For Each Problem Set Choose Evens or Odds.

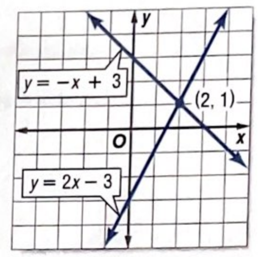


# 9 Solving Systems of Linear Equations

- Two or more equations that have common variables are called a **system of equations**. The solution of a system of equations in two variables is an ordered pair of numbers that satisfies both equations. A system of two linear equations can have zero, one, or an infinite number of solutions. There are three methods by which systems of equations can be solved: graphing, elimination, and substitution.

**Example 1** **Solve each system of equations by graphing. Then determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.**

**a.** $y = -x + 3$
$y = 2x - 3$

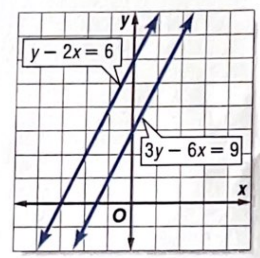



The graphs appear to intersect at (2, 1). Check this estimate by replacing $x$ with 2 and $y$ with 1 in each equation.

**Check:** $y = -x + 3$ $\qquad$ $y = 2x - 3$
$1 \stackrel{?}{=} -2 + 3$ $\qquad$ $1 \stackrel{?}{=} 2(2) - 3$
$1 = 1$ ✓ $\qquad$ $1 = 1$ ✓

The system has one solution at (2, 1).

**b.** $y - 2x = 6$
$3y - 6x = 9$




The graphs of the equations are parallel lines. Since they do not intersect, there are no solutions of this system of equations. Notice that the lines have the same slope but different $y$-intercepts. Equations with the same slope *and* the same $y$-intercepts have an infinite number of solutions.

- It is difficult to determine the solution of a system when the two graphs intersect at noninteger values. There are algebraic methods by which an exact solution can be found. One such method is **substitution**.

**Example 2** **Use substitution to solve the system of equations.**

$y = -4x$
$2y + 3x = 8$

Since $y = -4x$, substitute $-4x$ for $y$ in the second equation.

| | |
|---|---|
| $2y + 3x = 8$ | Second equation |
| $2(-4x) + 3x = 3$ | $y = -4x$ |
| $-8x + 3x = 8$ | Simplify. |
| $-5x = 8$ | Combine like terms. |
| $\frac{-5x}{-5} = \frac{8}{-5}$ | Divide each side by −5. |
| $x = -\frac{8}{5}$ | Simplify. |

Use $y = -4x$ to find the value of $y$.

| | |
|---|---|
| $y = -4x$ | First equation |
| $y = -4\left(-\frac{8}{5}\right)$ | $x = -\frac{8}{5}$ |
| $y = \frac{32}{5}$ | Simplify. |

The solution is $\left(-\frac{8}{5}, \frac{32}{5}\right)$.

- Sometimes adding or subtracting two equations together will eliminate one variable. Using this step to solve a system of equations is called **elimination**.

**Example 3** **Use elimination to solve the system of equations.**

$3x + 5y = 7$
$4x + 2y = 0$

Either $x$ or $y$ can be eliminated. In this example, we will eliminate $x$.

$3x + 5y = 7$ Multiply by 4. $12x + 20y = 28$

$4x + 2y = 0$ Multiply by $-3$. $+ \; -12x - 6y = 0$

$14y = 28$ Add the equations.

$\frac{14y}{14} = \frac{28}{14}$ Divide each side by 14.

$y = 2$ Simplify.

Now substitute 2 for $y$ in either equation to find the value of $x$.

$4x + 2y = 0$ Second equation
$4x + 2(2) = 0$ $y = 2$
$4x + 4 = 0$ Simplify.
$4x + 4 - 4 = 0 - 4$ Subtract 4 from each side.
$4x = -4$ Simplify.
$\frac{4x}{4} = \frac{-4}{4}$ Divide each side by 4.
$x = -1$ Simplify.

The solution is $(-1, 2)$.

