Do evens or odds and then do more if you need more practice.

Name:

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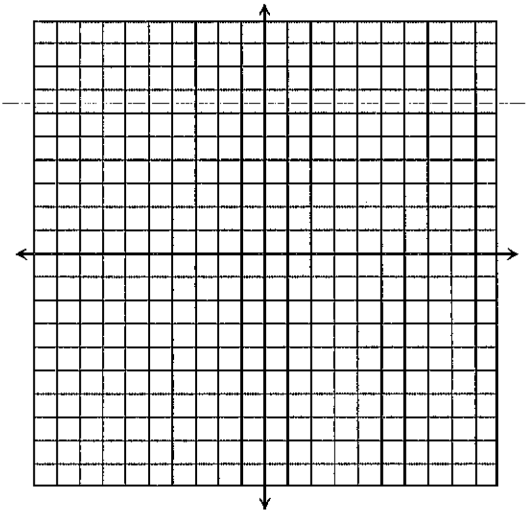
Main Ideas/Questions	Notes/Examples
TWO-VARIABLE Systems of Equations	• A system of equations is two more equations with the same variables. • The solution to a system of equations with two variables is the ordered pair (x, y) that satisfies both equations. [where the graphs intersect] • Linear systems can have one solution, no solution, or infinitely many solutions.

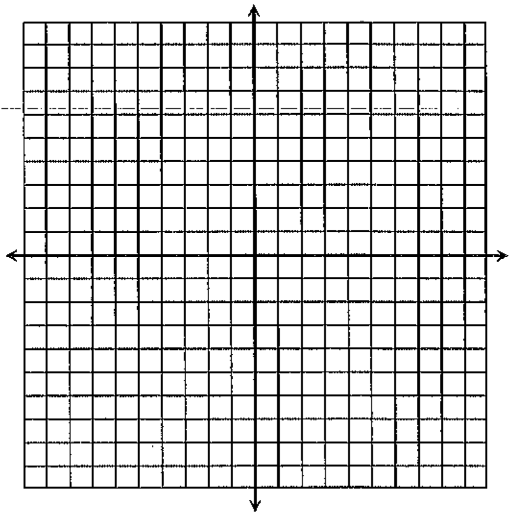
SOLVING GRAPHICALLY

Solve the system of equations by graphing.

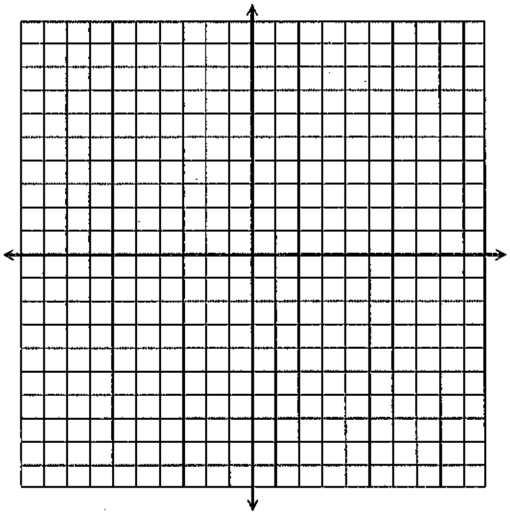
1. $\begin{cases} 3x - y = -5 \\ 4x + y = -2 \end{cases}$



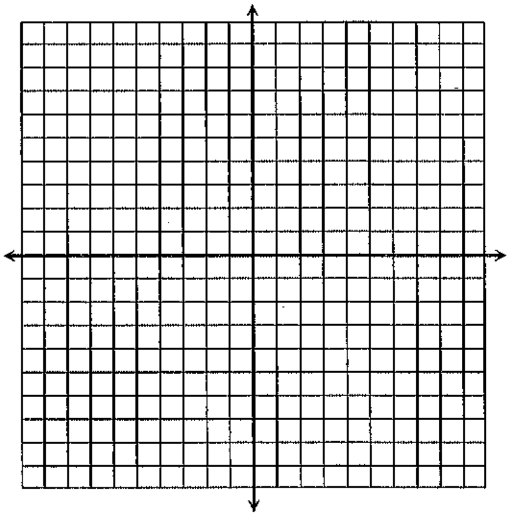
2. $\begin{cases} x - 2y = 8 \\ x = -6 \end{cases}$



3. $\begin{cases} 3x - 3y = -15 \\ x - y = 2 \end{cases}$



4. $\begin{cases} 2x + 3y = 9 \\ 6y = 18 - 4x \end{cases}$



SUBSTITUTION METHOD

To solve a system of equations by substitution:

❶ Solve either equation for one of the variables.

