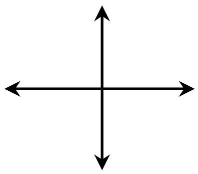
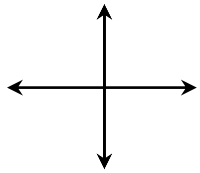
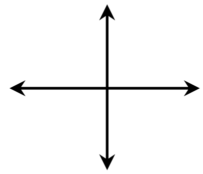
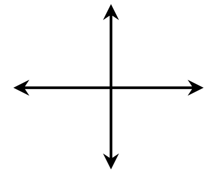
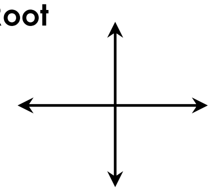
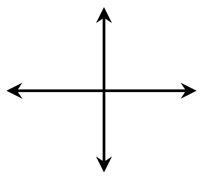
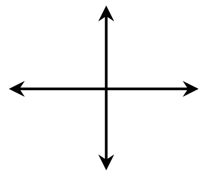
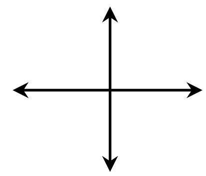
PRE-CALCULUS MID-YEAR TEST
Study Guide

Name: ______________________

Date: ______________________ **Per:** ________

Topic A: Basic Parent Functions and Transformations

Directons: For each function family, give the parent function and sketch the shape of the graph.

Linear	**Absolute Value**	**Quadratic**	**Cubic**
			
Square Root	**Cube Root**	**Reciprocal**	**Greatest Integer**
			

Recall the following transformations rules given a function $f(x)$:

Translations (Shifts)	**Reflections**	**Dilations** (compress/stretch)
$f(x+h)$ shifts left $f(x-h)$ shifts right	$-f(x)$ reflects over the x-axis	$a \cdot f(x)$ is a vertical compression when $\lvert a \rvert < 1$ and a vertical stretch when $\lvert a \rvert > 1$
$f(x)+k$ shifts up $f(x)-k$ shifts down	$f(-x)$ reflects over the y-axis	$f(b \cdot x)$ is a horizontal stretch when $\lvert b \rvert < 1$ and a horizontal compression when $\lvert b \rvert > 1$

Directions: Describe the transformations from the parent function.

1. $f(x) = -\dfrac{5}{x+7} - 2$

2. $f(x) = \sqrt[3]{-2(x-1)} + 5$

3. $f(x) = -\left[\!\left[\dfrac{x+4}{3} \right]\!\right] + 1$

4. The absolute value parent function is reflected in the x-axis, horizontally stretched by a factor of 3, and translated 5 units right. Write an equation to represent the **new function**.

5. The greatest integer parent function is vertically compressed by a factor of ¼, then translated 1 unit left and 7 units up. Write an equation to represent the **new function**.

6. Transformations were applied to the square root parent function such that it creates an endpoint at (3, 8). If the domain of the function is given by $(-\infty, 3]$, write an equation that could represent this **new function**.

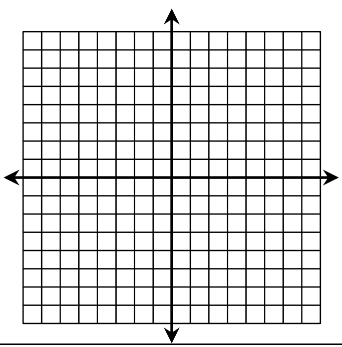
7. A certain function is vertically compressed by a factor of ½, horizontally stretched by a factor of 2, and translated left 6. If the new function is represented by $f(x) = 3(x+7)^3 - 4$, write an equation that could represent the **original function**.

Directions: Using the description of transformations from the parent function, **(a)** write a function, then **(b)** graph the function and state its domain and range.

8. The quadratic function is reflected in the x-axis, vertically stretched by a factor of 3, translated left 4 and up 6.

a.

b.



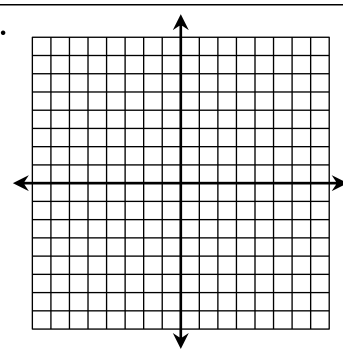
Domain:

Range:

9. The square root function is vertically stretched by a factor of 5, horizontally stretched by a factor of 2, then translated left 2 and down 8.

a.

b.



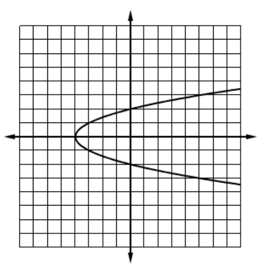
Domain:

Range:

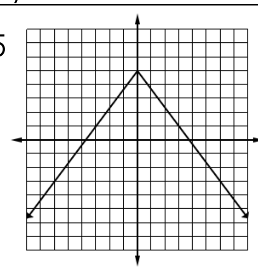
Topic B: More Characteristics of Functions & Graphs

Directions: Use the graph to determine if the relations given below are symmetric to the x-axis, y-axis, or origin. Confirm your answer algebraically.

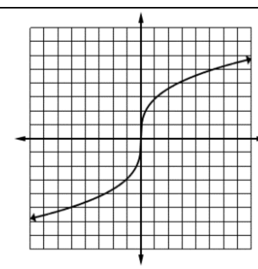
10. $y^2 - x = 4$



11. $y = -\left|\frac{4}{3}x\right| + 5$



12. $y = 2\sqrt[3]{3x}$



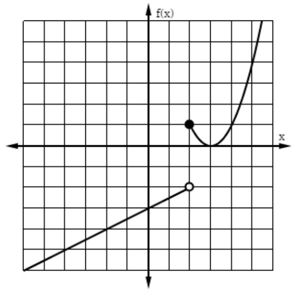
Directions: Determine algebraically if the function is even, odd, or neither. If even or odd, describe the symmetry.

13. $f(x) = \dfrac{3}{2x^2 - 9}$

14. $f(x) = \sqrt[3]{\dfrac{3}{5}x}$

Directions: Determine if the functions below are continuous. If discontinuous, identify the type and location of discontinuity.

15.



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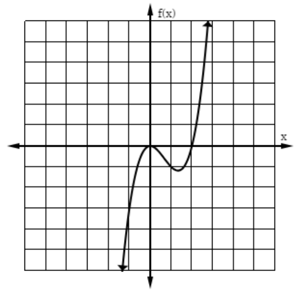

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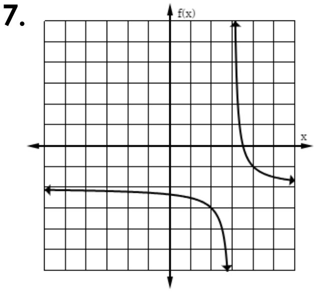

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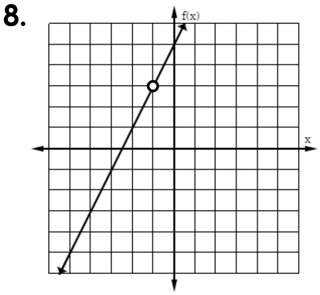


Topic C: Operations & Compositions of Functions

Directions: Use $f(x)=\dfrac{4x^2-1}{x+1}$, $g(x)=3x^2-2x-5$, and $h(x)=2x-1$ to find each function below.

19. $(g-h)(x)$

20. $\left(\dfrac{f}{h}\right)(x)$

21. $(f\cdot g)(x)$

22. $(f+h)(x)$

23. $(h\circ g)(x)$

24. $(f\circ h)(x)$

Directions: Use $f(x)=2x+7$, $g(x)=5x^2-3$, and $h(x)=|3x+9|$ to evaluate each function below.

25. $(h+g)(-2)$

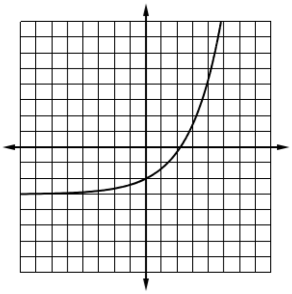
26. $(g+f)(4y-3)$

27. $(f\circ g)(3p)$

Topic D: Inverse Functions

Directions: Determine if the graph represents a one-to-one function.

28.



