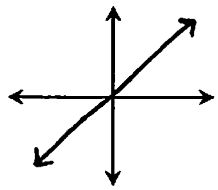
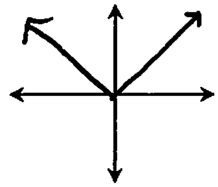
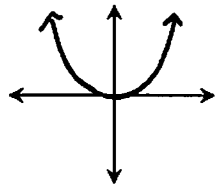
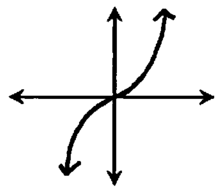
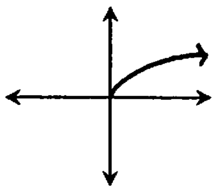
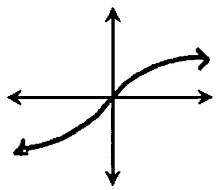
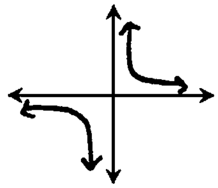
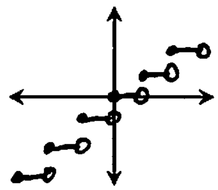
Name: ______________________ **PRE-CALCULUS REVIEW:** Packet #1

Topic A: Basic Parent Functions and Transformations

Directons: For each function family, give the parent function and sketch the shape of the graph.

Linear	Absolute Value	Quadratic	Cubic
			
Square Root	**Cube Root**	**Reciprocal**	**Greatest Integer**
			

Recall the following transformations rules given a function $f(x)$:

Translations (Shifts)	**Reflections**	**Dilations** (compress/stretch)
$f(x+h)$ shifts left	$-f(x)$ reflects over the x-axis	$a \cdot f(x)$ is a vertical compression when $\lvert a \rvert < 1$ and a vertical stretch when $\lvert a \rvert > 1$
$f(x-h)$ shifts right		
$f(x)+k$ shifts up	$f(-x)$ reflects over the y-axis	$f(b \cdot x)$ is a horizontal stretch when $\lvert b \rvert < 1$ and a horizontal compression when $\lvert b \rvert > 1$
$f(x)-k$ shifts down		

Directions: Describe the transformations from the parent function.

1. $f(x) = 3(x+7)^3 + 4$

- vert. stretch by 3
- translate left 7 and up 4

2. $f(x) = |-2(x-1)| - 9$

- Reflect in y-axis
- Horiz. Compression by 1/2
- Translate right 1 and down 9

3. $f(x) = -\dfrac{x+8}{6} - 5$

- Reflect in x-axis
- Vert. compression by 1/6
- Translate left 8, down 2

4. The greatest integer parent function is reflected in the y-axis, horizontally stretched by a factor of 4, and translated 3 units left. Write an equation to represent the **new function.**

$f(x) = [\![-\frac{1}{4}(x+3)]\!]$

5. The square root parent function is reflected in the x-axis, vertically compressed by a factor of ½, then translated 5 unit right and 1 unit down. Write an equation to represent the **new function.**

$f(x) = -\frac{1}{2}\sqrt{x-5} - 1$

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6. Transformations were applied to the quadratic parent function such that it creates an absolute maximum at (-7, -2). Write an equation that could represent this **new function**.

$f(x) = -(x+7)^2 - 2$

7. A certain function is reflected in the x-axis, vertically compressed by a factor of ¾, translated right 3 and up 7. If the new function is represented by $f(x) = 9|5(x-3)| + 2$, write an equation that could represent the **original function**.

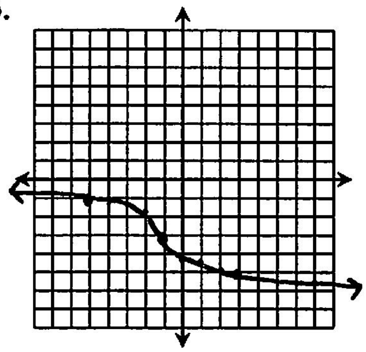
$f(x) = -12|5x| - 5$

Directions: Using the description of transformations from the parent function, **(a)** write a function, then **(b)** graph the function and state its domain and range.

8. The cube root function is reflected in the y-axis, horizontally compressed by a factor of ½, and translated left 1 and down 3.

a. $f(x) = \sqrt[3]{-2(x+1)} - 3$

b.



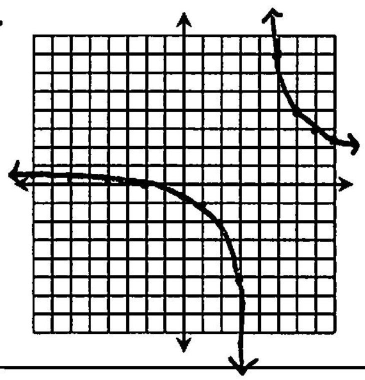
Domain: $\mathbb{R}$

Range: $\mathbb{R}$

9. The reciprocal function is vertically stretched by a factor of 3, horizontally stretched by a factor of 2, then translated right 4 and up 1.

a. $f(x) = \dfrac{3}{\frac{1}{2}(x-4)} + 1$

b.



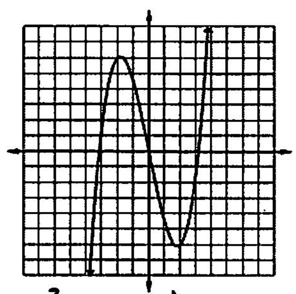
Domain: $\{x \mid x \neq 4\}$

Range: $\{y \mid y \neq 1\}$

Topic B: Symmetry; Even and Odd Functions

Directions: Use the graph to determine if the relations given below are symmetric to the x-axis, y-axis, or origin. Confirm your answer algebraically.

10. $y = \frac{1}{2}x^3 - 5x$

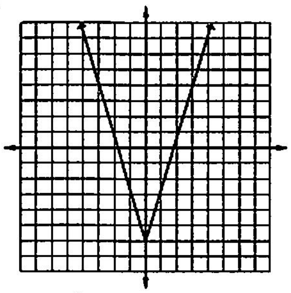


$-y = \frac{1}{2}(-x)^3 - 5(-x)$

$-y = -\frac{1}{2}x^3 + 5x$

$\boxed{\text{Symmetric to origin}}$

11. $y = \frac{2}{3}|5x| - 6$



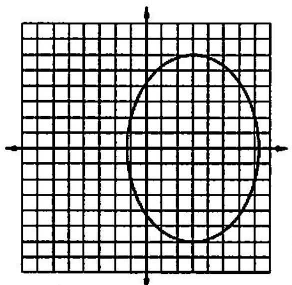
$y = \frac{2}{3}|5(-x)| - 6$

$y = \frac{2}{3}|-5x| - 6$

$y = \frac{2}{3}|5x| - 6$

$\boxed{\text{Symmetric to y-axis}}$

12. $2(x-3)^2 + y^2 = 36$



$2(x-3)^2 + (-y)^2 = 36$

$2(x-3)^2 + y^2 = 36$

$\boxed{\text{Symmetric to x-axis}}$

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Directions: Determine algebraically if the function is even, odd, or neither. If even or odd, describe the symmetry.

13. $f(x)=\sqrt[4]{2x^2}+9$

$f(-x)=\sqrt[4]{2(-x)^2}+9$

$f(-x)=\sqrt[4]{2x^2}+9$

$\boxed{\text{Even; symmetric to y-axis}}$

14. $f(x)=\dfrac{-x^3+7}{4}$

$f(-x)=\dfrac{-(-x)^3+7}{4}$

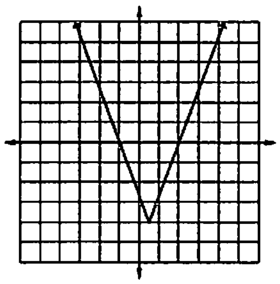
$f(-x)=\dfrac{x^3+7}{4}$

$\boxed{\text{Neither}}$

Topic C: Continuity

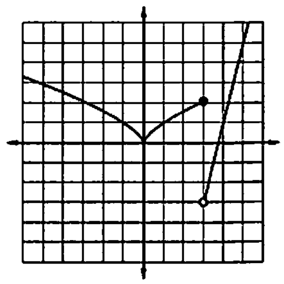
Directions: Determine if the functions below are continuous. If discontinuous, identify the type and location of discontinuity.

15.



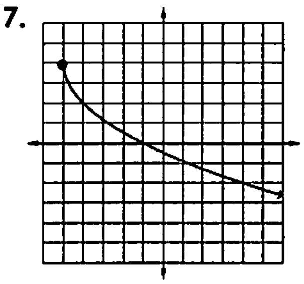
Continuous

16.



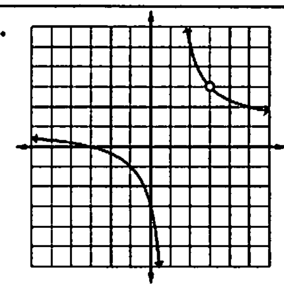
Discontinuous; $x=3$; jump

17.



Continuous

18.

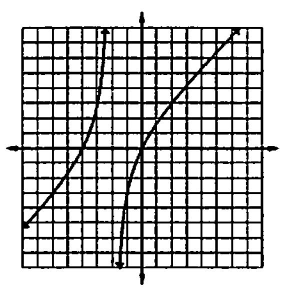


Discontinuous
$x=1$; infinite
$x=3$; removable

Topic D: Inverse Functions

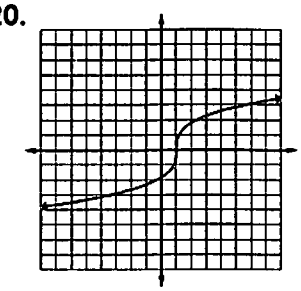
Directions: Determine if the graph represents a one-to-one function.

19.



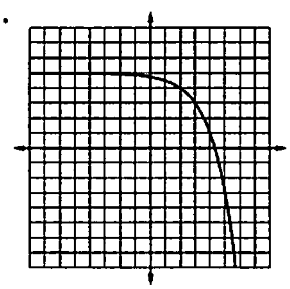
No

20.



Yes

21.



Yes

Directions: Find the inverse of each function. Then, graph both the function and its inverse.

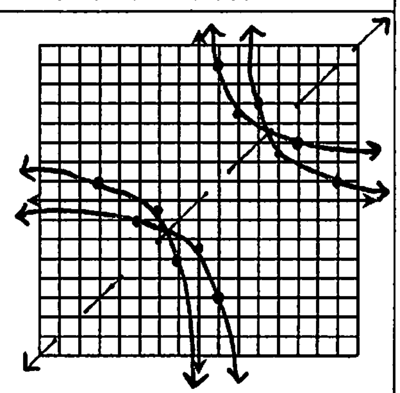
22. $f(x)=\dfrac{5}{x}+2$

$x=\dfrac{5}{y}+2$

$x-2=\dfrac{5}{y}$

$y(x-2)=5$

$y=\dfrac{5}{x-2}$



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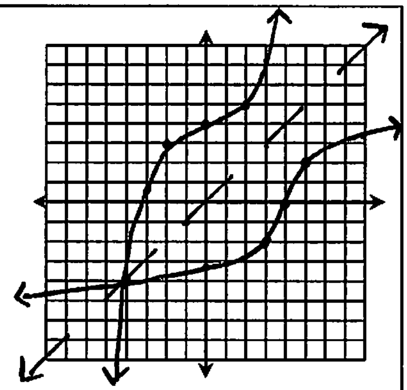
23. $f(x) = 2\sqrt[3]{x-4}$

$x = 2\sqrt[3]{y-4}$

$\frac{1}{2}x = \sqrt[3]{y-4}$

$\frac{1}{8}x^3 = y-4$

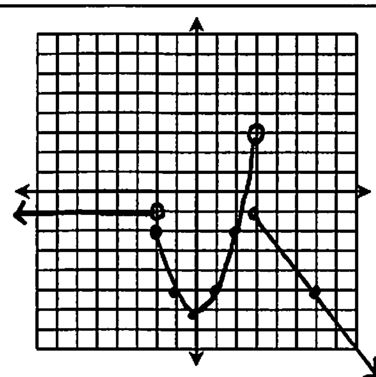
$\frac{1}{8}x^3 + 4 = y$



Topic E: Piecewise Functions

Directions: Graph the piecewise function. Identify the domain, range, and state the location and type of any discontinuities.