❷ Substitute this expression into the other equation so the equation is in terms of just one variable. Solve.

❸ Use substitution to solve for the other variable.

Solve each system of equations by subsitution.

5. $\begin{cases} x - 5y = 13 \\ -7x + 8y = -10 \end{cases}$

6. $\begin{cases} 2x - y = 16 \\ 12x + 5 = 6y \end{cases}$

7. $\begin{cases} 3x - 2y = 30 \\ 7x + 5y = -17 \end{cases}$

8. $\begin{cases} -4x - 3y = -21 \\ 5x + 9y = 63 \end{cases}$

ELIMINATION METHOD

To solve a system of equations by elimination:

❶ Align the equations.

❷ Multiple one or both equations by a number to result in a variable with the same coefficients.

❸ Subtract the equations to eliminate a variable. Solve.

❹ Use substitution to solve for the other variable.

Solve each system of equations by elimination.

9. $\begin{cases}3x+y=-25\\-7x-6y=62\end{cases}$

10. $\begin{cases}4x=5y+10\\8x+9y=58\end{cases}$

11. $\begin{cases}8x-2y=-32\\3x-11y=-53\end{cases}$

12. $\begin{cases}6y=9x+39\\-3x=13-2y\end{cases}$

APPLICATIONS

Set up a system of equations. Then solve by subsitution or elimination.

13. Jasmine has 35 coins in her wallet, all nickels and quarters. If the total value of the coins is $4.35, how many nickels and how many quarters does she have?

14. The admission fee at a carnival is $2.50 for children and $7.00 for adults. On a certain day, 1400 people attended the carnival and the admission fees totaled $5822. How many children and how many adults attended the carnival that day?

15. It took Brett 2 hours to fly from Philadelphia to Pittsburgh, a distance of 260 miles, flying into a headwind. Following the same flight path, his return flight was 1 hour and 15 minutes. If the wind speed and direction was the same for both flights, find his speed in still air and the wind speed.

Headwind is against the wind. Headwind speed is actual speed minus the wind speed.

Tailwind is with the wind. Tailwind speed is actual speed plus wind speed.

16. Marlena bought $1\frac{2}{3}$ pounds of French Roast coffee and $3\frac{1}{8}$ pounds of Columbian coffee and paid $31.25. Rowen bought $2\frac{5}{12}$ pounds of French Roast coffee and $\frac{9}{16}$ pounds of Columbian coffee and paid $15.15. Find the cost per pound of each type of coffee.

17. Alex invested $7500 in two accounts, one paying 5.5% and the other paying 8% simple interest per year. His annual interest was $540. How much did he invest into each account?

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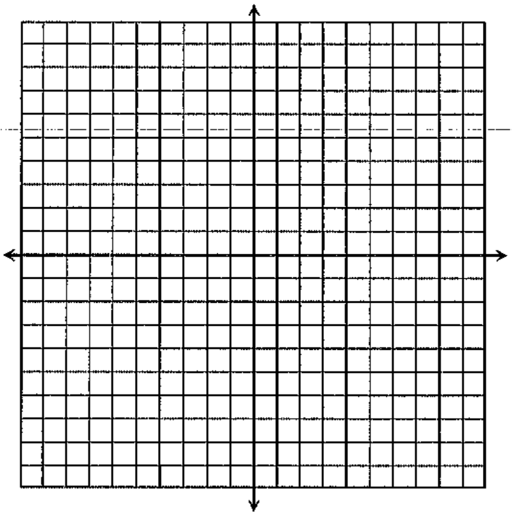
Main Ideas/Questions	Notes/Examples
NONLINEAR Systems of Equations	• A nonlinear system of equations is a system in which at least one equation is not linear. • There can be one solution, multiple solutions, no real solutions, or infinitely many to a system of nonlinear equations.

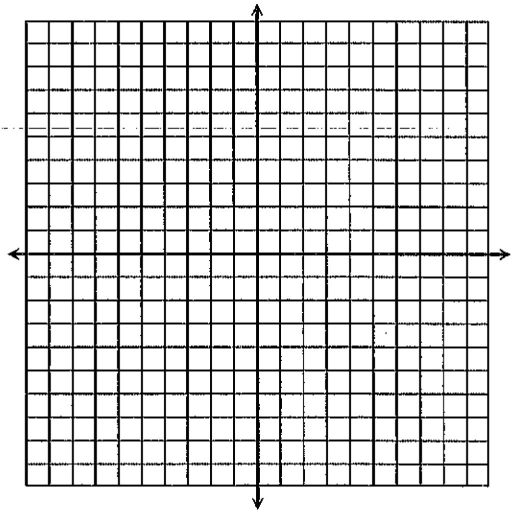
Solving **GRAPHICALLY**

Solve each system of equations by graphing.

