}{1 + \frac{4}{x} + \frac{7}{x^2}} \\
 &= \frac{\lim_{x \rightarrow \infty} 3x + 2 \cdot \lim_{x \rightarrow \infty} \frac{1}{x^2}}{\lim_{x \rightarrow \infty} 1 + 4 \cdot \lim_{x \rightarrow \infty} \frac{1}{x} + 7 \cdot \lim_{x \rightarrow \infty} \frac{1}{x^2}} \\
 &= \frac{3 \cdot \infty + 2 \cdot 0}{1 + 4 \cdot 0 + 7 \cdot 0} = \frac{-3\infty}{1} = \boxed{-\infty}
 \end{aligned}$$

$$\begin{aligned}
 38. \lim_{x \rightarrow \infty} \frac{12x^2 - 8x + 3}{5 - 3x^2} \rightarrow \lim_{x \rightarrow \infty} \frac{12 - \frac{8}{x} + \frac{3}{x^2}}{\frac{5}{x^2} - 3} \\
 &= \frac{\lim_{x \rightarrow \infty} 12 - 8 \cdot \lim_{x \rightarrow \infty} \frac{1}{x} + 3 \cdot \lim_{x \rightarrow \infty} \frac{1}{x^2}}{5 \cdot \lim_{x \rightarrow \infty} \frac{1}{x^2} - \lim_{x \rightarrow \infty} 3} \\
 &= \frac{12 - 8 \cdot 0 + 3 \cdot 0}{5 \cdot 0 - 3} = \frac{12}{-3} = \boxed{-4}
 \end{aligned}$$

### Topic F: Limits of Sequences

Directions: Evaluate the limit of each sequence, if it exists.

$$\begin{aligned}
 39. a_n = \frac{6n^2 + 1}{9n + 2n^2} \rightarrow \lim_{n \rightarrow \infty} \frac{6 + \frac{1}{n^2}}{\frac{9}{n} + 2} \\
 &= \frac{\lim_{n \rightarrow \infty} 6 + \lim_{n \rightarrow \infty} \frac{1}{n^2}}{9 \cdot \lim_{n \rightarrow \infty} \frac{1}{n} + \lim_{n \rightarrow \infty} 2} \\
 &= \frac{6 + 0}{9 \cdot 0 + 2} = \frac{6}{2} = \boxed{3}
 \end{aligned}$$

$$\begin{aligned}
 40. a_n = \frac{2n + 5}{n^2 - 7n} \rightarrow \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + \frac{5}{n^2}}{1 - \frac{7}{n}} \\
 &= \frac{2 \cdot \lim_{n \rightarrow \infty} \frac{1}{n} + 5 \cdot \lim_{n \rightarrow \infty} \frac{1}{n^2}}{\lim_{n \rightarrow \infty} 1 - 7 \cdot \lim_{n \rightarrow \infty} \frac{1}{n}} \\
 &= \frac{2 \cdot 0 + 5 \cdot 0}{1 - 7 \cdot 0} = \frac{0}{1} = \boxed{0}
 \end{aligned}$$

# Pre-Calculus Review

## QUIZ 6

Name: \_\_\_\_\_

Date: \_\_\_\_\_ Per: \_\_\_\_\_

1. A collection of nickels is arranged in a triangular array such that there are 21 nickels in the base row, 20 nickels in the next row, 19 nickels in the next, and so forth, with 1 nickel in the top row. Find the value of the coins in the collection.

$$S_{21} = 21 \left( \frac{21+1}{2} \right) = 231$$

- (A) \$11.55  
 B. \$11.60  
 C. \$11.65  
 D. \$11.70

2. The first and fourth term of a geometric sequence are 4 and -13.5. Find the sum of the first seven terms.

$$-13.5 = 4 \cdot r^{4-1}$$

$$\frac{-27}{8} = r^3$$

$$r = -\frac{3}{2}$$

$$S_7 = \frac{4 \left( 1 - \left( -\frac{3}{2} \right)^7 \right)}{1 - \left( -\frac{3}{2} \right)}$$

A.  $\frac{115}{4}$

B.  $-\frac{1261}{32}$

(C)  $\frac{463}{16}$

D.  $-\frac{133}{8}$

3. Given the sequence below, if  $S_n = -651,042$ , find the value of  $n$ .

$$a_n = -2, 10, -50, 250, \dots$$

$$-651042 = \frac{-2(1 - (-5)^n)}{1 - (-5)}$$

$$-3906252 = -2(1 - (-5)^n)$$

$$1953126 = 1 - (-5)^n$$

$$1953125 = 5^n$$

$$\frac{\log 1953125}{\log 5} = \frac{n \log 5}{\log 5}$$

$$n = 9$$

- A. 7  
 B. 8  
 (C) 9  
 D. 10

4. What is the sum of the series below?

$$S_n = \frac{1}{6} \left( \frac{1}{1-2/3} \right) \frac{1}{6} + \frac{1}{9} + \frac{2}{27} + \frac{4}{81} + \dots$$