24. $f(x) = \begin{cases} -1 & \text{if } x < -2 \\ x^2 - 6 & \text{if } -2 \le x < 3 \\ -\frac{4}{3}x + 3 & \text{if } x \ge 3 \end{cases}$

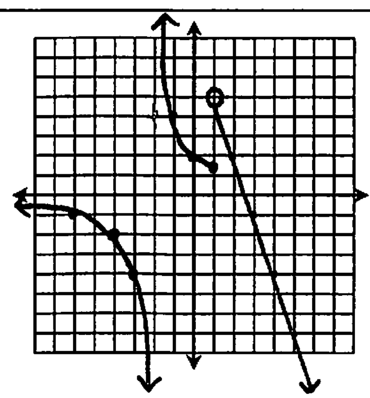


Domain: $\mathbb{R}$

Range: $\{y \mid y < 3\}$

Discontinuities:
$x = -2$; jump
$x = 3$; jump

25. $f(x) = \begin{cases} \frac{4}{x+2} & \text{if } x \le 1 \\ -3x + 8 & \text{if } x > 1 \end{cases}$

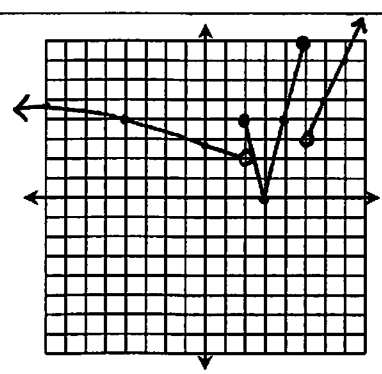


Domain: $\{x \mid x \neq -2\}$

Range: $\mathbb{R}$

Discontinuities:
$x = -2$; infinite
$x = 1$; jump

26. $f(x) = \begin{cases} \sqrt{8-2x} & \text{if } x < 2 \\ 4|x-3| & \text{if } 2 \le x \le 5 \\ 2x - 7 & \text{if } x > 5 \end{cases}$



Domain: $\mathbb{R}$

Range: $\{y \mid y \ge 0\}$

Discontinuities:
$x = 2$; jump
$x = 5$; jump

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Pre-Calculus Review

QUIZ 1

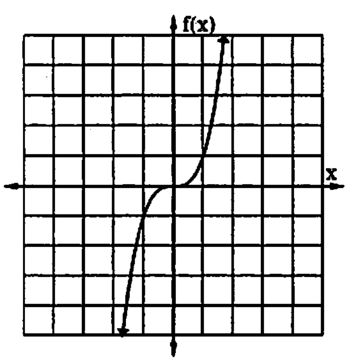
Name: ______________________________

Date: ______________________ Per: ______

1. The minimum point on the graph of the function $f(x)$ is (-4, -1). What is the minimum point on the graph of the function $f(x + 2)$?

   A. (-4, -3)
   B. (-4, 1)
   C. (-2, -1)
   D. (-6, -1)

2. If the function below is reflected in the x-axis, then horizontally compressed by a factor of 1/3, which equation represents the new function?



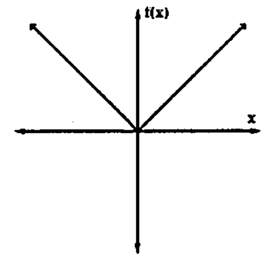
   A. $f(x) = -3x^3$
   B. $f(x) = \left(-\frac{1}{3}x\right)^3$
   C. $f(x) = -(3x)^3$
   D. $f(x) = -\left(\frac{1}{3}x\right)^3$

3. The function $f(x) = \sqrt{x}$ is transformed to create function g below. Which describes a transformation that took place?

$$g(x) = \frac{1}{2}\sqrt{-(x-1)}$$

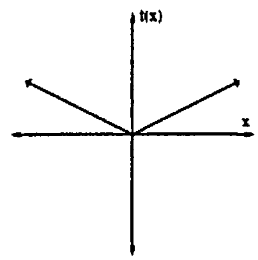
   A. a shift one unit to the left
   B. a shift one unit down
   C. a horizontal stretch of 2
   D. a reflection in the y-axis

4. The graph of $f(x)$ is shown below.

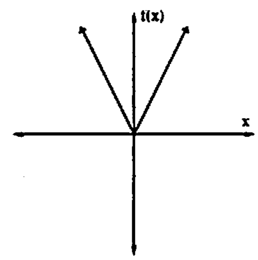


Which graph best represents $a \cdot f(-x)$ if $0 < a < 1$?

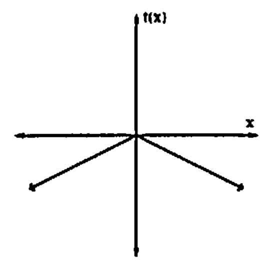
A.



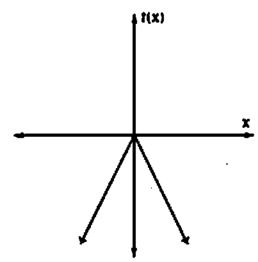
C.



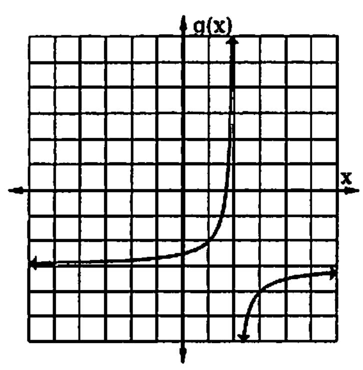
B.



D.



5. A function f was reflected in the x-axis, then translated three units left and two units down down to create function g shown below. Which function represents f?



   A. $f(x) = \frac{1}{x+1} + 5$
   B. $f(x) = \frac{1}{x-5} + 1$
   C. $f(x) = \frac{1}{x+1} - 3$
   D. $f(x) = \frac{1}{x-5} - 3$

6. If p and q are real numbers, which statement is true?

$$f(x) = (x-p)^2 + q$$

   A. The function decreases on the interval $[-q, \infty)$.
   B. The function increases on the interval $[p, \infty)$.
   C. The domain of the function is $[p, \infty)$.
   D. The range of the function is $[-q, \infty)$.

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7. What is the domain of the function below?

$$f(x) = \sqrt{2x+6}$$

A. $(-\infty, \infty)$
C. $[-3, \infty)$
B. $[6, \infty)$
D. $[3, \infty)$

8. What set of transformations would cause f and g to have the same range?

$$f(x) = -(x+5)^2 - 4; \; g(x) = (x-1)^2 - 2$$

A. Reflect g in the x-axis; Translate f two units up.
B. Reflect f in the x-axis; Translate g four units down.
C. Reflect g in the x-axis; Translate f six units up.
D. Reflect f in the x-axis; Translate g two units down.

9. Which function is odd?

A. $f(x) = \sqrt{x^2 - 3}$
C. $f(x) = |x+2|$
B. $f(x) = \dfrac{4}{x}$
D. $f(x) = 2x^4 - 8x^2 + 3$

10. Which equation, if true, will show that $y^2 = x - 4$ is symmetric to the x-axis?

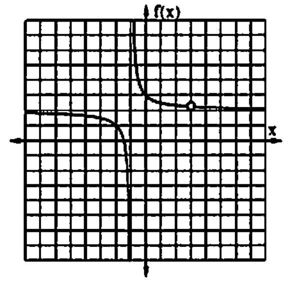
A. $(-y)^2 = x - 4$
C. $(-y)^2 = -x - 4$
B. $y^2 = -x - 4$
D. $y^2 = -(x - 4)$

11. Given the function $f(x)$ below, which statement is true?

$$f(x) = \begin{cases} |x+1|, & x < 1 \\ x^3 - 2, & x \geq 1 \end{cases}$$

A. f has a jump discontinuity at $x = 1$.
B. f has a infinite discontinuity at $x = 1$.
C. f has a removable discontinuity at $x = 1$.
D. f is continuous on the real numbers.

12. Given the graph of $f(x)$ below, which statement is true?

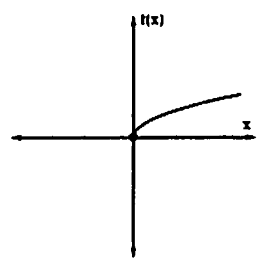


A. f has a jump discontinuity at $x = -1$.
B. f has a removable discontinuity at $x = -1$.
C. f has a removable discontinuity at $x = 3$.
D. f has an infinite discontinuity at $x = 3$.

13. The graph of $f(x) = x^3 + 5$ is reflected in the x-axis, then translated two units left. Which equation represents the inverse of the new function?

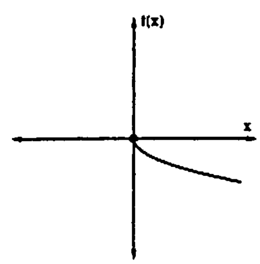
A. $f^{-1} = \sqrt[3]{-x+5} - 4$
C. $f^{-1} = \sqrt[3]{x+3}$
B. $f^{-1} = \sqrt[3]{-x+5} - 2$
D. $f^{-1} = \sqrt[3]{x+7}$

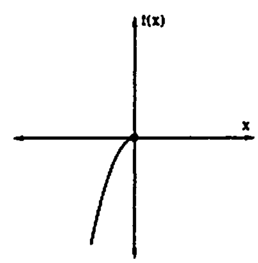
14. The graph of $f(x)$ is shown below.

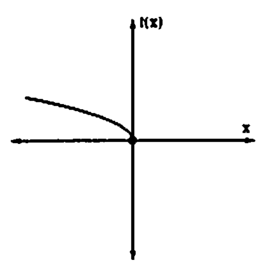


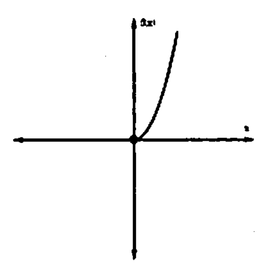
$f^{-1} = \sqrt[3]{x+7}$

Which graph best represents the inverse of $f(x)$?

A.


C.


B.


D.


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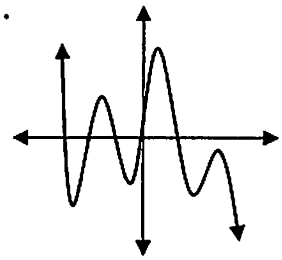
Name: ______________________________ **PRE-CALCULUS REVIEW:** Packet #2

Topic A: Graphs of Polynomial Functions

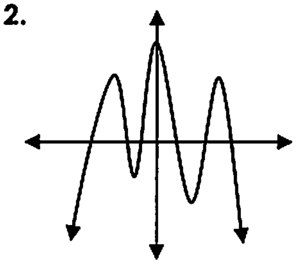
Directions: Given the graph of each polynomial function below, determine the sign of the leading coefficient and whether the function has an even or odd degree.

1.



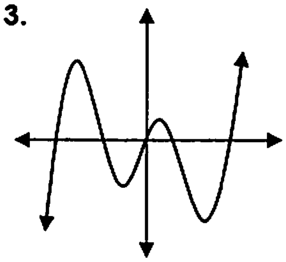
Negative, odd

2.



Negative, Even

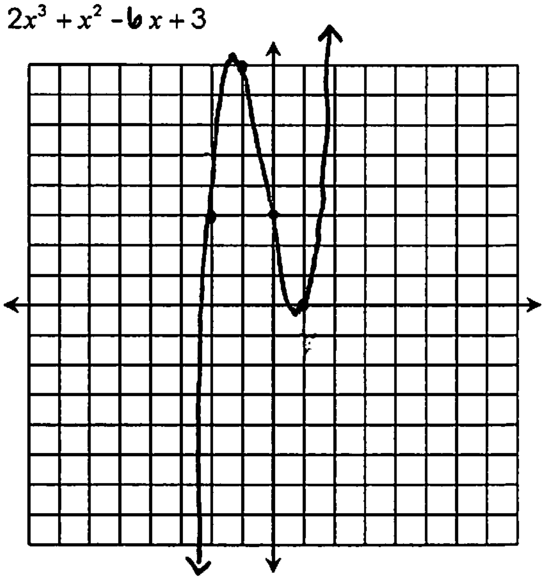
3.



Positive, odd

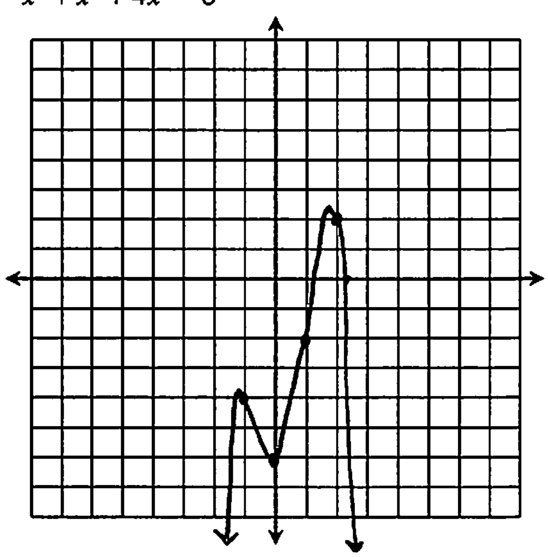
Directions: Graph each function, then identify its key characteristics.

4. $f(x) = 2x^3 + x^2 - 6x + 3$



Domain: $\mathbb{R}$	**Range:** $\mathbb{R}$
***x*-intercept(s):** (-2.19, 0), (0.69, 0), (1, 0)	
***y*-intercept:** (0, 3)	
Rel. Minimum(s): (0.85, -0.15)	
Rel. Maximum(s): (-1.18, 8.19)	
Inc. Interval(s): $(-\infty, -1.18)$, $(0.85, \infty)$	
Dec. Interval(s): (-1.18, 0.85)	
End Behavior: As $x \to \infty$, $f(x) \to \infty$; As $x \to -\infty$, $f(x) \to -\infty$	

5. $f(x) = -x^4 + x^3 + 4x^2 - 6$



Domain: $\mathbb{R}$	**Range:** $\{y \mid y \le 2.31\}$
***x*-intercept(s):** (1.28, 0), (2.25, 0)	
***y*-intercept:** (0, -6)	
Rel. Minimum(s): (0, -6)	
Rel. Maximum(s): (-1.09, -3.95), (1.84, 2.31)	
Inc. Interval(s): $(-\infty, -1.09)$, (0, 1.84)	
Dec. Interval(s): (-1.09, 0), $(1.84, \infty)$	
End Behavior: As $x \to \infty$, $f(x) \to -\infty$; As $x \to -\infty$, $f(x) \to -\infty$	

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Topic B: Zeros of Polynomial Functions

Directions: Find all zeros for each function. Simplify all irrational zeros and complex solutions.