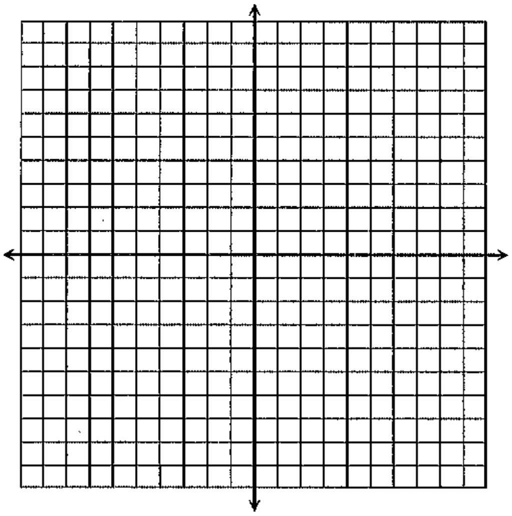
1. $\begin{cases} y = x^2 + 8x + 13 \\ y = 2x + 8 \end{cases}$



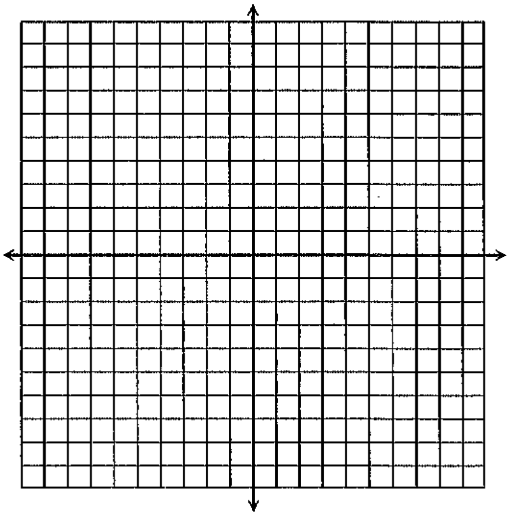
2. $\begin{cases} x^2 + y^2 = 25 \\ y^2 = 8x - 8 \end{cases}$



3. $\begin{cases} y = -2x^2 + 12x - 15 \\ x - 4y = -24 \end{cases}$



4. $\begin{cases} y = 2\sqrt{x+6} + 1 \\ x + 3y = 24 \end{cases}$



Solving
ALGEBRAICALLY

Solve each system of equations by substitution or elimination.

5. $\begin{cases} y = x^2 + x - 2 \\ y = 1 - x \end{cases}$

6. $\begin{cases} y = x^2 + 3x - 5 \\ y = 2x^2 + 5x - 4 \end{cases}$

7. $\begin{cases} x^2 + y^2 = 52 \\ 3x + 2y = 0 \end{cases}$

8. $\begin{cases} y^2 = 2x + 13 \\ x^2 + y^2 + 6x - 1 = 0 \end{cases}$

9. $\begin{cases} y = x^2 - 6x + 10 \\ y = -3x^2 - 12x - 5 \end{cases}$

10. $\begin{cases} 9x^2 + 16y^2 = 289 \\ x^2 - y^2 = 21 \end{cases}$

Name:

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Main Ideas/Questions	**Notes/Examples**

THREE-VARIABLE
Systems of Equations

- The solution to a system of equations with three variables is the ordered triple (x, y, z) that satisfies all equations.
- Three-variable systems can have one solution, no solution, or infinitely many solutions.

To solve a system with three variables:

❶ Eliminate a variable from a pair of equations, then, eliminate the same variable from another pair of equations.

❷ Solve the resulting system of equations from Step 1.

❸ Use substitution to find the value of the third variable.

EXAMPLES

Solve each system of equations.