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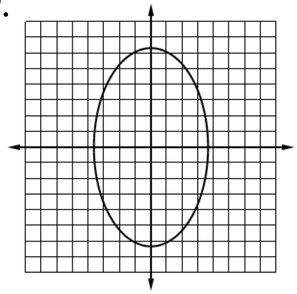

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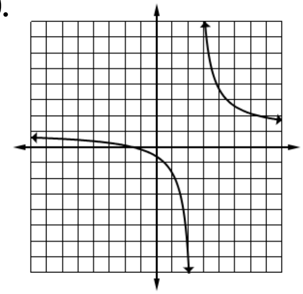
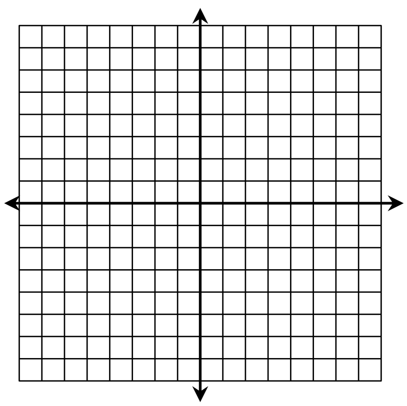
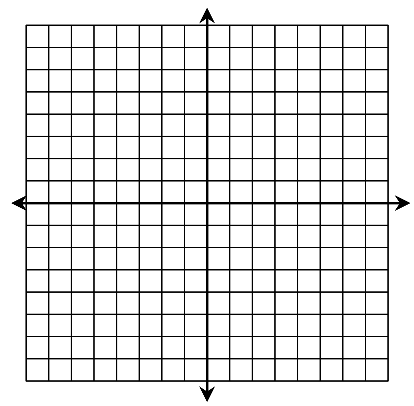


Directions: Find the inverse of each function. Then, graph both the function and its inverse.

31. $f(x) = \sqrt{-x+6}$



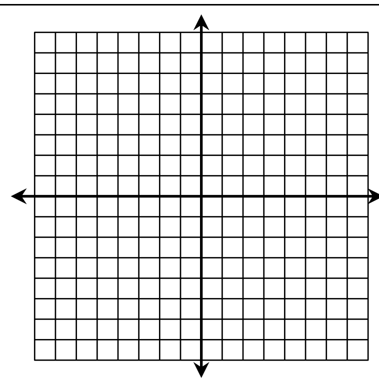
32. $f(x) = -\dfrac{3}{x+4}$



Topic E: Piecewise Functions

Directions: Graph the piecewise function. Identify the domain, range, and state the location and type of any discontinuities.

33. $f(x) = \begin{cases} -\frac{1}{2}x+3 & \text{if } x \le -3 \\ x^2 - 1 & \text{if } -3 < x < 1 \\ 3 & \text{if } x > 1 \end{cases}$



Domain:

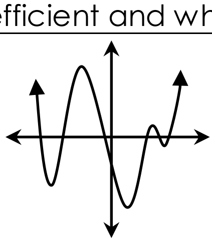
Range:

Discontinuities:

Topic F: Graphs of Polynomial Functions

Directions: Given the graph of the polynomial functions below, determine the sign of the leading coefficient and whether the function has an even or odd degree.

34.



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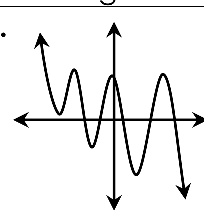
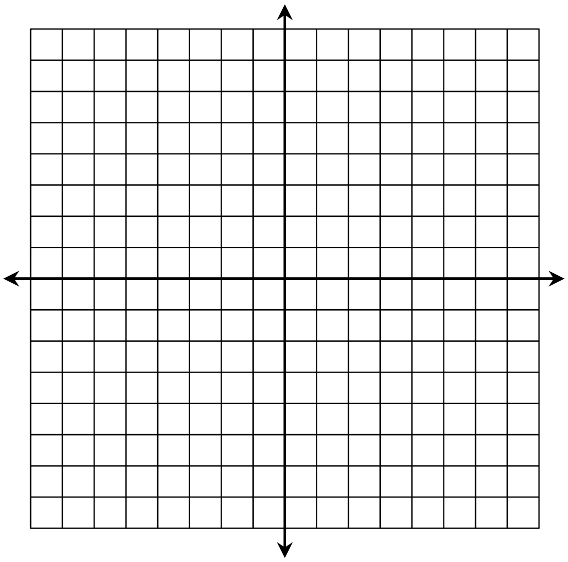


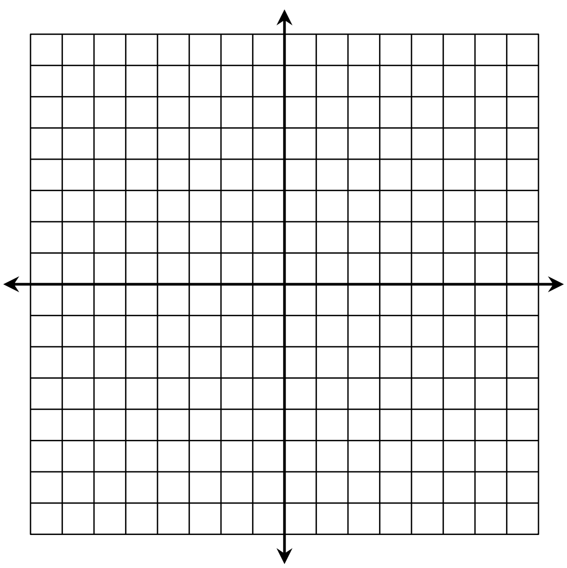
Directions: Graph each function, then identify its key characteristics.

36. $f(x) = -x^4 - x^3 + 4x^2 - 5$



Domain:	**Range:**
x-intercept(s):	
y-intercept:	
Rel. Minimum(s):	
Rel. Maximum(s):	
Inc. Interval(s):	
Dec. Interval(s):	
End Behavior:	

37. $f(x) = x^3 - 3x^2 + 6$



Domain:	**Range:**
x-intercept(s):	
y-intercept:	
Rel. Minimum(s):	
Rel. Maximum(s):	
Inc. Interval(s):	
Dec. Interval(s):	
End Behavior:	

Topic G: Zeros of Polynomial Functions

Directions: Write the function in factored form, identify the zeros and multiplicities, and describe the effect on the graph.

38. $f(x) = x^3 + 10x^2 + 25x$

Factored Form:

Zero	Multiplicity	Effect

39. $f(x) = x^3 + 2x^2 - 4x - 8$

Factored Form:

Zero	Multiplicity	Effect

Directions: Use the Remainder Theorem to evaluate $f(x)$ at $x = c$.	
40. $f(x) = 3x^4 - 2x^3 - 6x^2 + 7x - 9$; $c = -2$	**41.** $f(x) = -x^5 + 7x^3 - 18x + 21$; $c = 3$
Directions: Use the Factor Theorem to determine if the binomial is a linear factor of the function.	
42. $f(x) = x^3 + 4x^2 - 7x - 10$; $(x - 1)$	**43.** $f(x) = 3x^4 + 2x^3 - 20x^2 - 8x + 32$; $(x + 2)$
Directions: Use the Rational Zero Theorem to list all possible rational zeros of the given function.	
44. $f(x) = x^4 - 5x^2 + 17x - 54$	**45.** $f(x) = 3x^3 + 9x^2 - 11x + 36$
Directions: Use Descartes' Rule of Signs to give the possible number of positive and negative real zeros.	
46. $f(x) = 3x^5 + 4x^4 + 7x^3 + 11x^2 + x + 15$	**47.** $f(x) = -7x^4 - 2x^3 + 4x^2 + 9x - 23$
Directions: Find all zeros for each function. Simplify all irrational zeros and complex solutions. Then, give the complete factorization of the function.	
48. $f(x) = 4x^4 + 47x^2 - 12$	**49.** $f(x) = x^3 + 9x^2 + 8x - 60$

50. $f(x) = x^3 + 5x^2 + 23x + 51$

51. $f(x) = x^3 - 11x^2 + 34x - 30$

Directions: Write a polynomial function with the given zeros.

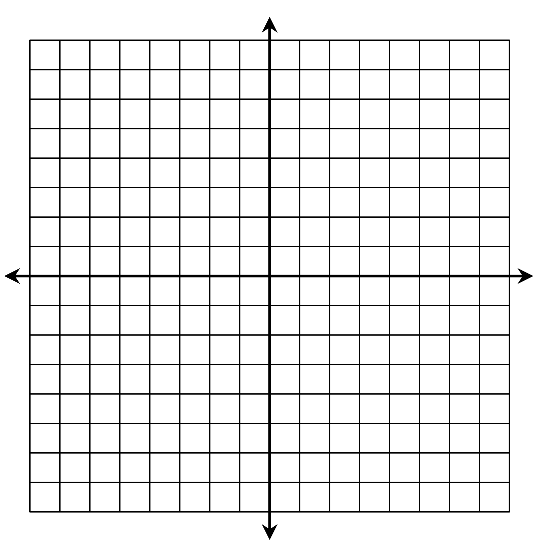
52. $\pm i, \frac{5}{2}$ (multiplicity 2)

53. $-4, \pm 3\sqrt{2}$

Topic H: Graphs of Rational Functions

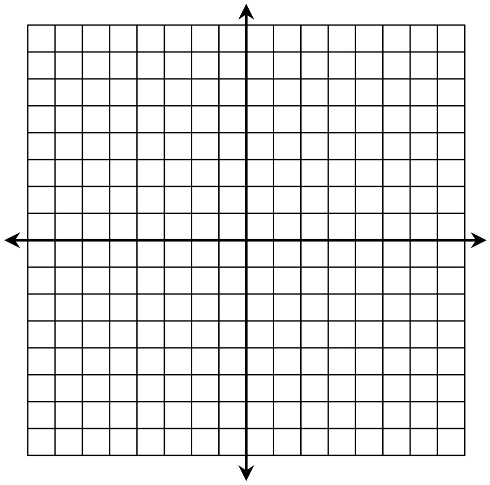
Directions: Graph each function, then identify its key characteristics.