6. $f(x) = 9x^4 + 68x^2 - 32$

$f(x) = (9x^2 - 4)(x^2 + 8)$

$x^2 = \frac{4}{9}$ | $x^2 = -8$

$x = \pm\frac{3}{2}$ | $x = \pm 2i\sqrt{2}$

$\boxed{x = \{\pm\frac{3}{2}, \pm 2i\sqrt{2}\}}$

7. $f(x) = 2x^3 - 11x^2 + 2x + 15$ $\quad \pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

$$\begin{array}{r|rrrr} -1 & 2 & -11 & 2 & 15 \\ & \downarrow & -2 & 13 & -15 \\ \hline & 2 & -13 & 15 & 0 \end{array}$$

$f(x) = (x+1)(2x^2 - 13x + 15)$

$f(x) = (x+1)(2x-3)(x-5)$

$\boxed{x = \{-1, \frac{3}{2}, 5\}}$

8. $f(x) = x^3 - 6x^2 + 15x - 28$ $\quad \pm1, \pm2, \pm4, \pm7, \pm14, \pm28$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 15 & -28 \\ & \downarrow & 4 & -8 & 28 \\ \hline & 1 & -2 & 7 & 0 \end{array}$$

$f(x) = (x-4)(x^2 - 2x + 7)$

$x = 4$ | $x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(7)}}{2(1)}$

$x = \frac{2 \pm \sqrt{-24}}{2}$

$x = \frac{2 \pm 2i\sqrt{6}}{2}$

$x = 1 \pm i\sqrt{6}$

$\boxed{x = \{4, 1 \pm i\sqrt{6}\}}$

9. $f(x) = x^3 - 6x^2 - 12x + 8$ $\quad \pm1, \pm2, \pm4, \pm8$

$$\begin{array}{r|rrrr} -2 & 1 & -6 & -12 & 8 \\ & \downarrow & -2 & 16 & -8 \\ \hline & 1 & -8 & 4 & 0 \end{array}$$

$f(x) = (x+2)(x^2 - 8x + 4)$

$x = -2$ | $x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(4)}}{2(1)}$

$x = \frac{8 \pm \sqrt{48}}{2}$

$x = \frac{8 \pm 4\sqrt{3}}{2}$

$x = 4 \pm 2\sqrt{3}$

$\boxed{x = \{-2, 4 \pm 2\sqrt{3}\}}$

Directions: Identify the zeros, their multiplicities, and describe the effect on the graph.

10. $f(x) = 2x^4 - 24x^3 + 72x^2$

$f(x) = 2x^2(x-6)^2$

Zero	Multiplicity	Effect
0	2	tangent
6	2	tangent

11. $f(x) = x^5 + 3x^4 - 9x^3 - 27x^2$

$f(x) = x^2(x+3)^2(x-3)$

Zero	Multiplicity	Effect
0	2	tangent
-3	2	tangent
3	1	intersect

Topic C: Graphs of Rational Functions

Directions: Graph each function, then identify its key characteristics.

12. $f(x) = \frac{3x^2 + 7x + 2}{x^2 + x - 2}$

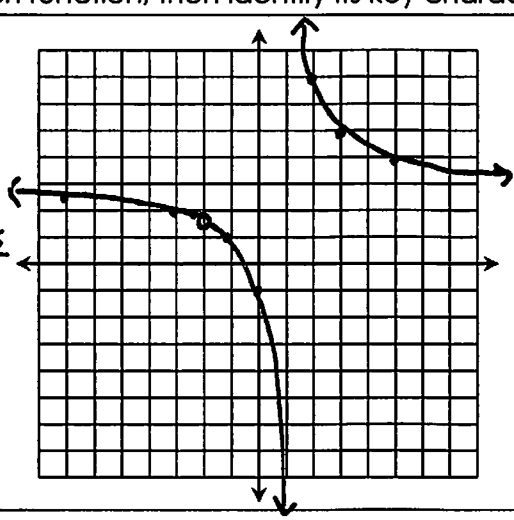
$= \frac{(3x+1)(x+2)}{(x+2)(x-1)}$

$= \frac{3x+1}{x-1}$

Hole $(x=-2)$: $\frac{3(-2)+1}{-2-1} = \frac{5}{3}$

VA: $x-1 = 0$; $x = 1$

HA: $y = \frac{3x}{x}$; $y = 3$



Domain: $\{x \mid x \neq -2, 1\}$

Range: $\{y \mid y \neq 3, 5/3\}$

x-intercept(s): $(-1/3, 0)$

y-intercept: $(0, -1)$

Vertical Asymptote: $x = 1$

Horizontal Asymptote: $y = 3$

Slant Asymptote: —

Hole(s): $(-2, 5/3)$

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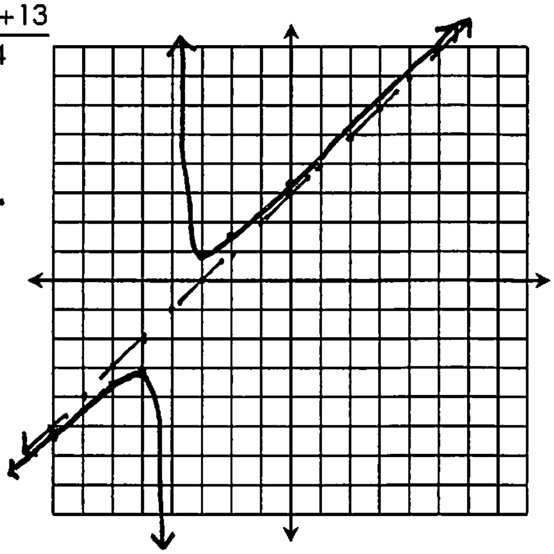
13. $f(x) = \dfrac{x^2+7x+13}{x+4}$

SA:

$$\begin{array}{r|rrr} -4 & 1 & 7 & 13 \\ & \downarrow & -4 & -12 \\ \hline & 1 & 3 & \cancel{1} \end{array}$$

VA: $x+4=0$;
$x=-4$

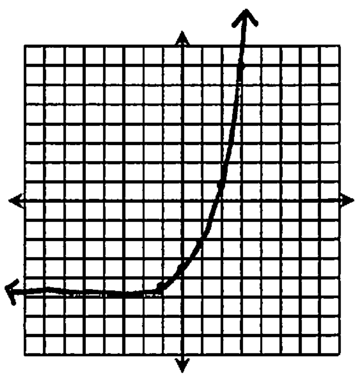


Domain: $\{x \mid x \neq -4\}$	**Range:** $\{y \mid y \leq -3 \text{ or } y \geq 1\}$
***x*-intercept(s):** —	
***y*-intercept:** $(0, 3.25)$	
Vertical Asymptote: $x = -4$	
Horizontal Asymptote: —	
Slant Asymptote: $y = x + 3$	
Hole(s): —	

Topic D: Graphs of Exponential & Logarithmic Functions

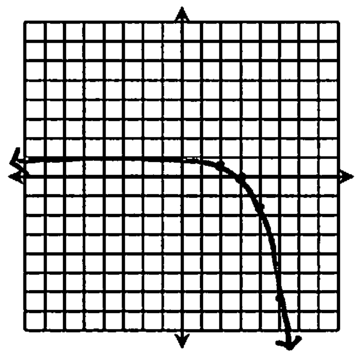
Directions: Graph each function, then identify its key characteristics.

14. $f(x) = \frac{3}{4} \cdot (2)^{x+1} - 5$



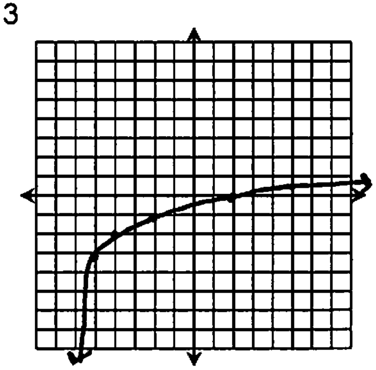
Domain: $\mathbb{R}$	**Range:** $\{y \mid y > -5\}$
***y*-intercept:** $(0, -3.5)$	**Asymptote:** $y = -5$

15. $f(x) = -e^{x-3} + 1$



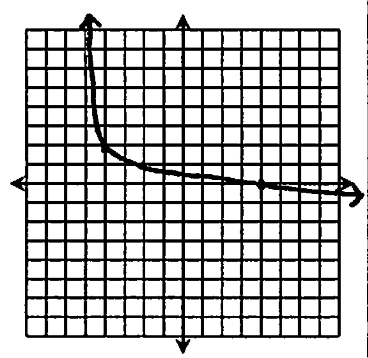
Domain: $\mathbb{R}$	**Range:** $\{y \mid y < 1\}$
***y*-intercept:** $(0, 0.95)$	**Asymptote:** $y = 1$

16. $f(x) = \log_2(x+6) - 3$



Domain: $\{x \mid x > -6\}$	**Range:** $\mathbb{R}$
***x*-intercept:** $(2, 0)$	**Asymptote:** $x = -6$

17. $f(x) = \log_{\frac{1}{3}}(x+5) + 2$



Domain: $\{x \mid x > -5$	**Range:** $\mathbb{R}$
***x*-intercept:** $(4, 0)$	**Asymptote:** $x = -5$

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Topic E: Solving Logarithmic & Exponential Equations

Directions: Solve each equation, rounding to the nearest ten-thousandths place when necessary.

18. $2\cdot\ln(2n)=\ln(n+3)+\ln(3n-2)$

$\ln(2n)^2=\ln(n+3)(3n-2)$

$4n^2=3n^2+7n-6$

$n^2-7n+6=0$

$(n-6)(n-1)=0$

$n=6,\ n=1$

$\boxed{n=\{1,6\}}$

19. $\log_7 3+\log_7(3y^2+8)=2$

$\log_7 3(3y^2+8)=2$

$9y^2+24=49$

$9y^2=25$

$y^2=25/9$

$y=\pm5/3$

$\boxed{y=\{\pm\frac{5}{3}\}}$

20. $\left(\frac{1}{16}\right)^{2x-1}=64^x\cdot\left(\frac{1}{4}\right)^{x+3}$

$4^{-2(2x-1)}=4^{3x}\cdot4^{-1(x+3)}$

$-4x+2=3x-x-3$

$-4x+2=2x-3$

$5=6x$

$x=5/6$

$\boxed{x=\frac{5}{6}}$

21. $9\cdot5^{-4c-3}+5=93$

$9\cdot5^{-4c-3}=88$

$5^{-4c-3}=88/9$

$\frac{(-4c-3)\cdot\log 5}{\log 5}=\frac{\log 88/9}{\log 5}$

$-4c-3=1.4167$

$-4c=4.4167$

$c=-1.1042$

$\boxed{c=-1.1042}$

22. The population of a certain town was 24,250. Fifteen years later, the population decreased to 10,625. If the population followed a continuous exponential decay model, find the rate at which the population decreased.

$10625=24250e^{r\cdot15}$

$.4381=e^{15r}$

$\ln(.4381)=15r$

$-.8252=15r$

$-.055=r$

$\boxed{\text{5.5\% decrease per year}}$

23. Myles was gifted \$3,000 as a graduation present. He deposited half into an account that earns 3.25% interest compounded monthly and the other half into an account that earns 2.5% interest compounded continuously. If Myles does not make any additional deposits or withdrawals, how long will it take for each account to double in value?

$3000=1500\left(1+\frac{.0325}{12}\right)^{12t}$

$2=1.0027^{12t}$

$\frac{\log 2}{\log 1.0027}=\frac{12t\cdot\log 1.0027}{\log 1.0027}$

$256.2777=12t$

$t=21.36$

$3000=1500e^{.025t}$

$2=e^{.025t}$

$\frac{\ln 2}{.025}=\frac{.025t}{.025}$

$t=27.73$

$\boxed{\begin{array}{l}\text{3.25\% account = 22 years}\\ \text{2.5\% account = 28 years}\end{array}}$

24. The number of radio commercials for a product affects the amount of people who purchase the product. Use the table below and a logistic function to determine how many commercials it takes for 17% of people to purchase the product.

Commercials	People (%)
0	1.7
20	4
40	5.2
60	9.1
80	16.4
100	20.7
120	24.3

$f(x)=\frac{28.89}{1+19.39e^{-.04x}}$

$17=\frac{28.89}{1+19.39e^{-.04x}}$

$1+19.39e^{-.04x}=1.70$

$e^{-.04x}=0.04$

$-.04x=\ln .04$

$x=83.06$

$\boxed{\text{84 Commercials}}$

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Pre-Calculus Review

QUIZ 2

Name: ______________________________

Date: ___________________________ **Per:** ________

1. Which statement is true about the graph of the function below?

$$f(x) = -5(x+2)^7(x-3)^4(x-1)^3$$

A. f is increasing at both its left and right ends

B. f is decreasing at both its left and right ends

C. f is increasing at its left end and decreasing at its right end

D. f is decreasing at its left end and increasing at its right end

2. Which statement is true about the graph of the function $f(x) = -3x^4 + 12x^2$?

A. f crosses the x-axis at $x = -2$, $x = 0$, and $x = 2$.

B. f touches, but does not cross the x-axis at $x = -2$ and $x = -2$.

C. f crosses the x-axis at $x = 0$ and touches but does not cross the x-axis at $x = -2$ and $x = 2$.

D. f crosses the x-axis at $x = -2$ and $x = 2$, then touches but does not cross the x-axis at $x = 0$.

3. Which function has the following characteristics?

- It has a relative maximum at (-1, -8).
- It has a y-intercept at (0, -9).
- It has zeros at $x = \{-3, 1\}$.