1. $\begin{cases} x+3y+2z=-13 \\ 5x+2y-2z=-11 \\ 7x-4y-z=29 \end{cases}$ A B

① A: $x+3y+2z=-13$

$(+)5x+2y-2z=-11$

$6x+5y\quad=-24$

B: $5x+2y-2z=-11$

$(7x-4y-z=29)2$

$5x+2y-2z=-11$

$+-14x+8y+2z=-58$

$-9x+10y=-69$

② $(6x+5y=-24)2$

$-9x+10y=-69$

$12x+10y=-48$

$-(-9x+10y=-69)$

$21x\quad=21$

$x=1$

③ $6(1)+5y=-24$

$5y=-30$

$y=-6$

$1+3(-6)+2z=-13$

$-17+2z=-13$

$2z=4$

$z=2$

$(1,-6,2)$

2. $\begin{cases} 4x-y+9z=-3 \\ 3x-4y-5z=-31 \\ -x+7y+11z=27 \end{cases}$

3. $\begin{cases} 2x-5y-6z=57 \\ -4x+7y-z=19 \\ 9x-y+4z=-75 \end{cases}$

4. $\begin{cases} 4x-5y+z=20 \\ 6x-y-2z=35 \\ -2x+8y-3z=-9 \end{cases}$

5. $\begin{cases} 8x-z=7 \\ 4x+3y=-31 \\ -7y-2z=73 \end{cases}$

APPLICATIONS

6. Mischa and Ben's combined SAT score was 2290. Ben and Ashley's combined score was 2530. Mischa and Ashley's combined score was 2660. Find each student's individual SAT score.

7. Jada has 33 coins in her wallet – all pennies, nickels, and quarters. The total value of the coins is $3.41. If the number of quarters is four less than the number of pennies, how many of each coin does she have?

8. Three solutions contain a certain acid. Solution A contains 15% acid, Solution B contains 5% acid, and Solution C contains 40% acid. A chemist combined all three solutions to create a 60-liter mixture containing 29% acid. If twice as much of Solution C as Solution A was used, how many liters of each solution was used for the mixture?

9. In last weekend's track meet, Lakeland High School had 30 individual-event placers score a total of 111 points. First-place finishers score 7 points, second-place finishers score 4 points, and third-place finishers score 1 point. The number of first- and second- place finishers was twice the number of third-place finishers. Find the number of runners that finished in each place.

10. Marvin inherited $25,000 from his grandparents. He placed the money in three different accounts that earn 2%, 6.25%, and 8.5% annual simple interest. The amount of money placed in the account that earns 2% was half the amount of money placed in the account that earns 6.25% interest. If he earned $1729 in interest combined after one year, how much money did he place in each account?

11. Beth bought $1\frac{1}{2}$ pounds of cashews, $2\frac{3}{4}$ pounds of almonds, and $\frac{2}{3}$ pounds of raisins and paid $23.75. Claire bought $\frac{3}{5}$ pounds of cashews, $1\frac{5}{16}$ pounds almonds, and 2 pounds of raisins and paid $14.25. If the cost per pound of cashews is 50¢ less than twice the cost per pound of almonds, find the cost per pound of each.

Name:	**Date:**
Topic:	**Class:**

Main Ideas/Questions | **Notes/Examples**

MATRIX

$$A=\begin{bmatrix}-3 & 6 & 24\\ 0 & -15 & -7\end{bmatrix}$$

A rectangular array of variables or constants in rows or columns, usually enclosed in brackets.

ELEMENTS

- Elements are the individual values within the matrix.
- Element axy is used to denote each element where x is the row number and y is the column number where the element is located.

Directions: Given matrix A, find the value of each element.

Note: Matrices are named using a capital letter.

$$A=\begin{bmatrix}-8 & 40 & 0 & -1 & 21\\ 27 & 32 & -29 & 6 & -2\\ 5 & -17 & 14 & 52 & -35\end{bmatrix}$$

1. $a_{\underset{\text{row}}{2}\overset{\text{column}}{5}}$ -2	**2.** a_{31}
3. a_{12}	**4.** a_{34}
5. a_{15}	**6.** a_{22}

DIMENSIONS

A matrix with m-rows and n-columns is said to be an "mxn" matrix

Directions: State the dimensions of each matrix.

7. $\begin{bmatrix}-5 & 34 & -18\end{bmatrix}$ 1×3	**8.** $\begin{bmatrix}1 & -8\\ -4 & 13\\ -6 & -2\\ 28 & 0\end{bmatrix}$
9. $\begin{bmatrix}-11 & 37 & 4 & -2\\ 0 & 9 & -5 & -3\\ -16 & 1 & 25 & 12\end{bmatrix}$	**10.** $\begin{bmatrix}7 & -3 & 0 & -23 & -1 & 5\\ -31 & 10 & -6 & -14 & 18 & -26\end{bmatrix}$

EQUAL MATRICES

Two matrices in which each element in one matrix is equal to the corresponding element in the other.