54. $f(x) = \dfrac{-2x+6}{x-1}$



Domain:	**Range:**
***x*-intercept(s):**	
***y*-intercept:**	
Vertical Asymptote:	
Horizontal Asymptote:	
Slant Asymptote:	
Hole(s):	

55. $f(x) = \dfrac{x^3 - 4x}{x^2 + x}$



Domain:	Range:
x-intercept(s):	
y-intercept:	
Vertical Asymptote:	
Horizontal Asymptote:	
Slant Asymptote:	
Hole(s):	

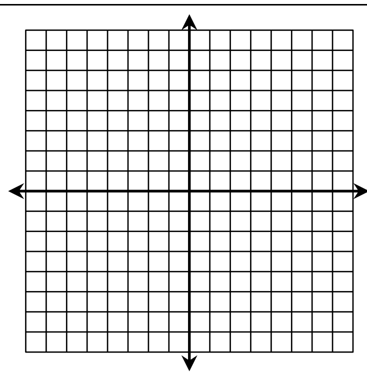
Topic I: Graphs of Exponential & Logarithmic Functions

Directions: Classify each function as exponential growth or decay.

56. $f(x) = -2 \cdot 5^x$	**57.** $f(x) = \frac{1}{2} \cdot \left(\frac{5}{4}\right)^x$	**58.** $f(x) = 4 \cdot \left(\frac{2}{7}\right)^x$

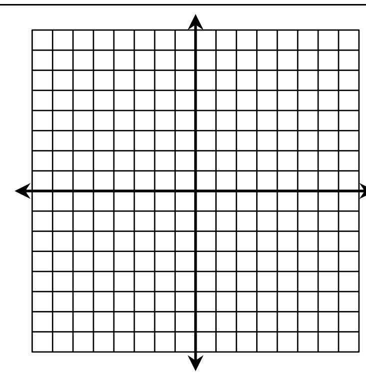
Directions: Graph each function, then identify its key characteristics.

59. $f(x) = 4 \cdot \left(\frac{1}{2}\right)^{x-1} + 1$



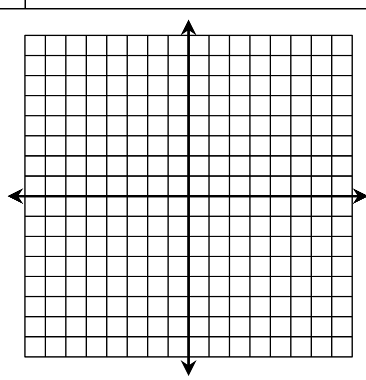
Domain:	Range:
y-intercept:	Asymptote:

60. $f(x) = 3 \cdot e^{x+2} - 4$



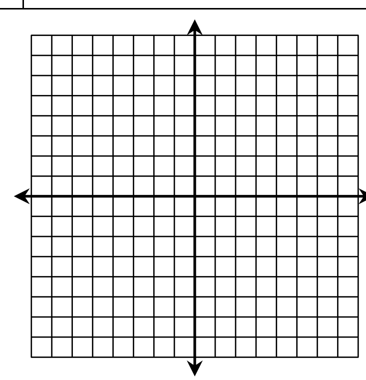
Domain:	Range:
y-intercept:	Asymptote:

61. $f(x) = \log_{\frac{1}{4}}(x-1) + 1$



Domain:	Range:
x-intercept:	Asymptote:

62. $f(x) = \log_4(x+4) + 2$



Domain:	Range:
x-intercept:	Asymptote:

Topic J: Properties of Logarithms/Simplifying Logarithms

Directions: Condense each expression into a single logarithm.

63. $3 \cdot \log_4 2c - \frac{1}{2}\log_4 64$	**64.** $\frac{1}{4}(\ln 256 + 3 \cdot \ln p) - 2 \cdot \ln q$

Directions: Expand each logarithm completely.

65. $\log_6 \frac{n\sqrt[5]{2m^2}}{5}$	**66.** $\ln(4a^2\sqrt{b})^3$

Topic K: Solving Logarithmic & Exponential Equations

Directions: Solve each equation, rounding to the nearest ten-thousandths place when necessary.

67. $\log_7 6 + \log_7(x+1) = 2 \cdot \log_7(4x+1)$	**68.** $\ln(w+6) - \ln(w+4) = 1$
69. $\left(\frac{1}{27}\right)^{3k-2} = \frac{1}{81} \cdot 243^{4-k}$	**70.** $-5 \cdot 8^{-5y-6} + 8 = -40$

71. Jason's boat was initially priced at $45,000. After 7 years, the boat is worth half of its original purchase price. Write and a continuous exponential decay function to model the price of the boat, then find the rate of depreciation.

72. Karen inherited $6,000 from her grandmother. She deposited half of this money into an investment account that earns 2.75% interest compounded quarterly. She deposited the other half of the money into an account that earns 4% interest compounded continuously. Assuming neither account had any additional deposits or withdrawals, find the total amount in the two accounts after 15 years.

Topic L: Trigonometric Functions

Directions: Convert the degrees to radians, and radians to degrees.

73. 198°	**74.** -480°	**75.** $\frac{2\pi}{9}$	**76.** $-\frac{13\pi}{8}$

Directions: Write each measure in Degree-Minute-Second form.

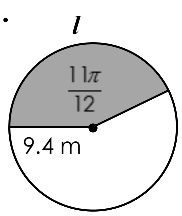
77. 154.861°	**78.** -236.255°

Directions: Write each angle measure in decimal degree form.

79. -28° 13’ 38”	**80.** 341° 47’ 22”

Directions: Find the length of the intercepted arc, l, and A, the area of the shaded sector.

81.



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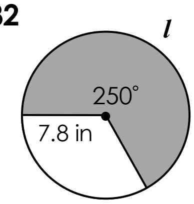


l =	A =	l =	A =

83. If (-3, 15) is a point on the terminal side of θ in standard form, find the exact values of the trigonometric functions of θ.

sin θ =	csc θ =
cos θ =	sec θ =
tan θ =	cot θ =

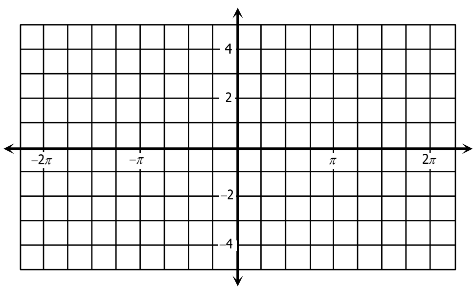
84. If $\csc\theta = -\frac{7}{5}$ and tan $\theta > 0$, find the exact values of the remaining trigonometric functions of θ.	sin θ =	csc $\theta = -\frac{7}{5}$
	cos θ =	sec θ =
	tan θ =	cot θ =

Directions: Use the unit circle to give the exact value of each trigonometric function.

85. $\sin\frac{5\pi}{3}$	**86.** $\cot\frac{\pi}{3}$	**87.** $\sec\frac{3\pi}{4}$	**88.** $\tan\left(-\frac{9\pi}{4}\right)$

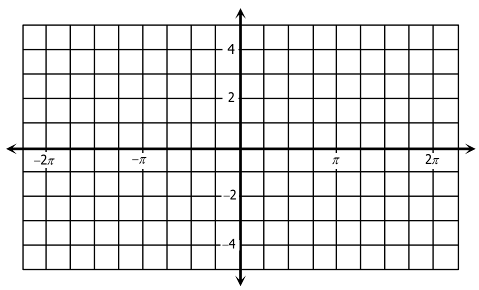
Directions: Graph each function and identify its key characteristics.

89. $f(x) = 3 \cdot \cos\frac{2}{3}x$



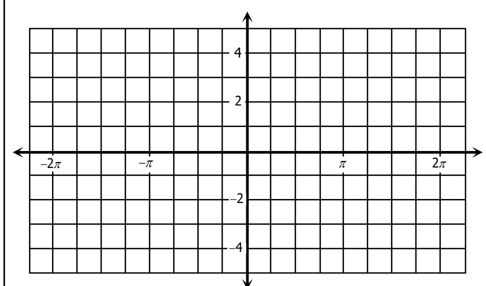
Amplitude:	Period:

90. $f(x) = \frac{1}{2} \cdot \tan 2x$



Amplitude:	Period:

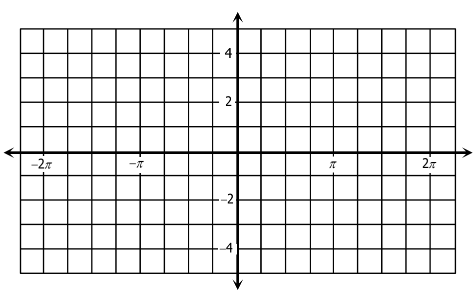
91. $f(x) = \frac{5}{2} \cdot \csc\frac{4}{3}x$



Amplitude:	Period:

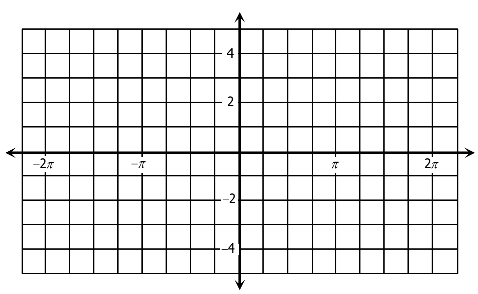
Directions: Graph each function, then give the amplitude, period, phase shift, and vertical shift.