A. $f(x) = x^4 - 4x^2 - 12x - 9$

B. $f(x) = x^4 - 4x^3 + 4x^2 - 9$

C. $f(x) = x^4 - 4x^3 - 2x^2 + 12x + 9$

D. $f(x) = x^4 + 4x^3 + 4x^2 - 9$

4. What are the zeros of the function below?

$$f(x) = x^3 + 3x^2 - 11x - 21$$

$$\begin{array}{r|rrrr} 3 & 1 & 3 & -11 & -21 \\ & \downarrow & 3 & 18 & 21 \\ \hline & 1 & 6 & 7 & 0 \end{array}$$

$f(x) = (x^2+6x+7)(x-3)$

$x = \frac{-6 \pm \sqrt{6^2-4(1)(7)}}{2(1)}$ | $x = 3$

$= \frac{-6 \pm \sqrt{8}}{2}$

$= \frac{-6 \pm 2\sqrt{2}}{2}$

$= -3 \pm \sqrt{2}$

A. $\{3, -3 \pm \sqrt{2}\}$

B. $\{-3, 3 \pm \sqrt{2}\}$

C. $\{3, -2 \pm 3\sqrt{2}\}$

D. $\{-3, 2 \pm 3\sqrt{2}\}$

5. What are the zeros of the function below?

$$f(x) = x^4 + x^2 - 72$$

$= (x^2+9)(x^2-8)$

$x^2 = -9$ | $x^2 = 8$

$x = \pm 3i$ | $x = \pm 2\sqrt{2}$

A. $\{\pm 2\sqrt{2}, \pm 3\}$

B. $\{\pm 2i\sqrt{2}, \pm 3i\}$

C. $\{\pm 2i\sqrt{2}, \pm 3\}$

D. $\{\pm 2\sqrt{2}, \pm 3i\}$

6. Which function has an x-intercept at (1, 0) and a horizontal asymptote of $y = -4/3$?

HA: $y = \frac{4x^2}{-3x^2} = -\frac{4}{3}$

$f(1) = \frac{4(1)^2 - 3(1) - 1}{-3(1)^2 - 3(1)} = \frac{0}{-6} = 0$

A. $f(x) = \dfrac{8x^2 + 5x - 3}{-6x^2 + 6x}$

B. $f(x) = \dfrac{4x^2 - 3x - 1}{-3x^2 - 3x}$

C. $f(x) = \dfrac{3x^2 + 7x + 4}{-4x^2 + 4x}$

D. $f(x) = \dfrac{12x^2 - 13x - 4}{-9x^2 + 9x}$

7. If the function below has a y-intercept at (0, -4) and a vertical asymptote at $x = 2$, what are the values of a and b?

$$f(x) = \frac{-16}{ax + b}$$

A. $a = 2$ and $b = -4$

B. $a = -2$ and $b = 4$

C. $a = -4$ and $b = 2$

D. $a = -2$ and $b = -4$

$f(x) = \frac{-16}{-2x+4}$

VA: $-2x + 4 = 0$

$x = 2$

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8. Which characteristics describes the graph of the function below?

$$f(x)=\frac{x^2-3x-4}{x-2}$$

$$\begin{array}{r|rrr} 2 & 1 & -3 & -4 \\ & \downarrow & 2 & -2 \\ \hline & 1 & -1 & \cancel{-6} \end{array}$$

A. There are zeros at $x = -4$ and $x = 1$

B. The y-intercept is (0, -2).

C. There is a vertical asymptote at $x = -1$.

D. There is a slant asymptote at $y = x - 1$.

9. Which statement is true regarding the two functions below?

VA: $x=4$ HA: $y=-4$

FUNCTION A	FUNCTION B
$f(x)=\log_3(x-4)-1$	$f(x)=2^{x-1}-4$

A. Function A has a vertical asymptote at $x = 4$ and Function B has a horizontal asymptote at $y = -4$.

B. Function A has a vertical asymptote at $x = -4$ and Function B has a horizontal asymptote at $y = 4$.

C. Function A has a vertical asymptote at $x = 1$ and Function B has a horizontal asymptote at $y = -1$.

D. The functions are inverses of each other.

10. Which function has the characteristics below?

- $f(x)$ has an x-intercept at (2, 0).
- As x approaches ∞, $f(x)$ approaches ∞.

A. $f(x)=\left(\frac{1}{2}\right)^{x-3}-2$

B. $f(x)=\ln(x-1)$

C. $f(x)=2^{x+2}-1$

D. $f(x)=\log_2(x+3)$

11. Find the solution to the equation below.

$$2\cdot\log(k-1)=\frac{1}{3}\log 8+\log(3k+5)$$

$(k-1)^2 = 2(3k+5)$

$k^2-2k+1 = 6k+10$

$k^2-8k-9=0$

$(k-9)(k+1)=0$

$k=9, \cancel{-1}$

A. $k=\{1, 9\}$

B. $k=\{-1, 9\}$

C. $k=9$

D. $k=1$

12. Find the solution to the equation below.

$$81^{5-3x}-27^x=0$$

$3^{4(5-3x)} = 3^{3x}$

$20-12x = 3x$

$20 = 15x$

$\frac{4}{3} = x$

A. 3

B. $\frac{16}{3}$

C. $\frac{4}{3}$

D. $\frac{3}{5}$

13. Which value is the closest to the solution of the equation below?

$$13^{6-2p}+5=85$$

$13^{6-2p} = 80$

$(6-2p)\cdot\log 13 = \log 80$

$6-2p = 1.7084$

$-2p = -4.2916$

$p = 2.1458$

A. 1.8147

B. 1.8695

C. 1.9722

D. 2.1458

14. Finn inherited $8,000 from his grandparents. He placed the money in an account that earns 6.75% interest compounded semiannually. If he makes no other deposits or withdrawals, how much interest will he have earned after 5 years?

$= 8000\left(1+\frac{.0675}{2}\right)^{2\cdot 5}$

$= 11{,}149.24$

A. $2,731.27

B. $2,895.92

C. $3,015.78

D. $3,149.24

15. In 2012, Jensyn deposited $500 into an account that earns 4% interest compounded continuously. If she makes no other deposits or withdrawals, in what year can she expect to triple her investment?

$1500 = 500e^{.04t}$

$3 = e^{.04t}$

$\ln 3 = .04t$

$t = 27.47 \rightarrow 2039$

A. 2038

B. 2039

C. 2040

D. 2041

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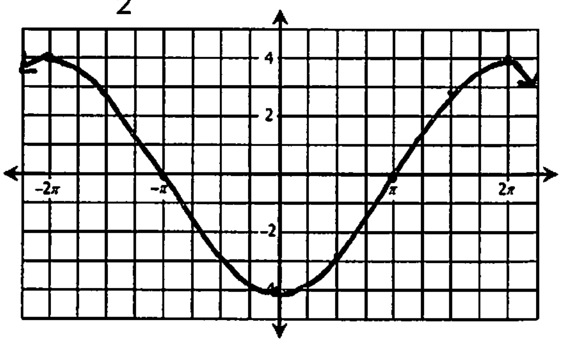
Name: ______________________________ **PRE-CALCULUS REVIEW:** Packet #3

Topic A: Graphing Trigonometric Functions

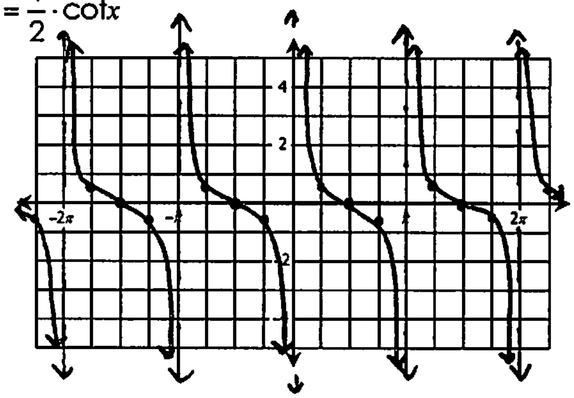
Directions: Graph each function and identify its key characteristics.

1. $f(x) = -4 \cdot \cos\frac{1}{2}x$



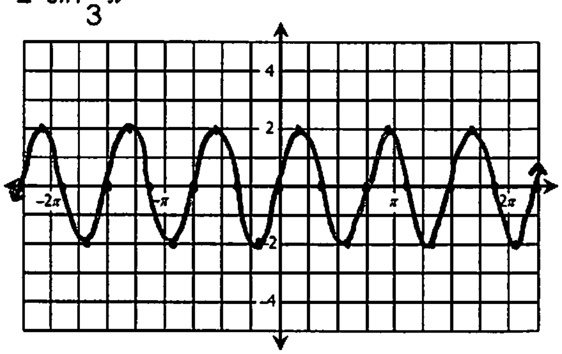
Amplitude:	Period:
4	$\frac{2\pi}{1/2} = 4\pi$

2. $f(x) = \frac{1}{2} \cdot \cot x$



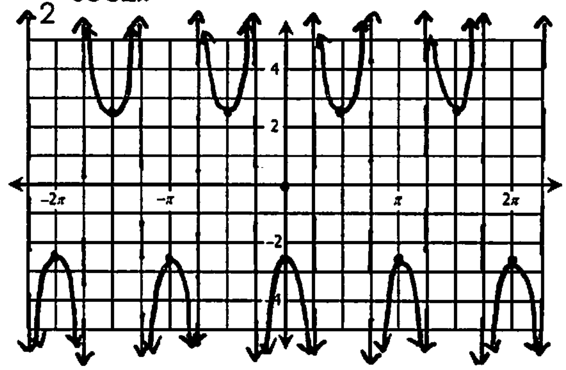
Amplitude:	Period:
undefined	π

3. $f(x) = 2 \cdot \sin\frac{8}{3}x$



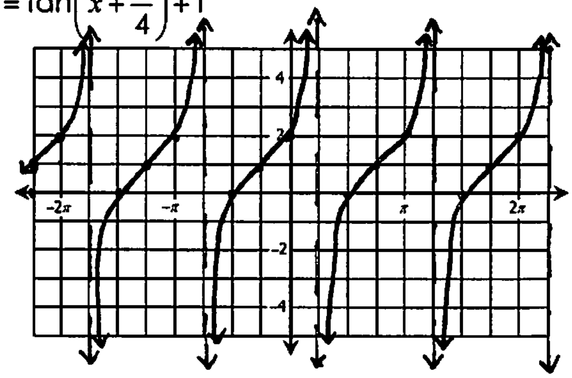
Amplitude:	Period:
2	$\frac{2\pi}{8/3} = \frac{3\pi}{4}$

4. $f(x) = -\frac{5}{2} \cdot \sec 2x$



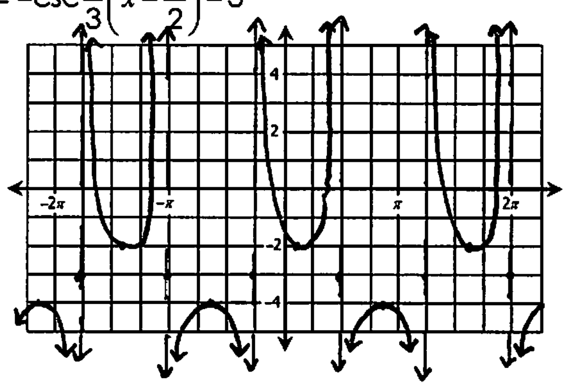
Amplitude:	Period:
undefined	$\frac{2\pi}{2} = \pi$

5. $f(x) = \tan\left(x + \frac{\pi}{4}\right) + 1$



Amplitude:	Period:
undefined	π
Phase Shift:	**Vertical Shift:**
Left $\pi/4$	Up 1

6. $f(x) = -\csc\frac{4}{3}\left(x - \frac{\pi}{2}\right) - 3$



Amplitude:	Period:
undefined	$\frac{2\pi}{4/3} = \frac{3\pi}{2}$
Phase Shift:	**Vertical Shift:**
Right $\pi/2$	Down 3

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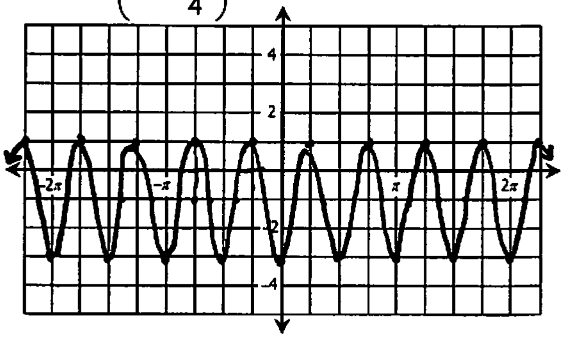
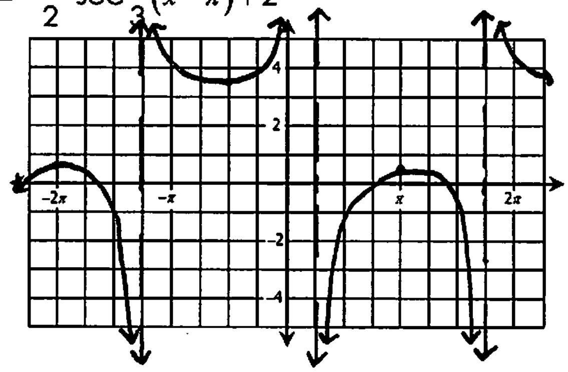
7. $f(x) = 2\cdot\cos 4\left(x+\frac{3\pi}{4}\right)-1$



Amplitude:	Period:
2	$\frac{2\pi}{4} = \frac{\pi}{2}$
Phase Shift:	**Vertical Shift:**
Left $\frac{3\pi}{4}$	Down 1

8. $f(x) = -\frac{3}{2}\cdot\sec\frac{2}{3}(x-\pi)+2$



Amplitude:	Period:
Undefined	$\frac{2\pi}{2/3} = 3\pi$
Phase Shift:	**Vertical Shift:**
Right π	Up 2

Topic B: Inverses & Compositions of Trigonometric Functions

Directions: Give the exact value, if it exists.