SPECIAL MATRICES

ROW MATRIX	COLUMN MATRIX	SQUARE MATRIX	ZERO MATRIX
$\begin{bmatrix}a & b & c\end{bmatrix}$	$\begin{bmatrix}x\\ y\\ z\end{bmatrix}$	$\begin{bmatrix}4 & 9\\ 6 & 2\end{bmatrix}$	$\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

ADDITION & SUBTRACTION

Matrices can be added or subtracted if and only if they have the same dimensions. Simply add or subtract corresponding elements.

Directions: Using the matrices below, perform each operation, if possible.

$$W = \begin{bmatrix} -1 & 9 \\ -11 & 15 \\ 8 & -20 \end{bmatrix} \quad X = \begin{bmatrix} 24 & -8 & -3 \\ -4 & 6 & -17 \end{bmatrix} \quad Y = \begin{bmatrix} -15 & 29 & 17 \\ 9 & -5 & -2 \end{bmatrix} \quad Z = \begin{bmatrix} -3 & -2 \\ -16 & 0 \\ 12 & 9 \end{bmatrix}$$

11. $X + Y$

$$\begin{bmatrix} 9 & 21 & 14 \\ 5 & 1 & -19 \end{bmatrix}$$

12. $Z - W$

13. $W + X$

Not Possible

14. $X - Y$

SCALAR MULTIPLICATION

Matrices can be multiplied by a constant, called a scalar. In this process, called scalar multiplication, each element in the matrix is multiplied by the scalar to produce the new matrix.

Directions: Using the same matrices above, perform each operation.

15. $3X$

$$\begin{bmatrix} 72 & -24 & -9 \\ -12 & 18 & -51 \end{bmatrix}$$

16. $\frac{3}{4}W$

17. $-5X + 2Y$

18. $4W - Z$

19. $-3(W + Z)$

20. $\frac{1}{2}(Y - X)$

Name: ______________________________

Date: ____________________ **Per:** ________

Unit 10: Systems and Matrices

Homework 4: Introduction to Matrices & Basic Operations

Directions: State the dimensions of each matrix.

1. $\begin{bmatrix} 9 & -11 & 4 & 7 \end{bmatrix}$

2. $\begin{bmatrix} 6 & 1 \\ 0 & 7 \\ -5 & 11 \end{bmatrix}$

3. $\begin{bmatrix} -8 & 7 & 3 \\ 6 & -1 & 9 \\ 0 & 11 & 5 \\ 1 & 7 & 3 \end{bmatrix}$

4. $\begin{bmatrix} -4 \\ 13 \\ -17 \end{bmatrix}$

Directions: Using the matrices below, perform each operation, if possible.

$$A=\begin{bmatrix} 1 & -4 \\ 5 & 3 \\ -7 & -1 \\ 8 & 2 \end{bmatrix},\ B=\begin{bmatrix} 0 & 3 \\ -8 & 6 \\ 1 & -2 \\ -7 & -5 \end{bmatrix},\ C=\begin{bmatrix} 4 & -3 & 9 & 2 \\ 7 & -9 & 0 & 6 \end{bmatrix},\ D=\begin{bmatrix} 11 & 7 & -2 & 0 \\ 8 & -5 & 1 & 10 \end{bmatrix},\ E=\begin{bmatrix} 5 & -1 \\ 7 & 2 \\ -4 & -9 \\ 13 & 3 \end{bmatrix}$$

5. $A-E$

6. $C+D$

7. $E-B$

8. $A+D$

Directions: Using the matrices below, perform each operation.

$$V=\begin{bmatrix} -6 & 7 \\ 2 & 1 \end{bmatrix},\ W=\begin{bmatrix} 11 & -8 & 3 \\ 6 & -1 & -4 \end{bmatrix},\ X=\begin{bmatrix} 6 & 9 \\ 11 & 5 \\ -1 & -8 \end{bmatrix},\ Y=\begin{bmatrix} 0 & -3 \\ 13 & 8 \end{bmatrix},\ Z=\begin{bmatrix} 12 & -5 \\ -7 & 3 \\ 1 & -2 \end{bmatrix}$$

9. $4W$

10. $-\frac{5}{3}Y$

11. $3Z-X$

12. $-2Y+3V$

13. $2(X+Z)$

14. $\frac{3}{2}(V-Y)$

Name:	**Date:**
Topic:	**Class:**

Main Ideas/Questions | **Notes/Examples**

MULTIPLYING MATRICES

- Matrices A and B can be multiplied if and only if the number of columns in A equals the number of rows in B
- If matrix A has dimensions m x n and matrix B has dimensions n x p, then the resulting dimensions of AB is m x p.