92. $f(x) = \frac{3}{2} \cdot \sec(x - \pi) + 2$



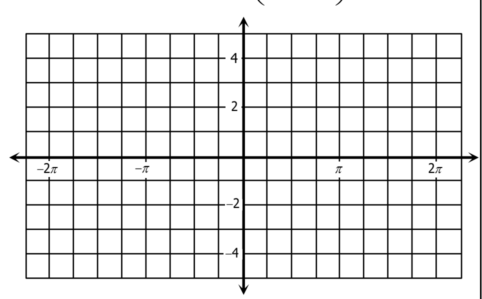
Amplitude:	Period:
Phase Shift:	**Vertical Shift:**

93. $f(x) = -4 \cdot \cot\left(x + \frac{3\pi}{4}\right) - 1$



Amplitude:	Period:
Phase Shift:	**Vertical Shift:**

94. $f(x) = 2 \cdot \sin\frac{1}{2}\left(x - \frac{\pi}{2}\right)$



Amplitude:	Period:
Phase Shift:	**Vertical Shift:**

Directions: Give the exact value, if it exists.

95. arctan 1	**96.** $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$	**97.** $\cos^{-1}\left[\sin\left(-\frac{5\pi}{6}\right)\right]$

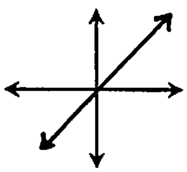
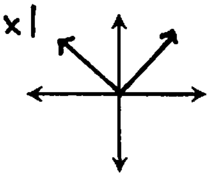
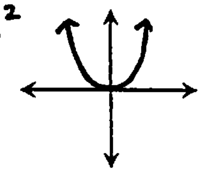
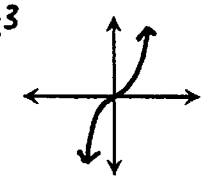
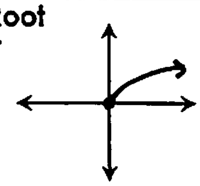
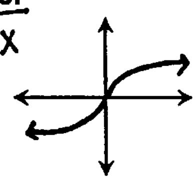
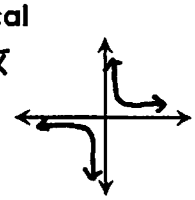
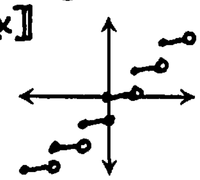
PRE-CALCULUS MID-YEAR TEST
Study Guide

Name: ______________________

Date: ______________ Per: ________

Topic A: Basic Parent Functions and Transformations

Directons: For each function family, give the parent function and sketch the shape of the graph.

Linear	**Absolute Value**	**Quadratic**	**Cubic**
$f(x)=x$ 	$f(x)=\|x\|$ 	$f(x)=x^2$ 	$f(x)=x^3$ 
Square Root	**Cube Root**	**Reciprocal**	**Greatest Integer**
$f(x)=\sqrt{x}$ 	$f(x)=\sqrt[3]{x}$ 	$f(x)=\frac{1}{x}$ 	$f(x)=[\![x]\!]$ 

Recall the following transformations rules given a function $f(x)$:

Translations (Shifts)	**Reflections**	**Dilations** (compress/stretch)
$f(x+h)$ shifts left	$-f(x)$ reflects over the x-axis	$a\cdot f(x)$ is a vertical compression when $\|a\|<1$ and a vertical stretch when $\|a\|>1$
$f(x-h)$ shifts right		
$f(x)+k$ shifts up	$f(-x)$ reflects over the y-axis	$f(b\cdot x)$ is a horizontal stretch when $\|b\|<1$ and a horizontal compression when $\|b\|>1$
$f(x)-k$ shifts down		

Directions: Describe the transformations from the parent function.

1. $f(x)=-\dfrac{5}{x+7}-2$

- Reflect in x-axis
- Vert. Stretch by 5
- Trans. left 7, down 2

2. $f(x)=\sqrt[3]{-2(x-1)}+5$

- Reflect in y-axis
- Horiz. compress by $\frac{1}{2}$
- Trans. right 1, up 5

3. $f(x)=-\left[\!\!\left[\dfrac{x+4}{3}\right]\!\!\right]+1$

- Reflect in x-axis
- Horiz. stretch by 3
- Trans. left 4, up 1

4. The absolute value parent function is reflected in the x-axis, horizontally stretched by a factor of 3, and translated 5 units right. Write an equation to represent the **new function.**

$f(x)=-\left|\frac{1}{3}(x-5)\right|$

5. The greatest integer parent function is vertically compressed by a factor of ¼, then translated 1 unit left and 7 units up. Write an equation to represent the **new function.**

$f(x)=\frac{1}{4}[\![x+1]\!]+7$

6. Transformations were applied to the square root parent function such that it creates an endpoint at (3, 8). If the domain of the function is given by $(-\infty, 3]$, write an equation that could represent this **new function.**

$f(x) = \sqrt{-(x-3)} + 8$

7. A certain function is vertically compressed by a factor of ½, horizontally stretched by a factor of 2, and translated left 6. If the new function is represented by $f(x) = 3(x+7)^3 - 4$, write an equation that could represent the **original function**.

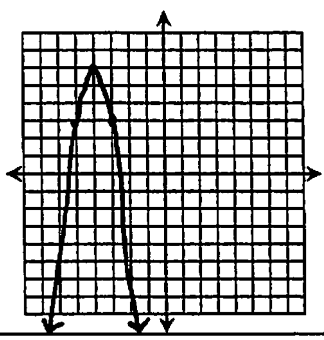
$f(x) = 6\left(2(x+1)\right)^3 - 4$

Directions: Using the description of transformations from the parent function, **(a)** write a function, then **(b)** graph the function and state its domain and range.

8. The quadratic function is reflected in the x-axis, vertically stretched by a factor of 3, translated left 4 and up 6.

a. $f(x) = -3(x+4)^2 + 6$

b.



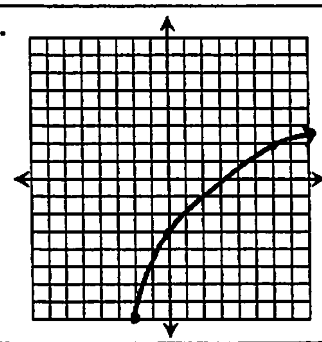
Domain: $\mathbb{R}$

Range: $\{y \mid y \leq 6\}$

9. The square root function is vertically stretched by a factor of 5, horizontally stretched by a factor of 2, then translated left 2 and down 8.

a. $f(x) = 5\sqrt{\frac{1}{2}(x+2)} - 8$

b.



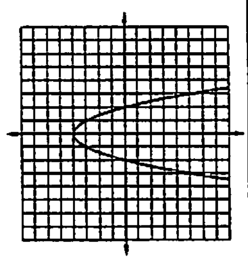
Domain: $\{x \mid x \geq -2\}$

Range: $\{y \mid y \geq -8\}$

Topic B: More Characteristics of Functions & Graphs

Directions: Use the graph to determine if the relations given below are symmetric to the x-axis, y-axis, or origin. Confirm your answer algebraically.

10. $y^2 - x = 4$

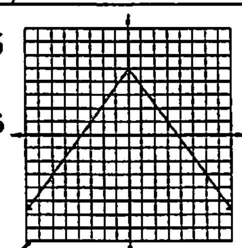


$(-y)^2 - x = 4$

$y^2 - x = 4$ ✓

$\boxed{\text{Symmetric to x-axis}}$

11. $y = -\left|\frac{4}{3}x\right| + 5$



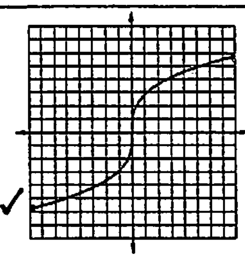
$y = -\left|\frac{4}{3}(-x)\right| + 5$

$y = -\left|-\frac{4}{3}x\right| + 5$

$y = -\left|\frac{4}{3}x\right| + 5$ ✓

$\boxed{\text{Symmetric to y-axis}}$

12. $y = 2\sqrt[3]{3x}$



$-y = 2\sqrt[3]{3(-x)}$

$-y = 2\sqrt[3]{-3x}$

$-y = -2\sqrt[3]{3x}$ ✓

$\boxed{\text{Symmetric to origin}}$

Directions: Determine algebraically if the function is even, odd, or neither. If even or odd, describe the symmetry.