9. arcsin (-1) $-\frac{\pi}{2}$	**10.** $\tan^{-1}(-\sqrt{3})$ $-\frac{\pi}{3}$	**11.** arccos (-2) DNE
12. $\arctan\left(\tan\frac{\pi}{2}\right)$ DNE	**13.** $\sin^{-1}\left(\cos\frac{7\pi}{6}\right)$ $-\frac{\pi}{3}$	**14.** cos (arctan 0) 1

Topic C: Using Identities to Find Exact Values

Directions: Find each exact value.

15. If $\cos\theta = \frac{1}{3}$ and $0 < \theta < \frac{\pi}{2}$, find $\cos 2\theta$.

$\cos 2\theta = 2(1/3)^2 - 1$

$= 2(1/9) - 1$

$= \frac{2}{9} - 1 = \boxed{-\frac{7}{9}}$

16. If $\tan\beta = -\frac{5\sqrt{7}}{7}$ and $\frac{\pi}{2} < \beta < \pi$, find $\sin 2\beta$.

$(5\sqrt{7})^2 + 7^2 = x^2$

$175 + 49 = x^2$

$224 = x^2$

$x = 4\sqrt{14}$

$\sin\theta = \frac{5\sqrt{2}}{8}$

$\cos\theta = -\frac{\sqrt{14}}{8}$

$\sin 2\theta = 2\left(\frac{5\sqrt{2}}{8}\right)\left(-\frac{\sqrt{14}}{8}\right)$

$= \frac{-10\sqrt{28}}{64}$

$= \frac{-20\sqrt{7}}{64}$

$= \boxed{\frac{-5\sqrt{7}}{16}}$

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17. $\sin 202.5°$ $\rightarrow \sin\left(\frac{405}{2}\right)$

$= -\sqrt{\frac{1-\cos 405}{2}}$ $= -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$

$= -\sqrt{\frac{2-\sqrt{2}}{4}}$

$= \boxed{-\frac{\sqrt{2-\sqrt{2}}}{2}}$

18. $\tan\frac{5\pi}{12}$ $\rightarrow \tan\left(\frac{5\pi/6}{2}\right)$

$= \frac{\sin\frac{5\pi}{6}}{1+\cos\frac{5\pi}{6}}$ $= \frac{\frac{1}{2}}{1+\left(-\frac{\sqrt{3}}{2}\right)}$

$= \frac{\frac{1}{2}}{\frac{2-\sqrt{3}}{2}}$

$= \frac{1}{2-\sqrt{3}}\frac{(2+\sqrt{3})}{(2+\sqrt{3})} = \frac{2+\sqrt{3}}{1} = \boxed{2+\sqrt{3}}$

19. If $\sin y = \frac{2\sqrt{6}}{5}$ and $\frac{\pi}{2} < y < \pi$, find $\cos\frac{y}{2}$.

$(2\sqrt{6})^2 + x^2 = 5^2$

$24 + x^2 = 25$

$x^2 = 1$

$x = 1$

$\cos x = \frac{1}{5}$

$\hookrightarrow \frac{\pi}{4} < \frac{y}{2} < \frac{\pi}{2}$ (Q1)

$\cos\frac{y}{2} = \sqrt{\frac{1+1/5}{2}}$

$= \sqrt{\frac{6/5}{2}}$

$= \frac{\sqrt{3}}{\sqrt{5}} = \boxed{\frac{\sqrt{15}}{5}}$

20. If $\tan\alpha = -\frac{\sqrt{3}}{3}$ and $\frac{3\pi}{2} < \alpha < 2\pi$, find $\tan\frac{\alpha}{2}$.

$(\sqrt{3})^2 + 3^2 = x^2$

$3+9 = x^2$

$12 = x^2$

$2\sqrt{3} = x$

$\sin\alpha = \frac{\sqrt{3}}{2\sqrt{3}} = -\frac{1}{2}$

$\cos\alpha = -\frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\hookrightarrow \frac{3\pi}{4} < \frac{\alpha}{2} < \pi$

$\tan\frac{\alpha}{2} = \frac{1-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$

$= \frac{\frac{2-\sqrt{3}}{2}}{-\frac{1}{2}}$

$= \boxed{-2+\sqrt{3}}$

21. $\cos 75°$ $\rightarrow \cos(45+30)$

$= \cos 45 \cdot \cos 30 - \sin 45 \cdot \sin 30$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

22. $\tan\frac{11\pi}{12}$ $\rightarrow \tan(225-60)$

$= \frac{\tan 225 - \tan 60}{1+\tan 225 \cdot \tan 60}$ $= \frac{1-\sqrt{3}}{1+1\cdot\sqrt{3}}$

$= \frac{1-\sqrt{3}}{1+\sqrt{3}}\frac{(1-\sqrt{3})}{(1-\sqrt{3})}$

$= \frac{4-2\sqrt{3}}{-2}$

$= \boxed{-2+\sqrt{3}}$

23. $3\left(\sin\frac{31\pi}{12} + \sin\frac{13\pi}{12}\right)$

$= 3\left[2\cdot\sin\left(\frac{\frac{31\pi}{12}+\frac{13\pi}{12}}{2}\right)\cdot\cos\left(\frac{\frac{31\pi}{12}-\frac{13\pi}{12}}{2}\right)\right.$

$= 6\sin\frac{11\pi}{6}\cdot\cos\frac{3\pi}{4}$

$= 6\left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{6\sqrt{2}}{4} = \boxed{\frac{3\sqrt{2}}{2}}$

24. $\cos\frac{13\pi}{12} - \cos\frac{7\pi}{12}$

$= -2\cdot\sin\left(\frac{\frac{13\pi}{12}+\frac{7\pi}{12}}{2}\right)\cdot\sin\left(\frac{\frac{13\pi}{12}-\frac{7\pi}{12}}{2}\right)$

$= -2\sin\frac{5\pi}{6}\cdot\sin\frac{\pi}{4}$

$= -2\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{-\frac{\sqrt{2}}{2}}$

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25. $\cos\frac{3\pi}{4}\cdot\sin\frac{\pi}{12}$ $=\frac{1}{2}\left[\sin\left(\frac{3\pi}{4}+\frac{\pi}{12}\right)-\sin\left(\frac{3\pi}{4}-\frac{\pi}{12}\right)\right]$ $=\frac{1}{2}\left[\sin\frac{5\pi}{6}-\sin\frac{2\pi}{3}\right]$ $=\frac{1}{2}\left[\frac{1}{2}-\frac{\sqrt{3}}{2}\right]=\frac{1}{2}\left[\frac{1-\sqrt{3}}{2}\right]=\boxed{\frac{1-\sqrt{3}}{4}}$	**26.** $4\cos\frac{\pi}{12}\cdot\cos\frac{\pi}{12}$ $=4\cdot\frac{1}{2}\left[\cos\left(\frac{\pi}{12}-\frac{\pi}{12}\right)+\cos\left(\frac{\pi}{12}+\frac{\pi}{12}\right)\right]$ $=2\left[\cos 0+\cos\frac{\pi}{6}\right]$ $=2\left[1+\frac{\sqrt{3}}{2}\right]=2\left[\frac{2+\sqrt{3}}{2}\right]=\boxed{2+\sqrt{3}}$

Topic D: Simplifying Trigonometric Expressions

Directions: Simplify each trigonometric expression.

27. $\frac{\cos^2 A-\cos^2 A\cdot\sec^2 A}{\sin^2 A}$ $=\frac{\cos^2 A(1-\sec^2 A)}{\sin^2 A}$ $=\frac{\cos^2 A}{\sin^2 A}\cdot -\tan^2 A$ $=\cot^2 A\cdot -\tan^2 A=\boxed{-1}$	**28.** $\frac{(\cot\beta+\csc\beta)(\cot\beta-\csc\beta)}{\csc\beta}$ $=\frac{\cot^2\beta-\csc^2\beta}{\csc\beta}$ $=\frac{-1}{\csc\beta}=\boxed{-\sin\beta}$
29. $\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}$ $=\frac{(1+\sin x)^2}{\cos x(1+\sin x)}+\frac{\cos^2 x}{\cos x(1+\sin x)}$ $=\frac{1+2\sin x+\sin^2 x+\cos^2 x}{\cos x(1+\sin x)}$ $=\frac{2+2\sin x}{\cos x(1+\sin x)}=\frac{2(1+\sin x)}{\cos x(1+\sin x)}=\frac{2}{\cos x}=\boxed{2\sec x}$	**30.** $\frac{\sec\theta\cdot\sin\theta}{\cot\theta+\tan\theta}=\frac{\frac{1}{\cos\theta}\cdot\sin\theta}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}$ $=\frac{\tan\theta}{\frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}}$ $=\frac{\tan\theta}{\frac{1}{\sin\theta\cos\theta}}$ $=\frac{\sin\theta}{\cos\theta}\cdot\sin\theta\cos\theta=\boxed{\sin^2\theta}$

Topic E: Solving Trigonometric Equations

Directions: Solve each equation for the given interval.

31. $10\sin\alpha\cdot\cos\alpha-2\sqrt{3}\sin\alpha=-7\sqrt{3}\sin\alpha;\ [0,\pi]$ $10\sin\alpha\cos\alpha+5\sqrt{3}\sin\alpha=0$ $5\sin\alpha(2\cos\alpha+\sqrt{3})=0$ $5\sin\alpha=0$; $2\cos\alpha=-\sqrt{3}$ $\sin\alpha=0$; $\cos\alpha=-\frac{\sqrt{3}}{2}$ $\alpha=0,\pi$; $\alpha=\frac{5\pi}{6}$ $\boxed{\alpha=0,\frac{5\pi}{6},\pi}$	**32.** $5\csc^2\theta-5=5\cot\theta;\ [\pi,2\pi]$ $5(\cot^2\theta+1)-5=5\cot\theta$ $5\cot^2\theta+5-5=5\cot\theta$ $5\cot^2\theta-5\cot\theta=0$ $5\cot\theta(\cot\theta-1)=0$ $\cot\theta=0$; $\cot\theta=1$ $\theta=\frac{3\pi}{2}$; $\theta=\frac{5\pi}{4}$ $\boxed{\theta=\frac{5\pi}{4},\frac{3\pi}{2}}$

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33. $\cos 2y + 3\cos^2 y - 1 = \cos^2 y + 1$; $[0, 2\pi]$

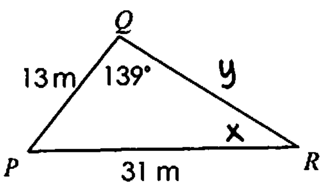
$$2\cos^2 y - 1 + 3\cos^2 y - 1 = \cos^2 y + 1$$
$$5\cos^2 y - 2 = \cos^2 y + 1$$
$$4\cos^2 y = 3$$
$$\cos^2 y = 3/4$$
$$\cos y = \pm\frac{\sqrt{3}}{2}$$
$$\boxed{y = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

34. $\sin 3x + \sin x = \cos x$; $\left[0, \frac{\pi}{2}\right]$

$$2 \cdot \sin\left(\frac{3x+x}{2}\right) \cdot \cos\left(\frac{3x-x}{2}\right) = \cos x$$
$$2 \cdot \sin 2x \cdot \cos x = \cos x$$
$$2\sin 2x \cos x - \cos x = 0$$
$$\cos x \,(2\sin 2x - 1) = 0$$
$$\cos x = 0 \qquad \sin 2x = 1/2$$
$$x = \frac{\pi}{2} \qquad 2x = \frac{\pi}{6}, \frac{5\pi}{6}$$
$$x = \frac{\pi}{12}, \frac{5\pi}{12}$$
$$\boxed{x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{\pi}{2}}$$

Topic F: Law of Sines & Cosines

35. Solve the triangle below. Round all answers to the nearest tenth.

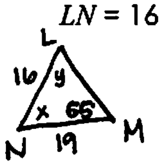


$$\frac{\sin 139}{31} = \frac{\sin x}{13}$$
$$\frac{31\sin x}{31} = \frac{13\sin 139}{31}$$
$$\sin x = .2751$$
$$x = 16$$

$$\frac{\sin 139}{31} = \frac{\sin 25}{y}$$
$$\frac{y\sin 139}{\sin 139} = \frac{31\sin 25}{\sin 139}$$
$$y = 20$$

QR =	20
$m\angle P$ =	25°
$m\angle R$ =	16°

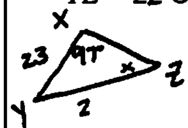
36. In ΔLMN, if $m\angle M = 55°$, $MN = 19$ in, and $LN = 16$ in, find $m\angle N$.



$$\frac{\sin 55}{16} = \frac{\sin y}{19}$$
$$\frac{16\sin y}{16} = \frac{19\sin 55}{16}$$
$$\sin y = .9727$$
$$y = 76.6°, 103.4°$$

$\boxed{m\angle N = 48.4° \text{ or } 21.6°}$

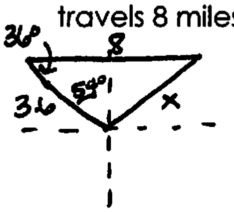
37. In ΔXYZ, if $m\angle X = 97°$, $XY = 23$ cm, and $YZ = 22$ cm, find $m\angle Z$.