$$= \frac{1}{6} (3)$$

- A. no sum  
 B.  $\frac{5}{9}$   
 C.  $\frac{2}{3}$   
 (D)  $\frac{1}{2}$

5. What is the sum of  $\sum_{n=1}^{\infty} 6 \cdot \left( -\frac{1}{4} \right)^{n-1}$ ?

$$S_n = 6 \left( \frac{1}{1 - (-1/4)} \right)$$

$$= 6 \left( \frac{4}{5} \right)$$

- A. no sum  
 B.  $\frac{24}{5}$   
 C.  $\frac{2}{3}$   
 D.  $\frac{15}{2}$

6. The first term in an infinite geometric sequence is -6. If the sum of the sequence is -24, find the common ratio.

$$-24 = -6 \left( \frac{1}{1-r} \right)$$

$$4 = \frac{1}{1-r}$$

$$4 - 4r = 1$$

$$-4r = -3$$

$$r = \frac{3}{4}$$

A.  $\frac{1}{2}$

B.  $-\frac{1}{4}$

C.  $-\frac{3}{4}$

(D)  $\frac{3}{4}$

7. Expand  $(3x - y)^4$ .

$$1(3x)^4(-y)^0 + 4(3x)^3(-y)^1 + 6(3x)^2(-y)^2 + 4(3x)^1(-y)^3 + 1(3x)^0(-y)^4$$

- A.  $81x^4 - 27x^3y + 9x^2y^2 - 3xy^3 + y^4$   
 (B)  $81x^4 - 108x^3y + 54x^2y^2 - 12xy^3 + y^4$   
 C.  $81x^4 - 162x^3y + 54y^2y^2 - 12xy^3 + 4y^4$   
 D.  $81x^4 - 243x^3y + 162x^2y^2 - 9xy^3 + 4y^4$

8. What is the coefficient of  $a^5b^2$  in the expansion of  $(2a + 3b)^7$ ?

1 7 21 35 35 21 7 1

$$21(2a)^5(3b)^2$$

$$21(32)(9) = 6048$$

- A. 10,206  
 B. 2,186  
 C. 6,048  
 D. 15,120

12. Evaluate the limit below.

$$\lim_{x \rightarrow 0} \frac{\cos(4x)}{2x}$$

- A. 2  
 B.  $\infty$   
 C.  $-\infty$   
 D. The limit does not exist.

9. Caleb would like to prove the statement below by induction. He verified an initial case, then assumed the statement is true for  $n = k$  for some positive integer  $k$ . What final step would complete his proof?

$$\begin{aligned} 7 + 11 + 15 + \dots + (4n + 3) &= 2n^2 + 5n \\ \dots + (4k + 3) + (4(k+1) + 3) &= 2(k+1)^2 + 5(k+1) \\ \dots + (4k + 3) + (4k + 7) &= 2k^2 + 4k + 2 + 5k + 5 \end{aligned}$$

- A.  $7 + 11 + 15 + \dots + (4k + 7) = 2k^2 + 9k + 7$   
 B.  $7 + 11 + 15 + \dots + (4k + 1) = 2k^2 + 9k + 7$   
 C.  $7 + 11 + 15 + \dots + (5k + 4) = 2k^2 + 9k + 2$   
 D.  $7 + 11 + 15 + \dots + (k + 1) = 2k^2 + 9k + 2$

13. Evaluate the limit below.

$$\lim_{x \rightarrow 5} \frac{\sqrt{x-1} - 2}{x-5} \cdot (\sqrt{x-1} + 2)$$

$$\lim_{x \rightarrow 5} \frac{(x-1) - 4}{(x-5)(\sqrt{x-1} + 2)}$$

$$\lim_{x \rightarrow 5} \frac{1}{\sqrt{x-1} + 2}$$

- A.  $\frac{1}{4}$   
 B.  $\frac{1}{2}$   
 C.  $\frac{1}{5}$   
 D.  $\frac{1}{8}$

10. Evaluate the limit below.

$$\lim_{x \rightarrow 4} \frac{x^2 + 10x + 24}{x^2 + 2x - 8} = \frac{(x+6)(x+4)}{(x+4)(x-2)}$$

$$\downarrow$$

$$\frac{-4+6}{-4-2} = \frac{2}{-6}$$

- A. -3  
 B.  $-\frac{1}{3}$   
 C. 5  
 D. -1

14. Evaluate the limit below.

$$\lim_{x \rightarrow \infty} \frac{2x^2 + x - 8}{1 - 4x^2} = \lim_{x \rightarrow \infty} \frac{2 + \frac{1}{x} - \frac{8}{x^2}}{\frac{1}{x^2} - 4}$$

$$= \lim_{x \rightarrow \infty} 2 + \lim_{x \rightarrow \infty} \frac{1}{x} - 8 \cdot \lim_{x \rightarrow \infty} \frac{1}{x^2} = \frac{2 + 0 - 8 \cdot 0}{0 - 4}$$

$$\lim_{x \rightarrow \infty} \frac{1}{x^2} = \lim_{x \rightarrow \infty} 4$$

- A. 2  
 B. -2  
 C.  $-\frac{1}{2}$   
 D. 0

11. Evaluate the limit below.

$$\lim_{x \rightarrow 3^+} \frac{x+3}{|x-3|}$$

- A.  $-\infty$   
 B.  $\infty$   
 C. -1  
 D. 3

15. Find the limit of the sequence below.

$$a_n = \frac{6n}{4n^2 - 1}$$

$$= \lim_{n \rightarrow \infty} \frac{\frac{6}{n}}{4 - \frac{1}{n^2}} = 6 \cdot \lim_{n \rightarrow \infty} \frac{1}{n} = \frac{6 \cdot 0}{4 - 0} = 0$$

- A.  $\infty$   
 B.  $-\infty$   
 C.  $\frac{3}{2}$   
 D. 0

# CREDITS

I use clipart and fonts in my products by:



Art with Jenny K



Many thanks to these talented artists!