13. $f(x) = \dfrac{3}{2x^2 - 9}$

$f(-x) = \dfrac{3}{2(-x)^2 - 9}$

$f(-x) = \dfrac{3}{2x^2 - 9}$

$\boxed{\text{Even; Symmetric to y-axis}}$

14. $f(x) = \sqrt[3]{\frac{3}{5}x}$

$f(-x) = \sqrt[3]{\frac{3}{5}(-x)}$

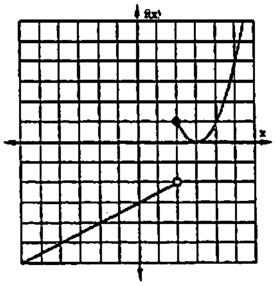
$f(-x) = \sqrt[3]{-\frac{3}{5}x}$

$f(-x) = -\sqrt[3]{\frac{3}{5}x}$

$\boxed{\text{Odd; Symmetric to origin}}$

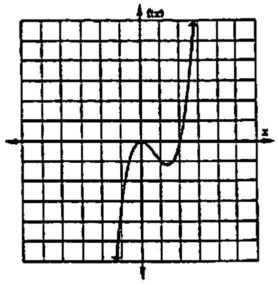
Directions: Determine if the functions below are continuous. If discontinuous, identify the type and location of discontinuity.

15.



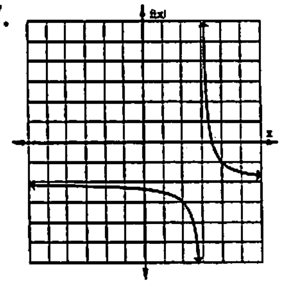
Discontinuous; $x=2$, jump

16.



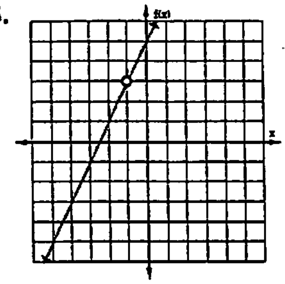
Continuous

17.



Discontinuous; $x=3$, infinite

18.



Discontinuous; $x=-1$, removable

Topic C: Operations & Compositions of Functions

Directions: Use $f(x)=\frac{4x^2-1}{x+1}$, $g(x)=3x^2-2x-5$, and $h(x)=2x-1$ to find each function below.

19. $(g-h)(x)$

$(3x^2-2x-5)-(2x-1)$

$\boxed{3x^2-4x-4}$

20. $\left(\frac{f}{h}\right)(x)$

$\dfrac{\frac{4x^2-1}{x+1}}{2x-1}$

$=\frac{(2x+1)(2x-1)}{x+1}\cdot\frac{1}{2x-1}$

$=\boxed{\frac{2x+1}{x+1};\ x\neq-1}$

21. $(f\cdot g)(x)$

$\frac{4x^2-1}{x+1}\cdot 3x^2-2x-5$

$=\frac{4x^2-1}{x+1}\cdot(3x-5)(x+1)$

$=(4x^2-1)(3x-5)$

$=\boxed{12x^3-20x^2-3x+5}$

22. $(f+h)(x)$

$\frac{4x^2-1}{x+1}+2x-1\cdot\frac{x+1}{x+1}$

$=\frac{4x^2-1}{x+1}+\frac{2x^2+x-1}{x+1}$

$=\boxed{\frac{6x^2+x-2}{x+1};\ x\neq-1}$

23. $(h\circ g)(x)$

$2(3x^2-2x-5)-1$

$=\boxed{6x^2-4x-11}$

24. $(f\circ h)(x)$

$\frac{4(2x-1)^2-1}{(2x-1)+1}$

$=\frac{4(4x^2-4x+1)-1}{2x}$

$=\boxed{\frac{16x^2-16x+3}{2x};\ x\neq0}$

Directions: Use $f(x)=2x+7$, $g(x)=5x^2-3$, and $h(x)=|3x+9|$ to evaluate each function below.

25. $(h+g)(-2)$

$h(-2)=|3(-2)+9|=3$

$g(-2)=5(-2)^2-3=17$

$=\boxed{20}$

26. $(g+f)(4y-3)$

$g(4y-3)=5(4y-3)^2-3$

$=5(16y^2-24y+9)-3$

$=80y^2-120y+42$

$f(4y-3)=2(4y-3)+7$

$=8y+1$

$=\boxed{80y^2-112y+43}$

27. $(f\circ g)(3p)$

$g(3p)=5(3p)^2-3$

$=45p^2-3$

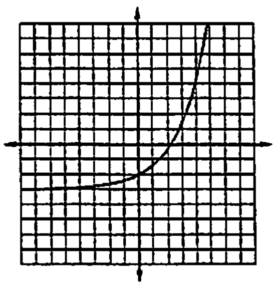
$f(45p^2-3)=2(45p^2-3)+7$

$\boxed{=90p^2+1}$

Topic D: Inverse Functions

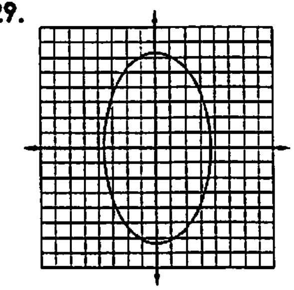
Directions: Determine if the graph represents a one-to-one function.

28.



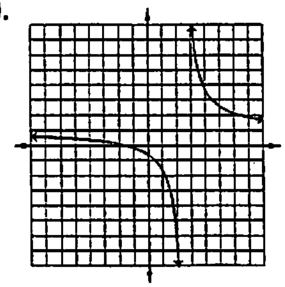
Yes

29.



No

30.



Yes

Directions: Find the inverse of each function. Then, graph both the function and its inverse.

31. $f(x)=\sqrt{-x+6}$

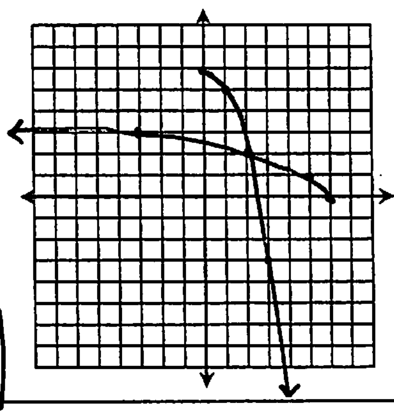
$x=\sqrt{-y+6}$

$x^2=-y+6$

$x^2-6=-y$

$-x^2+6=y$

$\boxed{f^{-1}(x)=-x^2+6,\ x\geq 0}$



32. $f(x)=-\dfrac{3}{x+4}$

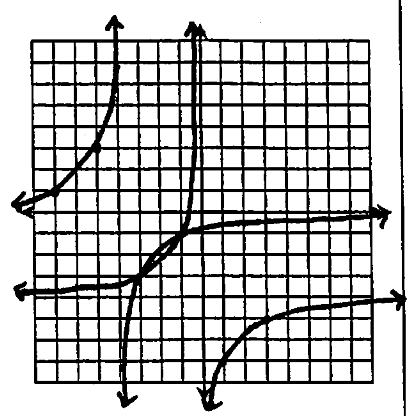
$x=\dfrac{-3}{y+4}$

$x(y+4)=-3$

$y+4=\dfrac{-3}{x}$

$y=\dfrac{-3}{x}-4$

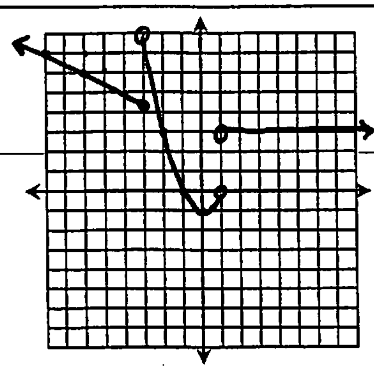
$\boxed{f^{-1}(x)=\dfrac{-3}{x}-4}$



Topic E: Piecewise Functions

Directions: Graph the piecewise function. Identify the domain, range, and state the location and type of any discontinuities.

33. $f(x)=\begin{cases}-\dfrac{1}{2}x+3 & \text{if } x\leq -3\\ x^2-1 & \text{if } -3<x<1\\ 3 & \text{if } x>1\end{cases}$



Domain:
$\{x \mid x\neq 1\}$

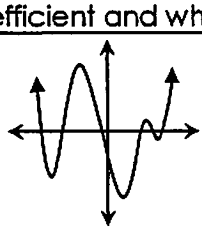
Range:
$\{y \mid y\geq -1\}$

Discontinuities:
$x=-3$, jump
$x=1$, removable

Topic F: Graphs of Polynomial Functions

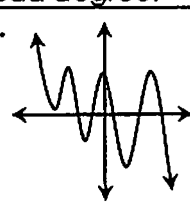
Directions: Given the graph of the polynomial functions below, determine the sign of the leading coefficient and whether the function has an even or odd degree.

34.