$$\frac{\sin 97}{22} = \frac{\sin x}{23}$$
$$\frac{23\sin 97}{22} = \frac{22\sin x}{22}$$
$$1.0377 = \sin x$$
$$x = \emptyset$$

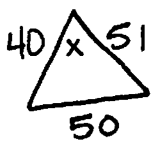
$\boxed{\text{No Solution}}$

38. A fox leaves his den and travels N 54° W. After traveling 3.6 miles, he changes direction and travels 8 miles due east before stopping. How far is the fox from his den?



$$x^2 = 3.6^2 + 8^2 - 2(3.6)(8)\cos 36$$
$$x^2 = 76.96 - 57.6\cos 36$$
$$x^2 = 30.3606$$
$$\boxed{x = 5.5 \text{ mi}}$$

39. Ed is staining his triangular deck. If the sides of the deck measure 40 feet, 51 feet, and 50 feet, and each can of stain covers 175 square feet, determine how many cans Ed will need.



$$50^2 = 40^2 + 51^2 - 2(40)(51)\cos x$$
$$2500 = 4201 - 4080\cos x$$
$$-1701 = -4080\cos x$$
$$.4169 = \cos x$$
$$x = 65.4°$$

$$A = 1/2(40)(51)(\sin 65.4)$$
$$= 927.42 \text{ ft}^2$$

$$\frac{927.42}{175} = 5.3$$

$\boxed{\text{6 cans of Stain}}$

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Pre-Calculus Review

QUIZ 3

Name: ______________________________

Date: ________________________ Per: ________

1. Which function has a period of 2π and an amplitude of 4?

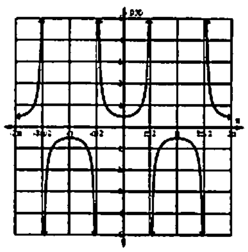
A. $f(x) = 4 \cdot \cos x$

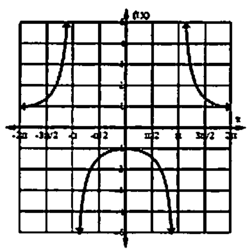
B. $f(x) = 4 \cdot \cos 2x$

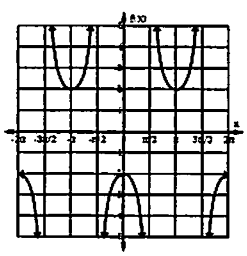
C. $f(x) = 2 \cdot \sin 4x$

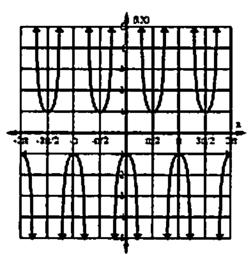
D. $f(x) = 4 \cdot \sin \frac{1}{2}x$

2. The graph of $f(x) = \sec x$ undergoes a reflection about the x-axis and a horizontal stretch by a factor of 2. Which graph shows this transformed function?

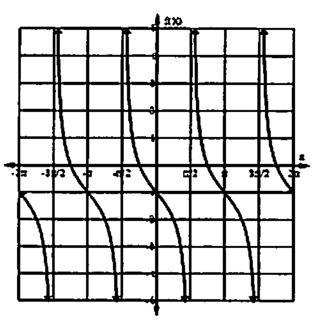
A.


C.


B.


D.


3. Which function is shown on the graph below?



A. $f(x) = \cot\left(x + \frac{\pi}{4}\right) - 1$

B. $f(x) = \cot\left(x + \frac{\pi}{2}\right) - 1$

C. $f(x) = -\tan\left(x + \frac{\pi}{2}\right) - 1$

D. $f(x) = -\tan\left(x + \frac{\pi}{4}\right) - 1$

4. The graph of the function $f(x) = \sin x$ undergoes a vertical stretch by a factor of 3, a phase shift of $\pi/4$ radians to the right, then a horizontal compression by a factor of ½. Which statement is true regarding the transformed function?

A. The y-intercept is (0, 3).

B. The function intersects the x-axis at $\left(\frac{\pi}{4} + \frac{k\pi}{2}, 0\right)$ where k is an integer.

C. The period is 2π.

D. The amplitude is 2.

5. Find the exact value of the expression below.

$$\sin\left[\arctan\left(-\frac{\sqrt{3}}{3}\right)\right]$$

A. $\frac{\sqrt{3}}{2}$

B. $-\frac{\sqrt{3}}{2}$

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

6. If $\sec x = -\frac{5}{2}$ and $\csc x > 0$, find the exact value of $\tan x$.

$\cos x = \frac{-2}{5}$

$2^2 + x^2 = 5^2$

$x = \sqrt{21}$

A. $\frac{\sqrt{21}}{2}$

B. $-\frac{\sqrt{21}}{2}$

C. $\frac{5\sqrt{21}}{21}$

D. $-\frac{5\sqrt{21}}{21}$

7. If $\cos\theta = -\frac{8}{17}$ and $\pi \le \theta < \frac{3\pi}{2}$, find the exact value of $\tan 2\theta$.

$8^2 + x^2 = 17^2$

$x = 15$

$\tan\theta = \frac{15}{8}$

$\frac{2(15/8)}{1 - (15/8)^2} = \frac{15/4}{-161/64} = \frac{-240}{161}$

A. $-\frac{289}{161}$

B. $-\frac{161}{289}$

C. $-\frac{240}{161}$

D. $-\frac{161}{240}$

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8. If $\tan A = -2\sqrt{6}$ and $\frac{3\pi}{2} \le A < 2\pi$, find the exact value of $\sin\frac{A}{2}$.

$= \sqrt{\frac{1-1/5}{2}} = \frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$

A. $\frac{2\sqrt{5}}{5}$

B. $-\frac{2\sqrt{5}}{5}$

C. $\frac{\sqrt{10}}{5}$

D. $-\frac{\sqrt{10}}{5}$

9. Find the exact value of $\tan\frac{19\pi}{12}$. $\rightarrow \tan\left(\frac{5\pi}{4}+\frac{\pi}{3}\right)$

$= \frac{1+\sqrt{3}}{1-(1)(\sqrt{3})} = \frac{1+\sqrt{3}}{1-\sqrt{3}} \frac{(1+\sqrt{3})}{(1+\sqrt{3})}$

$= \frac{4+2\sqrt{3}}{-2}$

$= -2-\sqrt{3}$

A. $-2-\sqrt{3}$

B. $2+\sqrt{3}$

C. $\sqrt{3}-2$

D. $2-\sqrt{3}$

10. Find the exact value of the expression below.

$$\sin\frac{3\pi}{4}\cdot\cos\frac{\pi}{12}$$

$= \frac{1}{2}\left[\sin\left(\frac{3\pi}{4}+\frac{\pi}{12}\right) + \sin\left(\frac{3\pi}{4}-\frac{\pi}{12}\right)\right] = \frac{1}{2}\left[\frac{1}{2}+\frac{\sqrt{3}}{2}\right]$

A. $\frac{-1+\sqrt{3}}{4}$

B. $\frac{-1-\sqrt{3}}{4}$

C. $\frac{1-\sqrt{3}}{4}$

D. $\frac{1+\sqrt{3}}{4}$

11. Find all solutions to the equation below on the given interval.

$$\cos x\cdot\sec x = \sqrt{2}\cos x;\ [\pi, 2\pi)$$

$1 = \sqrt{2}\cos x$

$\frac{\sqrt{2}}{2} = \cos x$

A. $\left\{\frac{5\pi}{4}\right\}$

B. $\left\{\frac{7\pi}{4}\right\}$

C. $\left\{\frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

D. $\left\{\frac{4\pi}{3}, \frac{7\pi}{3}\right\}$

12. Find all solutions to the equation below on the given interval.

$$3\cos^2\theta + 2 - \sin^2\theta = -4\cos\theta;\ [0, 2\pi)$$

$3\cos^2\theta + 2 - (1-\cos^2\theta) = -4\cos\theta$

$(2\cos\theta+1)(2\cos\theta+1) = 0$

A. $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

B. $\left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

C. $\left\{\frac{5\pi}{6}, \frac{7\pi}{6}\right\}$

D. $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

13. Find all solutions to the equation below on the given interval.

$$3\sin u = -4\sin^2 u - \cos 2u;\ [0, 2\pi)$$

$3\sin u = -4\sin^2 u - (1-2\sin^2 u)$

$(2\sin u+1)(\sin u+1) = 0$

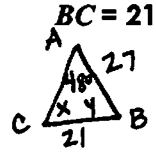
A. $\left\{\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$

B. $\left\{\frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}\right\}$

C. $\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}\right\}$

D. $\left\{\frac{3\pi}{4}, \pi, \frac{5\pi}{4}\right\}$

14. In ΔABC, $m\angle A = 48°$, $AB = 27$ feet, and $BC = 21$ feet. Which statement is true?



$\frac{\sin 48}{21} = \frac{\sin x}{27}$

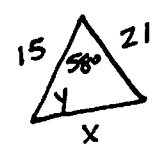
A. No triangle exists with these measurements.

B. $m\angle B$ can be 24.8° or 107.2°.

C. $m\angle B$ can be 59.2° or 24.8°.

D. $m\angle C$ can only be 24.8°.

15. A triangular greenspace is formed by three roads in a neighborhood: Derby Lane, Harvest Drive, and Cary Drive. The length of the greenspace along Derby Lane and Harvest Drive is 15 meters and 21 meters, respectively. If the angle formed by Derby Lane and Harvest Drive is 58°, find the angle formed by Derby Lane and Cary Drive.



A. 44.3°

B. 49.5°

C. 77.7°

D. 65.3°

$x^2 = 15^2 + 21^2 - 2(15)(21)\cos 58$

$x = 18.23$

$21^2 = 15^2 + 18.23^2 - 2(15)(18.23)\cos y$

$y = 77.7°$

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Name: ______________________________ **PRE-CALCULUS REVIEW:** Packet #4

Topic A: Vectors & Vector Operations

Directions: Use the initial and terminal points of the vectors below to give the component form, linear combination using standard vectors **i** and **j**, magnitude, and direction angle for each angle.

1. $\overline{CD}$ with $C(-7, 5)$ and $D(-5, 4)$

$\|\overrightarrow{CD}\| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$

$\tan\theta = -\frac{1}{2}$; $\theta = 333.43°$

Component Form:	Linear Combination:
$\langle 2, -1 \rangle$	$2i - j$
Magnitude:	**Direction Angle:**
$\sqrt{5}$ (2.24)	333.43°

2. $\overline{PQ}$ with $P(-1, -1)$ and $Q(5, 3)$

$\|\overrightarrow{PQ}\| = \sqrt{6^2 + 4^2} = \sqrt{52} = 2\sqrt{13}$

$\tan\theta = \frac{4}{6}$; $\theta = 33.69°$

Component Form:	Linear Combination:
$\langle 6, 4 \rangle$	$6i + 4j$
Magnitude:	**Direction Angle:**
$2\sqrt{13}$ (7.21)	33.69°

Directions: Find each of the following for $\mathbf{a} = \langle 8, 0 \rangle$, $\mathbf{b} = \langle -3, 7 \rangle$, and $\mathbf{c} = \langle 6, -9 \rangle$.

3. $\mathbf{b} - \mathbf{a}$

$\langle -3, 7 \rangle - \langle 8, 0 \rangle$

$= \boxed{\langle -11, 7 \rangle}$

4. $2\mathbf{a} + 3\mathbf{c}$

$2\langle 8, 0 \rangle + 3\langle 6, -9 \rangle$

$= \boxed{\langle 34, -27 \rangle}$

5. $-\frac{1}{3}\mathbf{c} - \frac{5}{2}\mathbf{a}$

$-\frac{1}{3}\langle 6, -9 \rangle - \frac{5}{2}\langle 8, 0 \rangle$

$= \boxed{\langle -22, 3 \rangle}$

Directions: Find the component form of each vector with the given magnitude and direction angle.

6. $\|\mathbf{k}\| = 14$, $\theta = 225°$

$k = \langle 14\cos 225°, 14\sin 225° \rangle$

$= \langle 14\left(-\frac{\sqrt{2}}{2}\right), 14\left(-\frac{\sqrt{2}}{2}\right) \rangle$

$= \boxed{\langle -7\sqrt{2}, -7\sqrt{2} \rangle}$

7. $\|\mathbf{q}\| = 5$, $\theta = 110°$

$q = \langle 5\cos 110°, 5\sin 110° \rangle$

$= \boxed{\langle -1.71, 4.70 \rangle}$

Directions: Convert each vector to trigonometric form.

8. $\mathbf{z} = 5\mathbf{i} - 5\mathbf{j}$

$\|z\| = \sqrt{5^2 + (-5)^2} = \sqrt{50} = \boxed{5\sqrt{2}}$

$\tan\theta = -5/5 = -1$; $\theta = 315°$

$z = \langle 5\sqrt{2}\cos 315°, 5\sqrt{2}\sin 315° \rangle$

$\boxed{z = 5\sqrt{2}\langle \cos 315°, \sin 315° \rangle}$

9. $\overline{EF}$ with $E(3, 1)$ and $F(2, -2)$ $\langle -1, -3 \rangle$

$\|\overrightarrow{EF}\| = \sqrt{(-1)^2 + (-3)^2} = \sqrt{10}$

$\tan\theta = -3/-1 = 3$; $\theta = 251.57°$

$\boxed{\overrightarrow{EF} = \sqrt{10}\langle \cos 251.57°, \sin 251.57° \rangle}$

Topic B: Dot Products & Angle Between Vectors

Directions: Find the dot product of the given vectors, then determine whether the vectors are orthogonal.