**Exercises** **Solve by graphing.**

1. $y = -x + 2$; $y = -\frac{1}{2}x + 1$
2. $y = 3x - 3$; $y = x + 1$
3. $y - 2x = 1$; $2y - 4x = 1$
4. $2x - 4y = -2$; $-6x + 12y = 6$
5. $4x + 3y = 12$; $3x - y = 9$
6. $3y + x = -3$; $y - 3x = -1$

**Solve by substitution.**

7. $-5x + 3y = 12$; $x + 2y = 8$
8. $x - 4y = 22$; $2x + 5y = -21$
9. $y + 5x = -3$; $3y - 2x = 8$
10. $y - 2x = 2$; $7y + 4x = 23$
11. $2x - 3y = -8$; $-x + 2y = 5$
12. $4x + 2y = 5$; $3x - y = 10$

**Solve by elimination.**

13. $-3x + y = 7$; $3x + 2y = 2$
14. $3x + 4y = -1$; $-9x - 4y = 13$
15. $-4x + 5y = -11$; $2x + 3y = 11$
16. $6x - 5y = 1$; $-2x + 9y = 7$
17. $3x - 2y = 8$; $5x - 3y = 16$
18. $4x + 7y = -17$; $3x + 2y = -3$

**Name an appropriate method to solve each system of equations. Then solve the system.**

19. $4x - y = 11$; $2x - 3y = 3$
20. $4x + 6y = 3$; $-10x - 15y = -4$
21. $3x - 2y = 6$; $5x - 5y = 5$
22. $3y + x = 3$; $-2y + 5x = 15$
23. $4x - 7y = 8$; $-2x + 5y = -1$
24. $x + 3y = 6$; $4x - 2y = -32$

# 10 Square Roots and Simplifying Radicals

- A radical expression is an expression that contains a square root. The expression is in simplest form when the following three conditions have been met.
- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.
- The **Product Property** states that for two numbers $a$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

**Example 1** **Simplify.**

**a.** $\sqrt{45}$

$\sqrt{45} = \sqrt{3 \cdot 3 \cdot 5}$ Prime factorization of 45

$= \sqrt{3^2} \cdot \sqrt{5}$ Product Property of Square Roots

$= 3\sqrt{5}$ Simplify.

**b.** $\sqrt{3} \cdot \sqrt{3}$

$\sqrt{3} \cdot \sqrt{3} = \sqrt{3 \cdot 3}$ Product Property

$= \sqrt{9}$ or 3 Simplify.

**c.** $\sqrt{6} \cdot \sqrt{15}$

$\sqrt{6} \cdot \sqrt{15} = \sqrt{6 \cdot 15}$ Product Property

$= \sqrt{3 \cdot 2 \cdot 3 \cdot 5}$ Prime factorization

$= \sqrt{3^2} \cdot \sqrt{10}$ Product Property

$= 3\sqrt{10}$ Simplify.

- For radical expressions in which the exponent of the variable inside the radical is *even* and the resulting simplified exponent is *odd*, you must use absolute value to ensure nonnegative results.

**Example 2** $\sqrt{20x^3y^5z^6}$

$\sqrt{20x^3y^5z^6} = \sqrt{2^2 \cdot 5 \cdot x^3 \cdot y^5 \cdot z^6}$ Prime factorization

$= \sqrt{2^2} \cdot \sqrt{5} \cdot \sqrt{x^3} \cdot \sqrt{y^5} \cdot \sqrt{z^6}$ Product Property

$= 2 \cdot \sqrt{5} \cdot x \cdot \sqrt{x} \cdot y^2 \cdot \sqrt{y} \cdot |z^3|$ Simplify.

$= 2xy^2|z^3|\sqrt{5xy}$ Simplify.

- The **Quotient Property** states that for any numbers $a$ and $b$, where $a \geq 0$ and $b \geq 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

**Example 3** **Simplify** $\sqrt{\frac{25}{16}}$.

$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}}$ Quotient Property

$= \frac{5}{4}$ Simplify.

- Rationalizing the denominator of a radical expression is a method used to eliminate radicals from the denominator of a fraction. To rationalize the denominator, multiply the expression by a fraction equivalent to 1 such that the resulting denominator is a perfect square.

**Example 4** **Simplify.**

a. $\frac{2}{\sqrt{3}}$

$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$.