Matrix A	**Matrix B**	**Matrix AB**
$m \times n$	$n \times p$	$m \times p$

$$A = \begin{bmatrix} -6 & 2 & 8 \\ 0 & -1 & 5 \\ 16 & 4 & -10 \\ -3 & 13 & -7 \end{bmatrix}$$

$$B = \begin{bmatrix} -11 & 4 & 19 & -2 & 6 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & -6 & 19 \\ 14 & 9 & -5 & -2 \end{bmatrix}$$

$$D = \begin{bmatrix} 8 \\ 7 \\ -1 \end{bmatrix}$$

$$E = \begin{bmatrix} 5 & -2 \\ -13 & 11 \\ 20 & 6 \\ -9 & -3 \\ 17 & -1 \end{bmatrix}$$

Using the matrices to the left, determine if the product is defined. If yes, give the dimensions of the resulting product.

1. AD 4×3 3×1 yes; 4×1	**2.** AC
3. BE	**4.** BD
5. CA	**6.** EC
7. EB	**8.** DB

HOW DO YOU

Multiply Matrices?

The element in the m^{th} row of and n^{th} column of matrix AB is the sum of the products of the corresponding elements in row m of matrix A and column n of matrix B. Show this pattern below.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, then $AB = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$

EXAMPLES

Find each product, if possible.

9. $\begin{bmatrix} -4 & -9 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 6 & -1 \\ -5 & 4 \end{bmatrix}$

$$\begin{bmatrix} (-4\cdot 6)+(-9\cdot -5) & (-4\cdot -1)+(-9\cdot 4) \\ (3\cdot 6)+(-2\cdot -5) & (3\cdot -1)+(32\cdot 4) \end{bmatrix}$$

$$\begin{bmatrix} -24+45 & 4-36 \\ 18+10 & -3-8 \end{bmatrix}$$

$$\begin{bmatrix} 21 & -32 \\ 28 & -11 \end{bmatrix}$$

10. $\begin{bmatrix} -2 & 0 \\ 6 & 4 \end{bmatrix} \cdot \begin{bmatrix} 8 & -3 \\ -2 & -9 \end{bmatrix}$

11. $\begin{bmatrix} -7 & 2 & -11 \\ 3 & -5 & -2 \end{bmatrix} \cdot \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}$

12. $\begin{bmatrix} 2 & -8 \end{bmatrix} \cdot \begin{bmatrix} 3 & 10 & -4 & -3 \\ -2 & -7 & 5 & -6 \end{bmatrix}$

13. $\begin{bmatrix} 9 & -5 \\ -2 & 4 \end{bmatrix} \cdot \begin{bmatrix} -3 & 13 & -5 \\ -1 & -7 & 2 \end{bmatrix}$

14. $\begin{bmatrix} 14 \\ -2 \\ -6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 9 & 5 \end{bmatrix}$

15. $\begin{bmatrix} 2 & -8 & -1 \\ 15 & -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 7 \\ -2 & 11 \\ 6 & -5 \end{bmatrix}$

16. $\begin{bmatrix} -7 & -5 \end{bmatrix} \cdot \begin{bmatrix} 16 \\ -1 \end{bmatrix}$

$A = \begin{bmatrix} 3 & -1 \\ 4 & 8 \end{bmatrix}$

$B = \begin{bmatrix} -2 & 10 \\ 7 & 6 \end{bmatrix}$

17. Given matrix A and B to the left, find AB and BA. What can you conclude from your results?

Properties of Matrix Multiplication

Associative Property of Matrix Multiplication	$(AB)C = A(BC)$
Associative Property of Scalar Multiplication	$K(AB) = KA(B) = A(KB)$
Left Distributive Property	$C(A+B) = CA + CB$
Right Distributive Property	$(A+B)C = AC + BC$

APPLICATIONS

18. The first table shows the number of reams of paper, packs of dry-erase markers, and boxes of staples requested by two teachers. The cost per item is given in the second table. Use matrix multiplication to show the total cost of the supplies for each teacher.

Teacher	Paper	Markers	Staples
Mr. Smith	6	4	5
Mrs. Wills	8	2	3

Supply Item	Cost
Paper	$15
Markers	$4
Staples	$2.50

Use for questions 19-22: The students at Oakhill High School are selling bags of popcorn for $10 each, tubs of cookie dough for $12 each, and baskets of fruit for $16 each to raise money for the school. The number of each item sold per class is shown below.