10. $\mathbf{c} = \langle 6, 4 \rangle$ and $\mathbf{d} = \langle -2, 3 \rangle$

$c \cdot d = 6(-2) + 4(3)$

$= \boxed{0}$

Orthogonal

11. $\mathbf{p} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{q} = -12\mathbf{i} + 9\mathbf{j}$

$p \cdot q = 3(-12) - 4(9)$

$= \boxed{-72}$

not orthogonal

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Directions: Find the angle between each vector pair.

12. $\mathbf{x} = \langle -7, 2 \rangle$ and $\mathbf{y} = \langle 5, 6 \rangle$

$$\cos\theta = \frac{-7(5) + 2(6)}{\sqrt{(-7)^2 + 2^2} \cdot \sqrt{5^2 + 6^2}} = \frac{-23}{\sqrt{3233}}$$

$$\boxed{\theta = 113.86^\circ}$$

13. $\mathbf{m} = -\mathbf{i}$ and $\mathbf{n} = 3\mathbf{i} + 10\mathbf{j}$

$$\cos\theta = \frac{-1(3) + 0(10)}{\sqrt{(-1)^2 + 0^2} \cdot \sqrt{3^2 + 10^2}} = \frac{-3}{\sqrt{109}}$$

$$\boxed{\theta = 106.7^\circ}$$

Topic C: Vector Applications

14. Leon kicks a soccer ball off the ground so that it travels with a velocity of 71 kilometers per hour at an angle of 41° with the ground. Find the magnitude of the horizontal and vertical components.

$$\cos 41^\circ = \frac{|x|}{71} \qquad \sin 41^\circ = \frac{|y|}{71}$$

$$|x| = \cos 41 \cdot 71 \qquad |y| = \sin 41 \cdot 71$$

$$\boxed{|x| = 53.58 \text{ km/hr}} \qquad \boxed{|y| = 46.58 \text{ km/hr}}$$

15. A blimp is traveling at a speed of 44 miles per hour in the direction of N 62° E. While in transit, the blimp encounters wind traveling at a velocity of 29 miles per hour traveling in the direction of N 4° W. Find the resultant speed and direction of the blimp.

$$V_1 = \langle 44\cos 28^\circ, 44\sin 28^\circ \rangle$$

$$V_2 = \langle 29\cos 94^\circ, 29\sin 94^\circ \rangle$$

$$V_1 + V_2 = \langle 36.83, 49.59 \rangle$$

$$\|V_1 + V_2\| = \sqrt{36.83^2 + 49.59^2} = \boxed{61.77 \text{ mph}}$$

$$\tan\theta = \frac{49.59}{36.83}; \quad \theta = 53.4^\circ \rightarrow \boxed{N\ 36.6^\circ E}$$

16. Two tug boats are pulling a freighter into a port. If Tugboat A is traveling in the direction N 37° W with a force of 31,000 kilograms, Tugboat B is traveling in the direction S 11° W with a force of 26,250 kilograms, find the magnitude and direction angle of the resultant force on the freighter.

$$V_1 = \langle 31000\cos 127^\circ, 31000\sin 127^\circ \rangle$$

$$V_2 = \langle 26250\cos 259^\circ, 26250\sin 259^\circ \rangle$$

$$V_1 + V_2 = \langle -23665, -1010.01 \rangle$$

$$\|V_1 + V_2\| = \sqrt{(-23665)^2 + (-1010.01)^2} = \boxed{23686.54 \text{ kg}}$$

$$\tan\theta = \frac{-1010.01}{-23665}; \quad \theta = 182.44^\circ \rightarrow \boxed{S\ 87.56^\circ W}$$

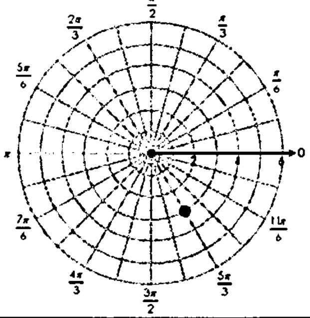
Topic D: Polar vs. Rectangular Coordinates

Directions: Given each point on the polar grids below, name two different pairs of polar coordinates if $0 \le \theta \le 2\pi$.

17.

$\left(3, \frac{5\pi}{3}\right), \left(3, -\frac{\pi}{3}\right),$

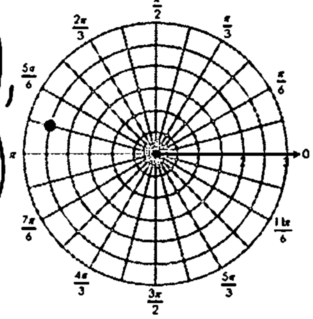
$\left(-3, \frac{2\pi}{3}\right), \left(-3, -\frac{4\pi}{3}\right)$



18.

$\left(2.5, \frac{11\pi}{12}\right), \left(2.5, -\frac{13\pi}{12}\right),$

$\left(-2.5, \frac{23\pi}{12}\right), \left(-2.5, -\frac{\pi}{12}\right)$



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Directions: Convert each pair of polar coordinates to rectangular coordinates. Round to the nearest hundredth if necessary.		
19. $\left(-7, \frac{\pi}{3}\right)$ $x = -7\cos\frac{\pi}{3} = -7\left(\frac{1}{2}\right)$ $y = -7\sin\frac{\pi}{3} = -7\left(\frac{\sqrt{3}}{2}\right)$ $\boxed{\left(-\frac{7}{2}, \frac{-7\sqrt{3}}{2}\right)}$	**20.** $\left(6, \frac{7\pi}{4}\right)$ $x = 6\cos\frac{7\pi}{4} = 6\left(\frac{\sqrt{2}}{2}\right)$ $y = 6\sin\frac{7\pi}{4} = 6\left(-\frac{\sqrt{2}}{2}\right)$ $\boxed{\left(3\sqrt{2}, -3\sqrt{2}\right)}$	**21.** $\left(3, -\frac{2\pi}{15}\right)$ $x = 3\cos\left(-\frac{2\pi}{15}\right)$ $y = 3\sin\left(-\frac{2\pi}{15}\right)$ $\boxed{(2.74, -1.22)}$

Directions: Convert each pair of rectangular coordinates to polar coordinates. Round to the nearest hundredth if necessary. If $0 \le \theta \le 2\pi$, give two possible solutions.		
22. $(5, 5)$ $r = \sqrt{5^2 + 5^2} = 5\sqrt{2}$ $\theta = \tan^{-1}(5/5) = \frac{\pi}{4}$ $\left(5\sqrt{2}, \frac{\pi}{4}\right), \left(-5\sqrt{2}, \frac{5\pi}{4}\right)$	**23.** $\left(-3\sqrt{3}, -3\right)$ $r = \sqrt{(-3\sqrt{3})^2 + (-3)^2} = 6$ $\theta = \tan^{-1}\left(\frac{-3}{3\sqrt{3}}\right) = \frac{7\pi}{6}$ $\left(6, \frac{7\pi}{6}\right), \left(-6, \frac{\pi}{6}\right)$	**24.** $(-2, 6)$ $r = \sqrt{(-2)^2 + 6^2} = 2\sqrt{10}$ $\theta = \tan^{-1}\left(\frac{6}{-2}\right) = 108.43°$ $(2\sqrt{10}, 108.43°)$, $(-2\sqrt{10}, 288.43°)$

Topic E: Polar Graphs

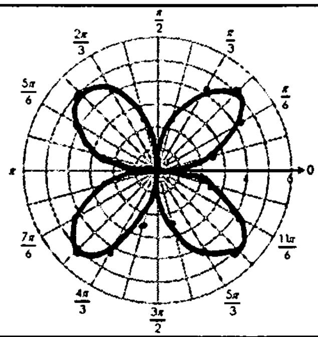
Directions: Classify the curve of each polar equation.

25. $r = 2 + 4\cos\theta$	**26.** $r^2 = 4\cos(2\theta)$	**27.** $r = 4 + 2\sin\theta$	**28.** $r = 7\cos(5\theta)$
Looped Limacon	Lemniscate	Convex Limacon	5-petal rose

Directions: Classify each of the following, then sketch the graph of each equation.

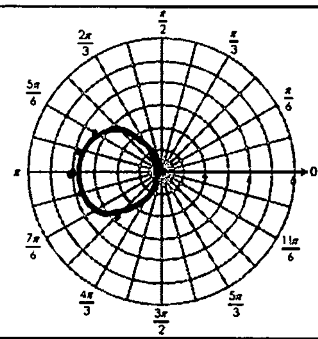
29. $r = 5\sin(2\theta)$

4-petal rose



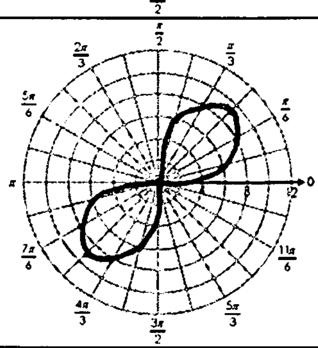
30. $r = -4\cos\theta$

Circle



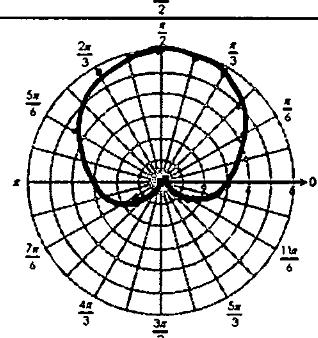
31. $r^2 = 81\sin(2\theta)$

Lemniscate



32. $r = 3 + 3\sin\theta$

Cardioid



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Topic F: Polar vs. Rectangular Equations

Directions: Convert the equations from polar to rectangular form.	
33. $y=\frac{x^2}{3}$ $r\sin\theta = \frac{(r\cos\theta)^2}{3}$ $3r\sin\theta = r^2\cos^2\theta$ $\frac{3\sin\theta}{\cos^2\theta} = r$ $3\cdot\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta} = r$ $\boxed{r = 3\tan\theta\sec\theta}$	**34.** $(x-4)^2+(y+1)^2=17$ $x^2-8x+y^2+2y=0$ $x^2+y^2-8x+2y=0$ $(r\cos\theta)^2+(r\sin\theta)^2-8r\cos\theta+2r\sin\theta=0$ $r^2\cos^2\theta+r^2\sin^2\theta=8r\cos\theta-2r\sin\theta$ $r^2(\cos^2\theta+\sin^2\theta)=r(8\cos\theta-2\sin\theta)$ $r^2=r(8\cos\theta-2\sin\theta)$ $\boxed{r=8\cos\theta-2\sin\theta}$
Directions: Convert the equations from rectangular to polar form.	
35. $r=-8\csc\theta$ $r\sin\theta = -8\csc\theta\sin\theta$ $r\sin\theta=-8$ $\boxed{y=-8}$	**36.** $r=-4\cos\theta+10\sin\theta$ $r^2=-4r\cos\theta+10r\sin\theta$ $x^2+y^2=-4x+10y$ $x^2+4x+y^2-10y=0$ $x^2+4x+4+y^2-10y+25=4+25$ $\boxed{(x+2)^2+(y-5)^2=29}$

Topic G: Polar Forms of Complex Numbers

Directions: Convert each complex number to polar form.	
37. $8-8i$ $r=\sqrt{8^2+(-8)^2}=\sqrt{128}=8\sqrt{2}$ $\tan\theta=\frac{8}{-8}=-1$; $\theta=\frac{7\pi}{4}$ $\boxed{=8\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)}$	**38.** $-\sqrt{15}+5i$ $r=\sqrt{(-\sqrt{15})^2+5^2}=\sqrt{40}=2\sqrt{10}$ $\tan\theta=\frac{5}{-\sqrt{15}}=\frac{-\sqrt{15}}{3}$; $\theta=127.76°$ $\boxed{=2\sqrt{10}\left(\cos 127.76°+i\sin 127.76°\right)}$
Directions: Convert the polar form of each complex number to rectangular form.	
39. $2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$ $=2\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$ $=\boxed{-1+i\sqrt{3}}$	**40.** $\sqrt{2}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$ $=\sqrt{2}\left(0+1i\right)$ $\boxed{=i\sqrt{2}}$

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Directions: Find each product or quotient. Express your answer in rectangular form.

41. $5\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\cdot 3\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$

$= 15\left[\cos\left(\frac{\pi}{2}+\frac{5\pi}{6}\right)+i\sin\left(\frac{\pi}{2}+\frac{5\pi}{6}\right)\right]$

$= 15\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) = \boxed{-\frac{15}{2}-\frac{15\sqrt{3}}{2}i}$

42. $4\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)\div 6\sqrt{2}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

$= \frac{\sqrt{2}}{3}\left[\cos\left(\frac{5\pi}{4}-\frac{\pi}{2}\right)+i\sin\left(\frac{5\pi}{4}-\frac{\pi}{2}\right)\right]$

$= \frac{\sqrt{2}}{3}\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)$

$= \boxed{-\frac{1}{3}+\frac{1}{3}i}$

Directions: Find each power. Express your answer in rectangular form.