$= \frac{2\sqrt{3}}{3}$ Simplify.

b. $\frac{\sqrt{13y}}{\sqrt{18}}$

$\frac{\sqrt{13y}}{\sqrt{18}} = \frac{\sqrt{13y}}{\sqrt{2 \cdot 3 \cdot 3}}$ Prime factorization

$= \frac{\sqrt{13y}}{3\sqrt{2}}$ Product Property

$= \frac{\sqrt{13y}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ Multiply by $\frac{\sqrt{2}}{\sqrt{2}}$.

$= \frac{\sqrt{26y}}{6}$ Product Property

- Sometimes, conjugates are used to simplify radical expressions. Conjugates are binomials of the form $p\sqrt{q} + r\sqrt{s}$ and $p\sqrt{q} - r\sqrt{s}$.

**Example 5** **Simplify** $\frac{3}{5 - \sqrt{2}}$.

$\frac{3}{5 - \sqrt{2}} = \frac{3}{5 - \sqrt{2}} \cdot \frac{5 + \sqrt{2}}{5 + \sqrt{2}}$ $\frac{5 + \sqrt{2}}{5 + \sqrt{2}} = 1$

$= \frac{3(5 + \sqrt{2})}{5^2 - (\sqrt{2})^2}$ $(a - b)(a + b) = a^2 - b^2$

$= \frac{15 + 3\sqrt{2}}{25 - 2}$ Multiply. $(\sqrt{2})^2 = 2$

$= \frac{15 + 3\sqrt{2}}{23}$ Simplify.

**Exercises** **Simplify.**

1. $\sqrt{32}$
2. $\sqrt{75}$
3. $\sqrt{50} \cdot \sqrt{10}$
4. $\sqrt{12} \cdot \sqrt{20}$
5. $\sqrt{6} \cdot \sqrt{6}$
6. $\sqrt{16} \cdot \sqrt{25}$
7. $\sqrt{98x^3y^6}$
8. $\sqrt{56a^2b^4c^5}$
9. $\sqrt{\frac{81}{49}}$
10. $\sqrt{\frac{121}{16}}$
11. $\sqrt{\frac{63}{8}}$
12. $\sqrt{\frac{288}{147}}$
13. $\frac{\sqrt{10p^3}}{\sqrt{27}}$
14. $\frac{\sqrt{108}}{\sqrt{2q^6}}$
15. $\frac{4}{5 - 2\sqrt{3}}$
16. $\frac{7\sqrt{3}}{5 - 2\sqrt{6}}$
17. $\frac{3}{\sqrt{48}}$
18. $\frac{\sqrt{24}}{\sqrt{125}}$
19. $\frac{3\sqrt{5}}{2 - \sqrt{2}}$
20. $\frac{3}{-2 + \sqrt{13}}$

# 11 Multiplying Polynomials

- The **Product of Powers** rule states that for any number $a$ and all integers $m$ and $n$, $a^m \cdot a^n = a^{m+n}$.

**Example 1** **Simplify each expression.**

**a.** $(4p^5)(p^4)$

$(4p^5)(p^4) = (4)(1)(p^5 \cdot p^4)$ Commutative and Associative Properties

$= (4)(1)(p^{5+4})$ Product of powers

$= 4p^9$ Simplify.

**b.** $(3yz^5)(-9y^2z^2)$

$(3yz^5)(-9y^2z^2) = (3)(-9)(y \cdot y^2)(z^5 \cdot z^2)$ Commutative and Associative Properties

$= -27(y^{1+2})(z^{5+2})$ Product of powers

$= -27y^3z^7$ Simplify.

- The Distributive Property can be used to multiply a monomial by a polynomial.

**Example 2** **Simplify $3x^3(-4x^2 + x - 5)$.**

$3x^3(-4x^2 + x - 5) = 3x^3(-4x^2) + 3x^3(x) - 3x^3(5)$ Distributive Property

$= -12x^5 + 3x^4 - 15x^3$ Multiply.

- To find the power of a power, multiply the exponents. This is called the **Power of a Power** rule.