Class	Popcorn	Cookie Dough	Fruit Basket
Freshman	60	84	45
Sophomore	75	58	28
Junior	40	63	18
Senior	91	47	32

19. If they earn 15% profit on each item they sell, use matrix multiplication to show the profit earned by each class. Give your answer as a matrix.

20. Which class earned the most profit?

21. Give the total profit for the school.

22. How much more profit did the seniors earn than the juniors?

Name: ______________________________ **Unit 10:** Systems and Matrices

Date: ____________________ **Per:** ________ **Homework 5:** Multiplying Matrices

**** This is a 2-page document! ****

Directions: Find each product, if possible.

1. $\begin{bmatrix} 4 & -1 & 0 \\ 8 & 4 & -3 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 5 \\ 1 \end{bmatrix}$

2. $\begin{bmatrix} 3 & -5 \\ 7 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 & -4 & -7 \\ 8 & 1 & 3 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 7 & 11 & 2 \\ -6 & 0 & -9 \\ 10 & -4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -3 \\ -1 \end{bmatrix}$

4. $\begin{bmatrix} 8 \\ -3 \\ 12 \\ -7 \end{bmatrix} \cdot \begin{bmatrix} 2 & 5 \end{bmatrix}$

5. $\begin{bmatrix} -5 & -3 \\ -6 & 7 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 \\ -7 & 9 \\ -1 & 13 \end{bmatrix}$

6. $\begin{bmatrix} 4 \\ -7 \end{bmatrix} \cdot \begin{bmatrix} 8 & -9 & 6 \end{bmatrix}$

7. $\begin{bmatrix} 10 & 1 & 6 \\ -4 & -1 & 9 \end{bmatrix} \cdot \begin{bmatrix} 12 & 5 \\ -2 & 3 \\ -2 & -4 \end{bmatrix}$

8. $\begin{bmatrix} 7 & -8 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} -5 & 1 \\ 0 & -2 \\ 3 & -3 \\ 7 & 1 \end{bmatrix}$

9. $\begin{bmatrix} 2 & -5 \\ -1 & 7 \end{bmatrix} \cdot \begin{bmatrix} 11 & -6 \\ 0 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 5 \\ 4 & -9 \end{bmatrix}$

10. $\begin{bmatrix} 14 & 0 \\ -8 & 6 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 5 \\ -1 \\ -7 \end{bmatrix} \cdot \begin{bmatrix} 3 & 11 \end{bmatrix}$

11. $\begin{bmatrix} 5 & 1 \\ 4 & -7 \end{bmatrix} \cdot \begin{bmatrix} -7 & 8 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} 9 & 0 \\ 11 & -4 \end{bmatrix} \cdot \begin{bmatrix} -6 & 6 \\ 2 & 10 \end{bmatrix}$

12. $\begin{bmatrix} 7 & -2 & 3 \\ -5 & -8 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 5 \\ 3 & -4 \\ -7 & 13 \end{bmatrix} + \begin{bmatrix} 6 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 4 & -1 \end{bmatrix}$

13. The first table shows the number of oil changes, tire rotations, and wheel alignments needed by two car rental companies. The cost per service is given in the second table. Use matrix multiplication to show the total cost of the vehicle services for each company.

Company	**Oil Change**	**Tire Rotation**	**Alignment**
Rentals 4 U	5	3	4
Car Care Plus	6	2	1

Service	**Cost**
Oil Change	$26.75
Tire Rotation	$33
Alignment	$89.25

14. A group of friends went back to school shopping, buying jeans for $28 each, t-shirts for $12 each, hoodies for $32 each, and sneakers for $65 each. The number of each item purchased per person is shown in the table below.

Person	**Jeans**	**T-Shirts**	**Hoodies**	**Sneakers**
Isabella	2	2	3	1
Penelope	3	1	3	2
Jeremy	4	3	1	2
Henry	3	2	2	2

a. If all items had an additional 8% tax added to their price, use matrix multiplication to show the total purchase price per person. Give your answer as a matrix.

b. Who spent the most money?

c. How much more did the boys spend than the girls?

Name:

Date:

Topic:

Class:

Main Ideas/Questions	**Notes/Examples**

DETERMINANT

- Every square matrix has a real number called its **determinant**.
- The determinant of matrix A is denoted as det (A) or |A|.