Positive,
Even

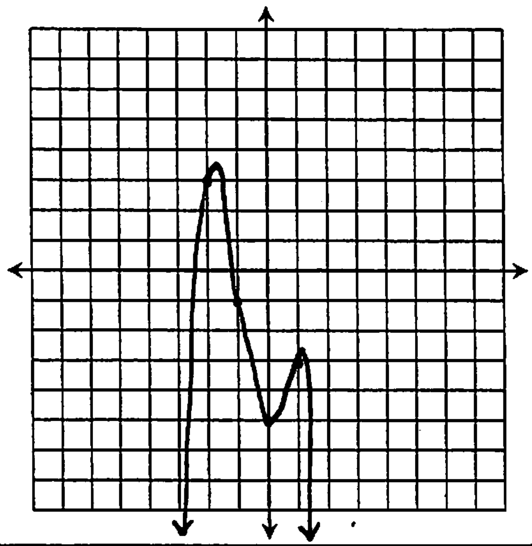
35.



Negative,
odd

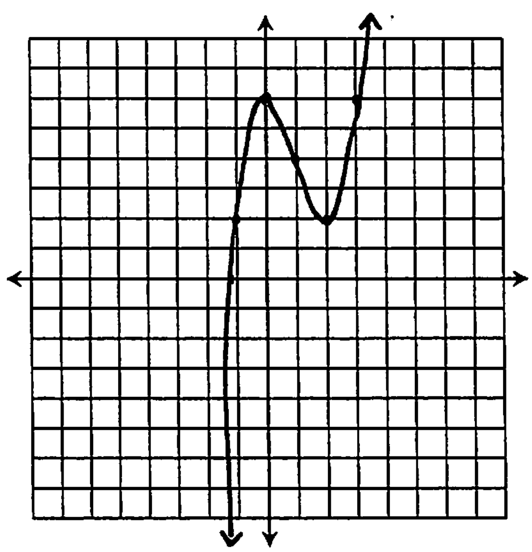
Directions: Graph each function, then identify its key characteristics.

36. $f(x) = -x^4 - x^3 + 4x^2 - 5$



Domain: $\mathbb{R}$	**Range:** $\{y \mid y \le 3.31\}$
x-intercept(s):	$(-2.32, 0)$, $(-1.14, 0)$
y-intercept:	$(0, -5)$
Rel. Minimum(s):	$(0, -5)$
Rel. Maximum(s):	$(-1.84, 3.31)$, $(1.09, -2.95)$
Inc. Interval(s):	$(-\infty, -1.84)$, $(0, 1.09)$
Dec. Interval(s):	$(-1.84, 0)$, $(1.09, \infty)$
End Behavior:	As $x \to \infty$, $f(x) \to -\infty$ As $x \to -\infty$, $f(x) \to -\infty$

37. $f(x) = x^3 - 3x^2 + 6$



Domain: $\mathbb{R}$	**Range:** $\mathbb{R}$
x-intercept(s):	$(-1.2, 0)$
y-intercept:	$(0, 6)$
Rel. Minimum(s):	$(2, 2)$
Rel. Maximum(s):	$(0, 6)$
Inc. Interval(s):	$(-\infty, 0)$, $(2, \infty)$
Dec. Interval(s):	$(0, 2)$
End Behavior:	As $x \to \infty$, $f(x) \to \infty$ As $x \to -\infty$, $f(x) \to -\infty$

Topic G: Zeros of Polynomial Functions

Directions: Write the function in factored form, identify the zeros and multiplicities, and describe the effect on the graph.

38. $f(x) = x^3 + 10x^2 + 25x$

$x(x^2 + 10x + 25)$

Factored Form: $f(x) = x(x+5)^2$

Zero	Multiplicity	Effect
-5	2	tangent
0	1	intersects

39. $f(x) = x^3 + 2x^2 - 4x - 8$

$x^2(x+2) - 4(x+2)$

Factored Form: $f(x) = (x-2)(x+2)^2$

Zero	Multiplicity	Effect
-2	2	tangent
2	1	intersects

Directions: Use the Remainder Theorem to evaluate $f(x)$ at $x = c$.

40. $f(x) = 3x^4 - 2x^3 - 6x^2 + 7x - 9$; $c = -2$

$$\begin{array}{r|rrrrr} -2 & 3 & -2 & -6 & 7 & -9 \\ & \downarrow & -6 & 16 & -20 & 26 \\ \hline & 3 & -8 & 10 & -13 & 17 \end{array}$$

$\boxed{17}$

41. $f(x) = -x^5 + 7x^3 - 18x + 21$; $c = 3$

$$\begin{array}{r|rrrrrr} 3 & -1 & 0 & 7 & 0 & -18 & 21 \\ & \downarrow & -3 & -9 & -6 & -18 & -108 \\ \hline & -1 & -3 & -2 & -6 & -36 & -87 \end{array}$$

$\boxed{-87}$

Directions: Use the Factor Theorem to determine if the binomial is a linear factor of the function.

42. $f(x) = x^3 + 4x^2 - 7x - 10$; $(x - 1)$

$$\begin{array}{r|rrrr} 1 & 1 & 4 & -7 & -10 \\ & \downarrow & 1 & 5 & -2 \\ \hline & 1 & 5 & -2 & -12 \end{array}$$

$\boxed{\text{No}}$

43. $f(x) = 3x^4 + 2x^3 - 20x^2 - 8x + 32$; $(x + 2)$

$$\begin{array}{r|rrrrr} -2 & 3 & 2 & -20 & -8 & 32 \\ & \downarrow & -6 & 8 & 24 & -32 \\ \hline & 3 & -4 & -12 & 16 & 0 \end{array}$$

$\boxed{\text{Yes}}$

Directions: Use the Rational Zero Theorem to list all possible rational zeros of the given function.

44. $f(x) = x^4 - 5x^2 + 17x - 54$

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm 27, \pm 54$

45. $f(x) = 3x^3 + 9x^2 - 11x + 36$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$

Directions: Use Descartes' Rule of Signs to give the possible number of positive and negative real zeros.

46. $f(x) = 3x^5 + 4x^4 + 7x^3 + 11x^2 + x + 15$

$f(-x) = -3x^5 + 4x^4 - 7x^3 + 11x^2 - x + 15$

Pos: 0

Neg: 5, 3, or 1

47. $f(x) = -7x^4 - 2x^3 + 4x^2 + 9x - 23$

$f(-x) = -7x^4 + 2x^3 + 4x^2 - 9x - 23$

Pos: 2 or 0

Neg: 2 or 0

Directions: Find all zeros for each function. Simplify all irrational zeros and complex solutions. Then, give the complete factorization of the function.

48. $f(x) = 4x^4 + 47x^2 - 12$

$f(x) = (x^2+12)(4x^2-1)$

$x^2 = -12 \quad | \quad x^2 = \frac{1}{4}$

$x = \pm 2i\sqrt{3} \quad | \quad x = \pm \frac{1}{2}$

zeros: $\{\pm 2i\sqrt{3}, \pm \frac{1}{2}\}$

$\boxed{f(x) = (x+2i\sqrt{3})(x-2i\sqrt{3})(2x+1)(2x-1)}$

49. $f(x) = x^3 + 9x^2 + 8x - 60$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 10, \pm 12, \pm 15, \pm 20, \pm 30, \pm 60$

$$\begin{array}{r|rrrr} -6 & 1 & 9 & 8 & -60 \\ & \downarrow & -6 & -18 & 60 \\ \hline & 1 & 3 & -10 & 0 \end{array}$$

$f(x) = (x+6)(x^2+3x-10)$

$\boxed{f(x) = (x+6)(x+5)(x-2)}$

zeros: $x = \{-6, -5, 2\}$

50. $f(x) = x^3 + 5x^2 + 23x + 51$ $\pm1, \pm3, \pm17, \pm51$

$$\begin{array}{r|rrrr} -3 & 1 & 5 & 23 & 51 \\ & \downarrow & -3 & -6 & -51 \\ \hline & 1 & 2 & 17 & 0 \end{array}$$

$f(x) = (x+3)(x^2+2x+17)$

$x=-3$ | $x = \frac{-2 \pm \sqrt{2^2 - 4(1)(17)}}{2(1)}$

$= \frac{-2 \pm 8i}{2}$

$= -1 \pm 4i$

Zeros: $x = \{-3, -1 \pm 4i\}$

$\boxed{f(x) = (x+3)(x-(-1+4i))(x-(-1-4i))}$

51. $f(x) = x^3 - 11x^2 + 34x - 30$ $\pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30$

$$\begin{array}{r|rrrr} 3 & 1 & -11 & 34 & -30 \\ & \downarrow & 3 & -24 & 30 \\ \hline & 1 & -8 & 10 & 0 \end{array}$$

$f(x) = (x-3)(x^2-8x+10)$

$x=3$ | $x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(10)}}{2(1)}$

$x = \frac{8 \pm \sqrt{24}}{2}$

$x = \frac{8 \pm 2\sqrt{6}}{2} = 4 \pm \sqrt{6}$

Zeros: $x = \{3, 4 \pm \sqrt{6}\}$

$\boxed{f(x) = (x-3)(x-(4+\sqrt{6}))(x-(4-\sqrt{6}))}$

Directions: Write a polynomial function with the given zeros.