**Example 3** **Simplify each expression.**

**a.** $(-3x^2y^4)^3$

$(-3x^2y^4)^3 = (-3)^3(x^2)^3(y^4)^3$ Power of a product

$= -27x^6y^{12}$ Power of a power

**b.** $(xy)^3(-2x^4)^2$

$(xy)^3(-2x^4)^2 = x^3y^3(-2)^2(x^4)^2$ Power of a product

$= x^3y^3(4)x^8$ Power of a power

$= 4x^3 \cdot x^8 \cdot y^3$ Commutative Property

$= 4x^{11}y^3$ Product of powers

- To multiply two binomials, find the sum of the products of

F the *First* terms,
O the *Outer* terms,
I the *Inner* terms, and
L the *Last* terms.

**Example 4** **Find each product.**

**a.** $(2x - 3)(x + 1)$

F O I L

$(2x - 3)(x + 1) = (2x)(x) + (2x)(1) + (-3)(x) + (-3)(1)$ FOIL method

$= 2x^2 + 2x - 3x - 3$ Multiply.

$= 2x^2 - x - 3$ Combine like terms.

**b.** $(x + 6)(x + 5)$

F O I L

$(x + 6)(x + 5) = (x)(x) + (x)(5) + (6)(x) + (6)(5)$ FOIL method

$= x^2 + 5x + 6x + 30$ Multiply.

$= x^2 + 11x + 30$ Combine like terms.

- The Distributive Property can be used to multiply any two polynomials.

**Example 5** Find $(3x - 2)(2x^2 + 7x - 4)$.

$(3x - 2)(2x^2 + 7x - 4) = 3x(2x^2 + 7x - 4) - 2(2x^2 + 7x - 4)$ Distributive Property

$= 6x^3 + 21x^2 - 12x - 4x^2 - 14x + 8$ Distributive Property

$= 6x^3 + 17x^2 - 26x + 8$ Combine like terms.

- Three special products are: $(a + b)^2 = a^2 + 2ab + b^2$,
  $(a - b)^2 = a^2 - 2ab + b^2$, and
  $(a + b)(a - b) = a^2 - b^2$.

**Example 6** Find each product.

a. $(2x - z)^2$

$(a - b)^2 = a^2 - 2ab + b^2$ Square of a difference

$(2x - z)^2 = (2x)^2 - 2(2x)(z) + (z)^2$ $a = 2x$ and $b = z$

$= 4x^2 - 4xz + z^2$ Simplify.

b. $(3x + 7)(3x - 7)$

$(a + b)(a - b) = a^2 - b^2$ Product of sum and difference

$(3x + 7)(3x - 7) = (3x)^2 - (7)^2$ $a = 3x$ and $b = 7$

$= 9x^2 - 49$ Simplify.

**Exercises** Find each product.

1. $(3q^2)(q^5)$
2. $(5m)(4m^3)$
3. $\left(\frac{9}{2}c\right)\left(8c^5\right)$
4. $(n^6)(10n^2)$
5. $(fg^8)(15f^2g)$
6. $(6j^4k^4)(j^2k)$
7. $(2ab^3)(4a^2b^2)$
8. $\left(\frac{8}{5}x^3y\right)\left(4x^3y^2\right)$
9. $-2q^2(q^2 + 3)$
10. $5p(p - 18)$
11. $15c(-3c^2 + 2c + 5)$
12. $8x(-4x^2 - x + 11)$
13. $4m^2(-2m^2 + 7m - 5)$
14. $8y^2(5y^3 - 2y + 1)$
15. $\left(\frac{3}{2}m^3n^2\right)^2$
16. $(-2c^3d^2)^2$
17. $(-5wx^5)^3$
18. $(6a^5b)^3$
19. $(k^2\ell)^3(13k^2)^2$
20. $(-5w^3x^2)^2(2w^5)^2$
21. $(-7y^3z^2)(4y^2)^4$
22. $\left(\frac{1}{2}p^2q^2\right)^2\left(4pq^3\right)^3$
23. $(m - 1)(m - 4)$
24. $(s - 7)(s - 2)$
25. $(x - 3)(x + 4)$
26. $(a + 3)(a - 6)$
27. $(5d + 3)(d - 4)$
28. $(q + 2)(3q + 5)$
29. $(2q + 3)(5q + 2)$
30. $(2a - 3)(2a - 5)$
31. $(d + 1)(d - 1)$
32. $(4a - 3)(4a + 3)$
33. $(s - 5)^2$
34. $(3f - g)^2$
35. $(2r - 5)^2$
36. $\left(t + \frac{8}{3}\right)^2$
37. $(x + 4)(x^2 - 5x - 2)$
38. $(x - 2)(x^2 + 3x - 7)$
39. $(3b - 2)(3b^2 + b + 1)$
40. $(2j + 7)(j^2 - 2j + 4)$