43. $\left[3\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^6$

$= 3^6\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)$

$= 729(0-i) = \boxed{-729i}$

44. $(\sqrt{3}-i)^4$

$r=\sqrt{(\sqrt{3})^2+(-1)^2}=2$

$\tan\theta=\frac{-1}{\sqrt{3}}=-\frac{\sqrt{3}}{3};\ \theta=\frac{11\pi}{6}$

$z=2\left(\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}\right)$

$z^4=2^4\left(\cos\frac{44\pi}{6}+i\sin\frac{44\pi}{6}\right)$

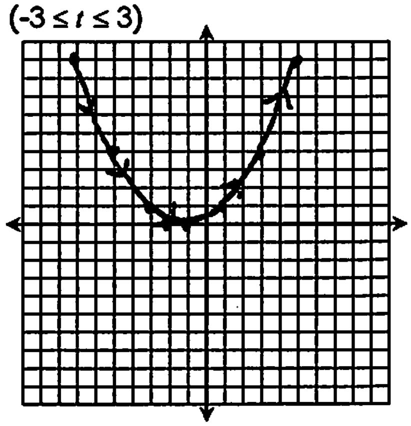
$z^4=16\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) = \boxed{-8-8i\sqrt{3}}$

Topic H: Graphing Parametric Equations

Directions: Sketch the curve given by each set of parametric equations over the given interval. Use arrows to indicate the direction of the curve as t increases.

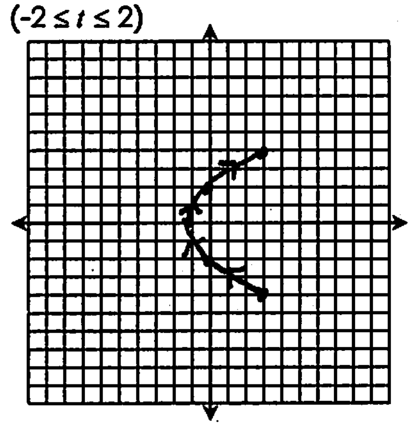
45. $x=2t-1,\ y=t^2\quad(-3\le t\le 3)$

t	x	y
-3	-7	9
-2	-5	4
-1	-3	1
0	-1	0
1	1	1
2	3	4
3	5	9



46. $x=t^2-1,\ y=2t\quad(-2\le t\le 2)$

t	x	y
-2	3	-4
-1	0	-2
0	-1	0
1	0	2
2	3	4



Topic I: Parametric vs. Rectangular Equations

Directions: Write each set of parametric equations in rectangular form. Note any restrictions on the domain.

47. $x=2t-3,\ y=-t+6$

$y-6=-t$

$-y+6=t$

$6-y=t$

$x=2(6-y)-3$

$x=12-2y-3$

$x=9-2y$

$x-9=-2y$

$\boxed{-\frac{1}{2}x+\frac{9}{2}=y}$

48. $x=t^2-4,\ y=3t$

$t=\frac{1}{3}y$

$x=\left(\frac{1}{3}y\right)^2-4$

$x=\frac{1}{9}y^2-4$

$x+4=\frac{1}{9}y^2$

$9(x+4)=y^2$

$\pm\sqrt{9(x+4)}=y^2$

$\boxed{y=\pm 3\sqrt{x+4}}$

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49. $x = 3\cos\theta,\ y = 5\sin\theta$ $\frac{x}{3} = \cos\theta\ ;\ \frac{y}{5} = \sin\theta$ $\cos^2\theta + \sin^2\theta = 1$ $(\frac{x}{3})^2 + (\frac{y}{5})^2 = 1$ $\boxed{\frac{x^2}{9} + \frac{y^2}{25} = 1}$	**50.** $x = \cos\theta + 4,\ y = \sin\theta - 1$ $x-4 = \cos\theta\ ;\ y+1 = \sin\theta$ $\cos^2\theta + \sin^2\theta = 1$ $\boxed{(x-4)^2 + (y+1)^2 = 1}$

Topic J: Applications of Parametric Equations

Use for questions 51 and 52: A projectile was launched at an angle of elevation of 82° at an initial velocity of 36 meters per second from a height of 14 meters above ground.

51. Find the horizontal distance the ball has traveled at 2.5 seconds. $x = t \cdot 36\cos 82°$ $x = 2.5 \cdot 36\cos 82°$ $\boxed{= 12.53\text{ m}}$	**52.** Find the vertical distance the ball has traveled at 2.5 seconds. $y = t \cdot 36\sin 82° - 4.9t^2 + 14$ $y = 2.5 \cdot 36\sin 82° - 4.9(2.5)^2 + 14$ $\boxed{= 72.5\text{ m}}$

Use for questions 53 and 54: A soccer ball is kicked down the field. The ball travels at an angle of elevation of 53° at an initial velocity of 16 meters per second.

53. What was the maximum height of the ball? $t = \frac{-16\sin 53}{-9.8} = 1.3\text{ sec}$ $y = t \cdot 16\sin 53° - 4.9t^2$ $y = 1.3 \cdot 16\sin 53° - 4.9(1.3)^2$ $\boxed{= 8.33\text{ m}}$	**54.** What was the horizontal distance the ball traveled at its maximum height? $x = t \cdot 16\cos 53°$ $x = 1.3 \cdot 16\cos 53°$ $\boxed{= 12.52\text{ m}}$

Use for questions 55 and 56: A frog leaped from the ground at an angle of elevation of 53°, leaping at an initial velocity of 8 feet per second.

55. How long was the frog in the air? $y = t \cdot 8\sin 39° - 16t^2$ $0 = t \cdot 8\sin 39° - 16t^2$ $t = \frac{-8\sin 39° \pm \sqrt{(8\sin 39°)^2 - 4(-16)(0)}}{2(-16)}$ $t = \frac{-8\sin 39° \pm \sqrt{(8\sin 39)^2}}{-32}$ $t = \{0,\ 0.31\}$ $\boxed{0.31\text{ sec}}$	**56.** What was the horizontal distance traveled by the frog? $x = t \cdot 8\cos 39°$ $x = 0.31 \cdot 8\cos 39°$ $\boxed{= 1.93\text{ ft}}$

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Pre-Calculus Review

QUIZ 4

Name: ______________________________

Date: __________________________ Per: ________

1. Given $R(4, -1)$ and $S(-2, 7)$, give the magnitude and approximate direction angle for $\overline{RS}$.

$\langle -6, 8 \rangle$

$\|\overline{RS}\| = \sqrt{(-6)^2 + 8^2} = 10$

$\tan\theta = \frac{8}{-6} = -\frac{4}{3}$; $\theta = 126.87°$

A. $2\sqrt{10}$; 71.57°
B. $2\sqrt{10}$; 108.43°
C. 10; 126.87°
D. 10; 143.13°

2. Vector a has a magnitude of 4 units and a direction of -30°. Vector b has a magnitude of 2 units and a direction of 90°. Which is the component form of vector c if $c = a + 2b$?

$a = \langle 4\cos(-30°), 4\sin(-30°) \rangle$
$b = \langle 2\cos 90°, 2\sin 90° \rangle$

A. $\langle 2, -2\sqrt{3} \rangle$
B. $\langle -2, 2\sqrt{3} \rangle$
C. $\langle -2\sqrt{3}, 2 \rangle$
D. $\langle 2\sqrt{3}, 2 \rangle$

3. Find the approximate measure of the angle formed between the vectors given below.

$$u = -i + 7j;\ v = 2i + 3j$$

$\cos\theta = \frac{-1(2) + 7(3)}{\sqrt{(-1)^2 + 7^2} \cdot \sqrt{2^2 + 3^2}}$

$= \frac{19}{\sqrt{650}}$

A. 37.78°
B. 38.19°
C. 41.82°
D. 48.34°

4. Two soccer players kick a ball at the same time. One player kicks the ball with a force of 148 newtons in the direction of N 40° E. The other player kicks with a force of 115 newtons in the direction of N 70° W. What is the resultant magnitude and direction of the resultant force on the ball?

$v_1 = \langle 148\cos 50°, 148\sin 50° \rangle$
$v_2 = \langle 115\cos 160°, 115\sin 160° \rangle$

A. 153.26 newtons; N 4.84° W
B. 161.54 newtons; N 4.84° W
C. 153.26 newtons; N 4.16° E
D. 161.54 newtons; N 4.16° E

5. A helicopter flying with a velocity of 125 miles per hour makes a 36° angle with the horizontal. What is the approximate vertical component of the helicopters velocity?

$\sin 36° = \frac{|y|}{125}$

$|y| = \sin 36 \cdot 125$

A. 73.47 mph
B. 83.92 mph
C. 95.28 mph
D. 101.13 mph

6. Which point maps to the same position on a polar coordinate as point P below?

$$P\left(-1, \frac{13\pi}{18}\right)$$

A. $\left(1, \frac{5\pi}{18}\right)$
B. $\left(1, -\frac{5\pi}{18}\right)$
C. $\left(-1, -\frac{5\pi}{18}\right)$
D. $\left(-1, \frac{13\pi}{9}\right)$

7. What are the rectangular coordinates of the polar coordinates given below?

$$\left(-10, -\frac{4\pi}{3}\right)$$

$x = -10\cos\left(-\frac{4\pi}{3}\right)$
$= -10(-1/2) = 5$

$y = -10\sin\left(-\frac{4\pi}{3}\right)$
$= -10\left(\frac{\sqrt{3}}{2}\right) = -5\sqrt{3}$

A. $(5\sqrt{3}, -5)$
B. $(-5\sqrt{3}, 5)$
C. $(5, -5\sqrt{3})$
D. $(-5, 5\sqrt{3})$

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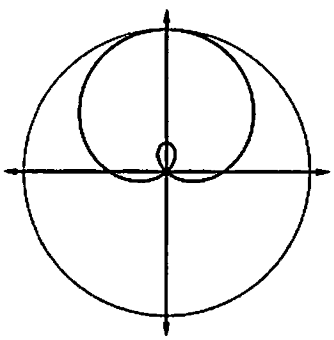
8. Which statement best describes the graph of the polar equation below?

$$r = \cos 3\theta$$

A. A rose with 6 petals
B. A rose with 3 petals.
C. A cardioid.
D. A lemniscate.

9. The graph of the equation $r = a + b \sin \theta$ is shown below. Which gives possible values for a and b?



A. $a = -2; b = -3$
B. $a = 2; b = -3$
C. $a = 3; b = 2$
D. $a = 2; b = 3$

10. A circle has a radius of 4 and a center at (0, 4) in the rectangular plane. What is the polar equation for this circle?

A. $r = 8\cos\theta$
B. $r = 4\cos\theta$
C. $r = 8\sin\theta$
D. $r = 4\sin\theta$

11. Which expression is the equivalent polar form of the expression below?

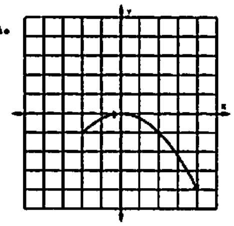
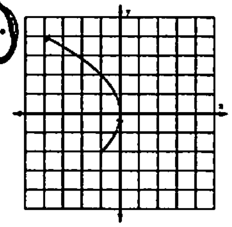
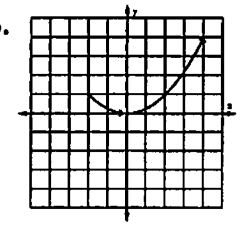
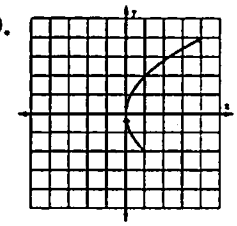
$$(1 - i\sqrt{3})^5$$

$= 2^5 \left(\cos\frac{25\pi}{3} + i\sin\frac{25\pi}{3}\right)$

A. $16\left(\cos\frac{10\pi}{3} + i\sin\frac{10\pi}{3}\right)$
B. $32\left(\cos\frac{25\pi}{3} + i\sin\frac{25\pi}{3}\right)$
C. $243\left(\cos\frac{10\pi}{3} + i\sin\frac{10\pi}{3}\right)$
D. $243\left(\cos\frac{25\pi}{3} + i\sin\frac{25\pi}{3}\right)$

12. Which graph represents the pair of parametric equations below on the given interval?

$$x = -\frac{t^2}{4},\ y = t;\quad -2 \le t \le 4$$

A.

C.

B.

D.


13. Give the rectangular form of the pair of parametric equations below.

$$x = t^2 - 2,\ y = 5t$$ $\rightarrow t = \frac{1}{5}y$

$x = \left(\frac{1}{5}y\right)^2 - 2$

$x + 2 = \frac{y^2}{25}$

$25(x+2) = y^2$

A. $y = \pm 5\sqrt{x+2}$
B. $y = \pm 5\sqrt{x-2}$
C. $y = \pm 2\sqrt{x-5}$
D. $y = \pm 2\sqrt{x+5}$

14. Which set of parametric equations represent the rectangular equation shown below?

$$\frac{x^2}{81} + \frac{y^2}{16} = 1$$

$\frac{x}{9} = \cos\theta$

$\frac{y}{4} = \sin\theta$

A. $x = 4\sin\theta,\ y = 9\cos\theta$
B. $x = 9\sin\theta,\ y = 4\cos\theta$
C. $x = 4\cos\theta,\ y = 9\sin\theta$
D. $x = 9\cos\theta,\ y = 4\sin\theta$

15. A referee threw a flag at a speed of 30 feet per second at an angle of elevation of 55°. If the flag left his hand at a height of 4 feet, how far from the point where the referee is standing does the flag land?

$y = t \cdot 30\sin 55° - 16t^2 + 4$

$t = \frac{-30\sin 55° \pm \sqrt{(30\sin 55°)^2 - 4(-16)(4)}}{2(-16)}$

$t = \{\cancel{-.15}, 1.68\}$

$x = 1.68 \cdot 30\cos 55°$

A. 23.55 ft
B. 24.73 ft
C. 26.14 ft
D. 28.91 ft

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