52. $\pm i, \frac{5}{2}$ (multiplicity 2)

$(x+i)(x-i)(2x-5)^2$

$= (x^2+1)(4x^2-20x+25)$

$= 4x^4 - 20x^3 + 25x^2 + 4x^2 - 20x + 25$

$\boxed{f(x) = 4x^4 - 20x^3 + 29x^2 - 20x + 25}$

53. $-4, \pm 3\sqrt{2}$

$(x+4)(x-3\sqrt{2})(x+3\sqrt{2})$

$= (x+4)(x^2-18)$

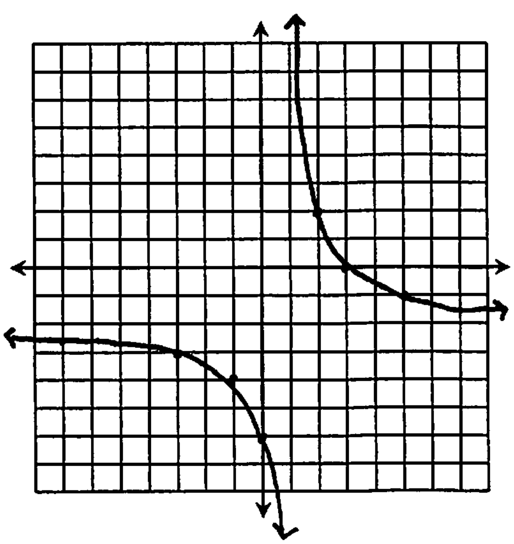
$= x^3 - 18x + 4x^2 - 72$

$\boxed{f(x) = x^3 + 4x^2 - 18x - 72}$

Topic F: Graphs of Rational Functions

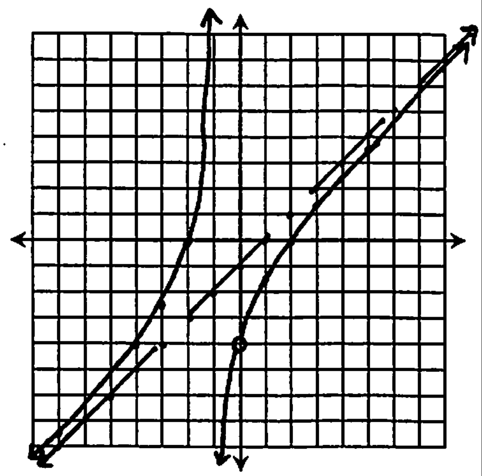
Directions: Graph each function, then identify its key characteristics.

54. $f(x) = \frac{-2x+6}{x-1}$



Domain: $\{x \mid x \neq -1\}$	**Range:** $\{y \mid y \neq -2\}$
***x*-intercept(s):** $(3,0)$	
***y*-intercept:** $(0,-6)$	
Vertical Asymptote: $x=1$	
Horizontal Asymptote: $y=-2$	
Slant Asymptote: None	
Hole(s): None	

55. $f(x) = \dfrac{x^3 - 4x}{x^2 + x}$



Domain: $\{x \mid x \neq 0, 1\}$	**Range:** $\mathbb{R}$
x-intercept(s): $(-2, 0), (2, 0)$	
y-intercept: None	
Vertical Asymptote: $x = -1$	
Horizontal Asymptote: None	
Slant Asymptote: $y = x - 1$	
Hole(s): $(0, -4)$	

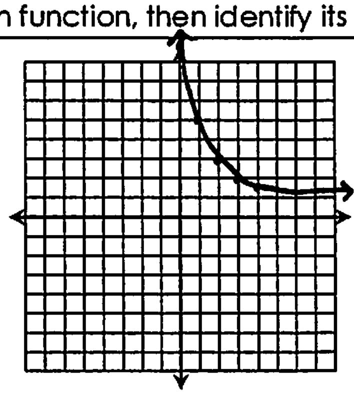
Topic I: Graphs of Exponential & Logarithmic Functions

Directions: Classify each function as exponential growth or decay.

56. $f(x) = -2 \cdot 5^x$ Growth	**57.** $f(x) = \dfrac{1}{2} \cdot \left(\dfrac{5}{4}\right)^x$ Growth	**58.** $f(x) = 4 \cdot \left(\dfrac{2}{7}\right)^x$ Decay

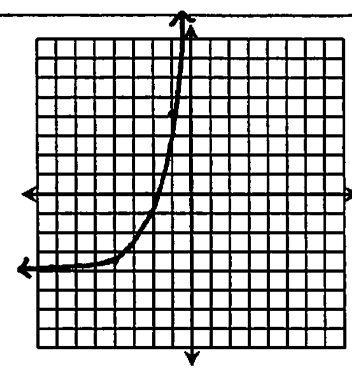
Directions: Graph each function, then identify its key characteristics.

59. $f(x) = 4 \cdot \left(\dfrac{1}{2}\right)^{x-1} + 1$



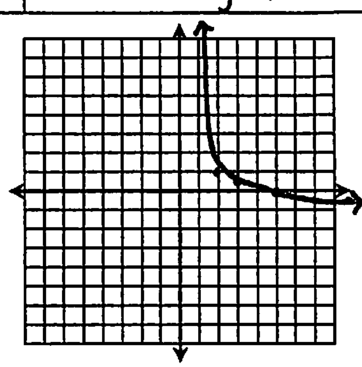
Domain: $\mathbb{R}$	**Range:** $\{y \mid y > 1\}$
y-intercept: $(0, 9)$	**Asymptote:** $y = 1$

60. $f(x) = 3 \cdot e^{x+2} - 4$



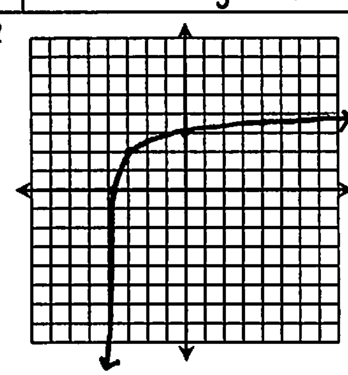
Domain: $\mathbb{R}$	**Range:** $\{y \mid y > -4\}$
y-intercept: $(0, 18.17)$	**Asymptote:** $y = -4$

61. $f(x) = \log_{\frac{1}{4}}(x - 1) + 1$



Domain: $\{x \mid x > 1\}$	**Range:** $\mathbb{R}$
x-intercept: $(5, 0)$	**Asymptote:** $x = 1$

62. $f(x) = \log_4(x + 4) + 2$



Domain: $\{x \mid x > -4\}$	**Range:** $\mathbb{R}$
x-intercept: $(-3.94, 0)$	**Asymptote:** $x = -4$

Topic J: Properties of Logarithms/Simplifying Logarithms

Directions: Condense each expression into a single logarithm.

63. $3 \cdot \log_4 2c - \frac{1}{2}\log_4 64$

$\log_4 \frac{(2c)^3}{64^{1/2}} = \log_4 \frac{8c^3}{8}$

$= \boxed{\log_4 c^3}$

64. $\frac{1}{4}(\ln 256 + 3 \cdot \ln p) - 2 \cdot \ln q$

$\ln \frac{(256 \cdot p^3)^{1/4}}{q^2} = \boxed{\ln \frac{4\sqrt[4]{p^3}}{q^2}}$

Directions: Expand each logarithm completely.

65. $\log_6 \frac{n\sqrt[5]{2m^2}}{5}$

$\boxed{\log_6 n + \frac{1}{5}\log_6 2 + \frac{2}{5}\log_6 m - \log_6 5}$

66. $\ln(4a^2\sqrt{b})^3$

$\ln 64 + \ln a^6 + \ln b^{3/3}$

$= \boxed{\ln 64 + 6\ln a + \frac{2}{3}\ln b}$

Topic K: Solving Logarithmic & Exponential Equations

Directions: Solve each equation, rounding to the nearest ten-thousandths place when necessary.

67. $\log_7 6 + \log_7 (x+1) = 2 \cdot \log_7 (4x+1)$

$\log_7 6(x+1) = \log_7 (4x+1)^2$

$6x+6 = 16x^2+8x+1$

$0 = 16x^2+2x-5$

$0 = (8x+5)(2x-1)$

$\cancel{x = -\frac{5}{8}}$ $\boxed{x = \frac{1}{2}}$

68. $\ln(w+6) - \ln(w+4) = 1$

$\ln \frac{w+6}{w+4} = 1$

$\frac{w+6}{w+4} = e^1$

$2.71828(w+4) = w+6$

$1.71828w = -4.873127$

$\boxed{w = -2.836}$

69. $\left(\frac{1}{27}\right)^{3k-2} = \frac{1}{81} \cdot 243^{4-k}$

$3^{-3(3k-2)} = 3^{-4} \cdot 3^{5(4-k)}$

$-9k+6 = -4+20-5k$

$-4k = 10$

$\boxed{k = -\frac{5}{2}}$

70. $-5 \cdot 8^{-5y-6} + 8 = -40$

$8^{-5y-6} = 9.6$

$(-5y-6)\log 8 = \log 9.6$

$-5y-6 = 1.087678$

$-5y = 7.087678$

$\boxed{y = -1.4175}$

71. Jason's boat was initially priced at \$45,000. After 7 years, the boat is worth half of its original purchase price. Write and a continuous exponential decay function to model the price of the boat, then find the rate of depreciation.