## 12 Dividing Polynomials

- The **Quotient of Powers** rule states that for any nonzero number $a$ and all integers $m$ and $n$, $\frac{a^m}{a^n} = a^{m-n}$.
- To find the power of a quotient, find the power of the numerator and the power of the denominator.

**Example 1** **Simplify.**

**a.** $\frac{x^5y^8}{-xy^3}$

$$\frac{x^5y^8}{-xy^3} = \left(\frac{x^5}{-x}\right)\left(\frac{y^8}{y^3}\right)$$ Group powers that have the same base.

$$= -(x^{5-1})(y^{8-3})$$ Quotient of powers

$$= -x^4y^5$$ Simplify.

**b.** $\left(\frac{4z^3}{3}\right)^3$

$$\left(\frac{4z^3}{3}\right)^3 = \frac{(4z^3)^3}{3^3}$$ Power of a quotient

$$= \frac{4^3(z^3)^3}{3^3}$$ Power of a product

$$= \frac{64z^9}{27}$$ Power of a product

**c.** $\frac{w^{-2}x^4}{2w^{-5}}$

$$\frac{w^{-2}x^4}{2w^{-5}} = \frac{1}{2}\left(\frac{w^{-2}}{w^{-5}}\right)x^4$$ Group powers that have the same base.

$$= \frac{1}{2}(w^{-2-(-5)})x^4$$ Quotient of powers

$$= \frac{1}{2}w^3x^4$$ Simplify.

- You can divide a polynomial by a monomial by separating the terms of the numerator.

**Example 2** **Simplify** $\frac{15x^3 - 3x^2 + 12x}{3x}$.

$$\frac{15x^3 - 3x^2 + 12x}{3x} = \frac{15x^3}{3x} - \frac{3x^2}{3x} + \frac{12x}{3x}$$ Divide each term by $3x$.

$$= 5x^2 - x + 4$$ Simplify.

- Division can sometimes be performed using factoring.

**Example 3** **Find** $(n^2 - 8n - 9) \div (n - 9)$.

$$(n^2 - 8n - 9) \div (n - 9) = \frac{n^2 - 8n - 9}{(n - 9)}$$ Write as a rational expression.

$$= \frac{(n - 9)(n + 1)}{(n - 9)}$$ Factor the numerator.

$$= \frac{\cancel{(n - 9)}(n + 1)}{\cancel{(n - 9)}}$$ Divide by the GCF.

$$= n + 1$$ Simplify.

- When you cannot factor, you can use a long division process similar to the one you use in arithmetic.

**Example 4** **Find $(n^3 - 4n^2 - 9) \div (n - 3)$.**

In this case, there is no $n$ term, so you must rename the dividend using 0 as the coefficient of the missing term.

$(n^3 - 4n^2 + 9) \div (n - 3) = (n^3 - 4n^2 + 0n + 9) \div (n - 3)$

Divide the first term of the dividend, $n^3$, by the first term of the divisor, $n$.

$$\begin{array}{rll} & n^2 - n - 3 & \\ n - 3 \,\big)\!\! & \overline{n^3 - 4n^2 + 0n + 12} & \\ & \underline{(-)\, n^3 - 3n^2} & \text{Multiply } n^2 \text{ and } n - 3. \\ & -n^2 + 0n & \text{Subtract and bring down } 0n. \\ & \underline{(-) -n^2 + 3n} & \text{Multiply } -n \text{ and } n - 3. \\ & -3n + 12 & \text{Subtract and bring down 12.} \\ & \underline{(-) -3n + \;\,9} & \text{Multiply } -3 \text{ and } n - 3. \\ & 3 & \text{Subtract.} \end{array}$$

Therefore, $(n^3 - 4n^2 + 9) \div (n - 3) = n^2 - n - 3 + \frac{3}{n - 3}$. Since the quotient has a nonzero remainder, $n - 3$ is not a factor of $n^3 - 4n^2 + 9$.

**Exercises** **Find each quotient.**

1. $\frac{a^2c^2}{2a}$
2. $\frac{5q^5r^3}{q^2r^2}$
3. $\frac{b^2d^5}{8b^{-2}d^3}$
4. $\frac{5p^{-3}x}{2p^{-7}}$
5. $\frac{3r^{-3}s^2t^4}{2r^2st^{-3}}$
6. $\frac{3x^3y^{-1}z^5}{xyz^2}$
7. $\left(\frac{w^4}{6}\right)^3$
8. $\left(\frac{-3q^2}{5}\right)^3$
9. $\left(\frac{-2y^2}{7}\right)^2$
10. $\left(\frac{5m^2}{3}\right)^4$
11. $\frac{4z^2 - 16z - 36}{4z}$
12. $(5d^2 + 8d - 20) \div 10d$
13. $(p^3 - 12p^2 + 3p + 8) \div 4p$
14. $(b^3 + 4b^2 + 10) \div 2b$
15. $\frac{a^3 - 6a^2 + 4a - 3}{a^2}$
16. $\frac{8x^2y - 10xy^2 + 6x^3}{2x^2}$
17. $\frac{s^2 - 2s - 8}{s - 4}$
18. $(r^2 + 9r + 20) \div (r + 5)$
19. $(t^2 - 7t + 12) \div (t - 3)$
20. $(c^2 + 3c - 54) \div (c + 9)$
21. $(2q^2 - 9q - 5) \div (q - 5)$
22. $\frac{3z^2 - 2z - 5}{z + 1}$
23. $\frac{(m^3 + 3m^2 - 5m + 1)}{m - 1}$
24. $(d^3 - 2d^2 + 4d + 24) \div (d + 2)$
25. $(2j^3 + 5j + 26) \div (j + 2)$
26. $\frac{2x^3 + 3x^2 - 176}{x - 4}$
27. $(x^2 + 6x - 3) \div (x + 4)$
28. $\frac{h^3 + 2h^2 - 6h + 1}{h - 2}$

Prerequisite Skills

# 13 Factoring to Solve Equations

- Some polynomials can be factored using the Distributive Property.

**Example 1** **Factor $5t^2 + 15t$.**

Find the greatest common factor (GCF) of $5t^2$ and $15t$.

$5t^2 = 5 \cdot t \cdot t$, $15t = 3 \cdot 5 \cdot t$ — GCF: $5 \cdot t$ or $5t$

$5t^2 + 15t = 5t(t) + 5t(3)$ — Rewrite each term using the GCF.

$= 5t(t + 3)$ — Distributive Property

- To factor polynomials of the form $x^2 + bx + c$, find two integers $m$ and $n$ so that $mn = c$ and $m + n = b$. Then write $x^2 + bx + c$ using the pattern $(x + m)(x + n)$.

**Example 2** **Factor each polynomial.**

**a.** $x^2 + 7x + 10$

Both $b$ and $c$ are positive.

In this equation, $b$ is 7 and $c$ is 10. Find two numbers with a product of 10 and with a sum of 7.

| Factors of 10 | Sum of Factors |
|---|---|
| 1, 10 | 11 |
| 2, 5 | 7 |

$x^2 + 7x + 10 = (x + m)(x + n)$ — The correct factors are 2 and 5.

$= (x + 2)(x + 5)$ — Write the pattern; $m = 2$ and $n = 5$.

**b.** $x^2 - 8x + 15$

$b$ is negative and $c$ is positive.

In this equation, $b$ is $-8$ and $c$ is 15. This means that $m + n$ is negative and $mn$ is positive. So $m$ and $n$ must both be negative.

| Factors of 15 | Sum of Factors |
|---|---|
| −1, −15 | −16 |
| −3, −5 | −8 |

$x^2 - 8x + 15 = (x + m)(x + n)$ — The correct factors are $-3$ and $-5$.

$= (x - 3)(x - 5)$ — Write the pattern; $m = -3$ and $n = -5$.

- To factor polynomials of the form $ax^2 + bx + c$, find two integers $m$ and $n$ with a product equal to $ac$ and with a sum equal to $b$. Write $ax^2 + bx + c$ using the pattern $ax^2 + mx + nx + c$. Then factor by grouping.