$22500 = 45000 \cdot e^{r(0.7)}$

$0.5 = e^{7r}$

$\ln 0.5 = 7r$

$-0.69315 = 7r$

$r = -0.09902$

$\boxed{\text{9.9\% depreciation per year}}$

72. Karen inherited \$6,000 from her grandmother. She deposited half of this money into an investment account that earns 2.75% interest compounded quarterly. She deposited the other half of the money into an account that earns 4% interest compounded continuously. Assuming neither account had any additional deposits or withdrawals, find the total amount in the two accounts after 15 years.

$A = 3000\left(1 + \frac{.0275}{4}\right)^{4 \cdot 15}$ $\qquad$ $A = 3000e^{.04(15)}$

$A = 4525.38$ $\qquad$ $A = 5466.36$ $\qquad$ $\boxed{\$9991.74}$

Topic L: Trigonometric Functions

Directions: Convert the degrees to radians, and radians to degrees.

73. 198° $\quad \frac{11\pi}{10}$	**74.** -480° $\quad -\frac{8\pi}{3}$	**75.** $\frac{2\pi}{9}$ $\quad 40°$	**76.** $-\frac{13\pi}{8}$ $\quad -292.5°$

Directions: Write each measure in Degree-Minute-Second form.

77. 154.861°

$.861(60) = 51.66$

$.66(60) = 39.6$

$\boxed{154° \, 51' \, 40''}$

78. -236.255°

$.255(60) = 15.3$

$.3(60) = 18$

$\boxed{-236° \, 15' \, 18''}$

Directions: Write each angle measure in decimal degree form.

79. -28° 13' 38"

$13/60 = .2167$

$38/3600 = .0106$

$\boxed{-28.2273°}$

80. 341° 47' 22"

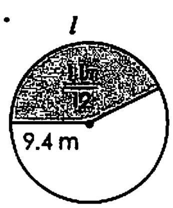
$47/60 = .7833$

$22/3600 = .0061$

$\boxed{341.7894°}$

Directions: Find the length of the intercepted arc, l, and A, the area of the shaded sector.

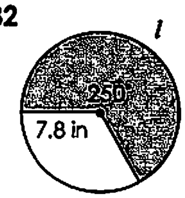
81.



$S = 9.4\left(\frac{11\pi}{12}\right)$

$A = \frac{1}{2}\left(\frac{11\pi}{12}\right)(9.4)^2$

82



$S = 7.8\left(\frac{25\pi}{18}\right)$

$A = \frac{1}{2}\left(\frac{25\pi}{18}\right)(7.8)^2$

l = 27.1 m	A = 127.2 m^2	l = 34 in	A = 132.7 in^2

83. If (-3, 15) is a point on the terminal side of θ in standard form, find the exact values of the trigonometric functions of θ.

$3^2 + 15^2 = r^2$

$234 = r^2$

$3\sqrt{26} = r$

$x = -3$

$y = 15$

$r = 3\sqrt{26}$

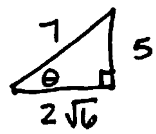
$\sin\theta = \frac{5\sqrt{26}}{26}$	$\csc\theta = \frac{\sqrt{26}}{5}$
$\cos\theta = -\frac{\sqrt{26}}{26}$	$\sec\theta = -\sqrt{26}$
$\tan\theta = -5$	$\cot\theta = -\frac{1}{5}$

84. If $\csc\theta = -\frac{7}{5}$ and $\tan\theta > 0$, find the exact values of the remaining trigonometric functions of θ.

$x^2 + 5^2 = 7^2$

$x^2 = 24$

$x = 2\sqrt{6}$



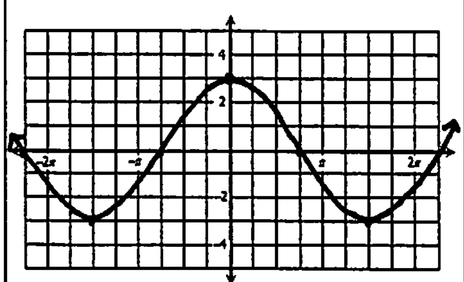
$\sin\theta = -\frac{5}{7}$	$\csc\theta = -\frac{7}{5}$
$\cos\theta = -\frac{2\sqrt{6}}{7}$	$\sec\theta = -\frac{7\sqrt{6}}{12}$
$\tan\theta = \frac{5\sqrt{6}}{12}$	$\cot\theta = \frac{2\sqrt{6}}{5}$

Directions: Use the unit circle to give the exact value of each trigonometric function.

85. $\sin\frac{5\pi}{3}$ $-\frac{\sqrt{3}}{2}$	**86.** $\cot\frac{\pi}{3}$ $\frac{\sqrt{3}}{3}$	**87.** $\sec\frac{3\pi}{4}$ $-\sqrt{2}$	**88.** $\tan\left(-\frac{9\pi}{4}\right)$ -1

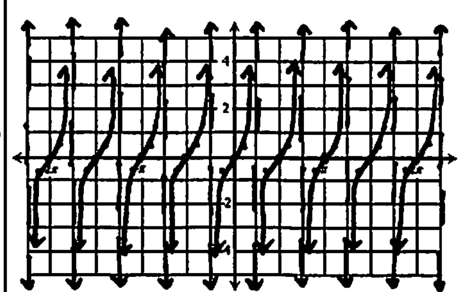
Directions: Graph each function and identify its key characteristics.

89. $f(x) = 3\cdot\cos\frac{2}{3}x$



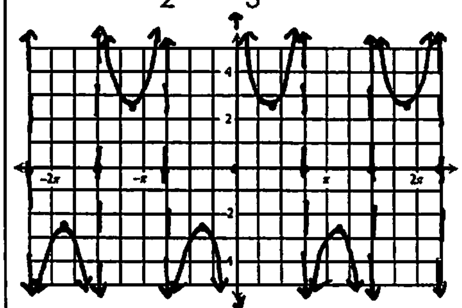
Amplitude:	Period:
3	3π

90. $f(x) = \frac{1}{2}\cdot\tan 2x$



Amplitude:	Period:
undef.	$\frac{\pi}{2}$

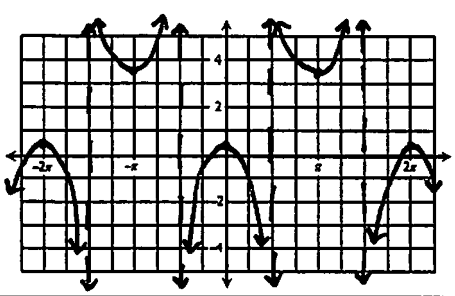
91. $f(x) = \frac{5}{2}\cdot\csc\frac{4}{3}x$



Amplitude:	Period:
Undef.	$\frac{3\pi}{2}$

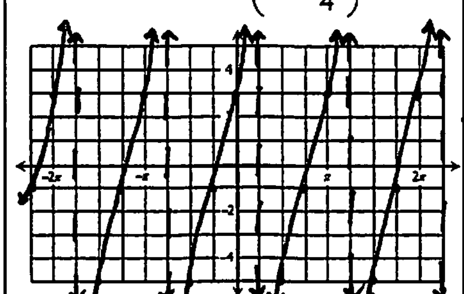
Directions: Graph each function, then give the amplitude, period, phase shift, and vertical shift.

92. $f(x) = \frac{3}{2}\cdot\sec(x-\pi)+2$



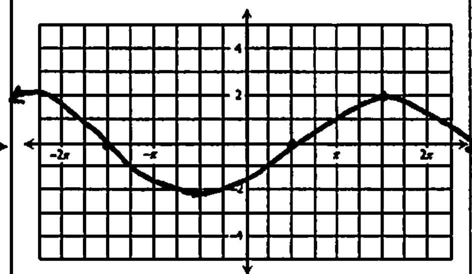
Amplitude:	Period:
undef.	2π
Phase Shift:	**Vertical Shift:**
Right π	up 2

93. $f(x) = -4\cdot\cot\left(x+\frac{3\pi}{4}\right)-1$



Amplitude:	Period:
undef.	π
Phase Shift:	**Vertical Shift:**
Left $\frac{3\pi}{4}$	down 1

94. $f(x) = 2\cdot\sin\frac{1}{2}\left(x-\frac{\pi}{2}\right)$



Amplitude:	Period:
2	4π
Phase Shift:	**Vertical Shift:**
Right $\frac{\pi}{2}$	None

Directions: Give the exact value, if it exists.

95. arctan 1

$\frac{\pi}{4}$

96. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$\frac{\pi}{3}$

96. $\cos^{-1}\left[\sin\left(-\frac{5\pi}{6}\right)\right]$

$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$