**c.** $5x^2 - 19x - 4$

$b$ is negative and $c$ is negative.

In this equation, $a$ is 5, $b$ is $-19$, and $c$ is $-4$.
Find two numbers with a product of $-20$ and with a sum of $-19$.

| Factors of −20 | Sum of Factors |
|---|---|
| −2, 10 | 8 |
| 2, −10 | −8 |
| −1, 20 | 19 |
| 1, −20 | −19 |

The correct factors are 1 and −20.

$5x^2 - 19x - 4 = 5x^2 + mx + nx - 4$ — Write the pattern.

$= 5x^2 + x + (-20)x - 4$ — $m = 1$ and $n = -20$

$= (5x^2 + x) - (20x + 4)$ — Group terms with common factors.

$= x(5x + 1) - 4(5x + 1)$ — Factor the GCF from each group.

$= (x - 4)(5x + 1)$ — Distributive Property

- Here are some special products.

**Perfect Square Trinomials**

$$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$$

**Difference of Squares**

$$a^2 - b^2 = (a + b)(a - b)$$

**Example 3** **Factor each polynomial.**

**a.** $9x^2 + 6x + 1$ ← The first and last terms are perfect squares, and the middle term is equal to $2(3x)(1)$.

$9x^2 + 6x + 1 = (3x)^2 + 2(3x)(1) + 1^2$ Write as $a^2 + 2ab + b^2$.

$= (3x + 1)^2$ Factor using the pattern.

**b.** $x^2 - 9 = 0$ ← This is a difference of squares.

$x^2 - 9 = x^2 - (3)^2$ Write in the form $a^2 - b^2$.

$= (x - 3)(x + 3)$ Factor the difference of squares.

- The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$. Since 0 times any number is equal to zero, this implies that we can use factoring to solve equations.

**Example 4** **Solve $x^2 - 5x + 4 = 0$ by factoring.**

Factor the polynomial. This expression is of the form $x^2 + bx + c$.

$x^2 - 5x + 4 = 0$ Original equation

$(x - 1)(x - 4) = 0$ Factor the polynomial.

If $ab = 0$, then $a = 0$, $b = 0$, or both equal 0. Let each factor equal 0.

$x - 1 = 0$ or $x - 4 = 0$

$x = 1$ $x = 4$

**Exercises** **Factor each polynomial.**

1. $u^2 - 12u$
2. $w^2 + 4w$
3. $7j^2 - 28j$
4. $2g^2 + 24g$
5. $6x^2 + 2x$
6. $5t^2 - 30t$
7. $z^2 + 10z + 21$
8. $n^2 + 8n + 15$
9. $h^2 + 8h + 12$
10. $x^2 + 14x + 48$
11. $m^2 + 6m - 7$
12. $b^2 + 2b - 24$
13. $q^2 - 9q + 18$
14. $p^2 - 5p + 6$
15. $a^2 - 3a - 4$
16. $k^2 - 4k - 32$
17. $n^2 - 7n - 44$
18. $y^2 - 3y - 88$
19. $3z^2 + 4z - 4$
20. $2y^2 + 9y - 5$
21. $5x^2 + 7x + 2$
22. $3s^2 + 11s - 4$
23. $6r^2 - 5r + 1$
24. $8a^2 + 15a - 2$
25. $w^2 - \frac{9}{4}$
26. $c^2 - 64$
27. $r^2 + 14r + 49$
28. $b^2 + 18b + 81$
29. $j^2 - 12j + 36$
30. $4t^2 - 25$

**Solve each equation by factoring.**

31. $10r^2 - 35r = 0$
32. $3x^2 + 15x = 0$
33. $k^2 + 13k + 36 = 0$
34. $w^2 - 8w + 12 = 0$
35. $c^2 - 5c - 14 = 0$
36. $z^2 - z - 42 = 0$
37. $2y^2 - 5y - 12 = 0$
38. $3b^2 - 4b - 15 = 0$
39. $t^2 + 12t + 36 = 0$
40. $u^2 + 5u + \frac{25}{4} = 0$
41. $q^2 - 8q + 16 = 0$
42. $a^2 - 6a + 9 = 0$