Determinant of a 2 x 2 Matrix

The determinant of a 2 x 2 matrix is called a **second-order determinant.**
To find the determinant of a 2 x 2 matrix, use the rule:

Given $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $|A| = ad - cb$.

Find each determinant.

1. $\begin{bmatrix} -4 & 3 \\ 5 & -10 \end{bmatrix}$

$(-4)(-10) - (5)(3)$

$40 - 15$

$\boxed{25}$

2. $\begin{bmatrix} -6 & -3 \\ 11 & -2 \end{bmatrix}$

3. $\begin{bmatrix} 5 & -7 \\ -5 & -9 \end{bmatrix}$

4. $\begin{bmatrix} 2 & 6 \\ 4 & -8 \end{bmatrix}$

Determinant of a 3 x 3 MATRIX

The determinant of a 3 x 3 matrix is called a **third-order determinant.**
To find the determinant of a 3 x 3 matrix, use the steps below:

❶ Rewrite the first two columns to the right of the matrix.

❷ Find the sum of the products of each downward diagonal.

❸ Find the sum of the products of each upward diagonal.

❹ Subtract the upward diagonal sum from the downward diagonal sum.

Find each determinant.

5. $$\begin{array}{ccccc} D & D & D & & \\ 3 & -7 & 2 & 3 & -7 \\ -5 & 4 & -5 & -5 & 4 \\ 1 & 5 & -1 & 1 & 5 \\ u & u & u & & \end{array}$$

D - u

$-27 + 102 = 75$

Down: $3 \cdot 4 \cdot -1 = -12$

$-7 \cdot -5 \cdot 1 = 35$

$2 \cdot -5 \cdot 5 = -50$

$-12 + 35 - 50 = -27$

Up: $1 \cdot 4 \cdot 2 = 8$

$5 \cdot -5 \cdot 3 = -75$

$-1 \cdot -5 \cdot -7 = -35$

$8 - 75 - 35 = -102$

6. $\begin{bmatrix} 6 & -3 & 6 \\ -1 & 4 & 2 \\ -3 & -6 & -4 \end{bmatrix}$

7. $\begin{bmatrix} -3 & 4 & -7 \\ 2 & 2 & 5 \\ -3 & 8 & -4 \end{bmatrix}$

8. $\begin{bmatrix} -8 & -8 & 5 \\ 3 & 2 & -6 \\ -6 & -2 & 0 \end{bmatrix}$

APPLICATION:
Area of a Triangle

Given a triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3), the area of the triangle can be found using the formula:

$$A = \frac{1}{2}\left|\,\textbf{det}(X)\right| \text{ where } X = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$

9. Triangle PQR has vertices $P(-5, -2)$, $Q(3, 9)$ and $R(7, -4)$. Find the area of the triangle.

10. Wellspring is 14 miles north and 3 miles east of Chester. Hillford is 8 miles east and 5 miles north of Chester. Find the area of the land formed between these three towns.

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IDENTITY MATRICES

- The **identity matrix,** denoted I, is a square matrix, that when multiplied by another matrix, equals that same matrix.
- If A is an $n \times n$ matrix and I is an $n \times n$ identity matrix, then $A \cdot I = A$ and $I \cdot A = A$.
- An identity matrix contains 1's along the main diagonal and 0's for the remaining elements.

2 x 2 Identity Matrix:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3 x 3 Identity Matrix:

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

INVERSE MATRICES

- Two $n \times n$ matrices are inverses of each other if and only if their product (in both orders) is the identity matrix
- If matrix A has an inverse, denoted as B, then $AB = I$ AND $BA = I$.

Determine whether the pair of matrices are inverses.

1. $A = \begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}, B = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} -5+6 & -2+2 \\ 15-15 & 6-5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} -5+6 & 10-10 \\ -3+3 & 6-5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Yes

2. $R = \begin{bmatrix} -8 & 3 \\ 4 & -2 \end{bmatrix}, S = \begin{bmatrix} -6 & 2 \\ 16 & -5 \end{bmatrix}$

3. $C = \begin{bmatrix} -3 & -2 \\ -4 & 8 \end{bmatrix}$, $D = \begin{bmatrix} -\frac{1}{4} & -\frac{1}{16} \\ -\frac{1}{8} & \frac{3}{32} \end{bmatrix}$

DOES AN INVERSE EXIST?

Not all matrices have an inverse. Matrix A does not have an inverse if

the determinant $|A| = 0$

Inverse of a 2 X 2 MATRIX

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (where $|A| \neq 0$)

Find the inverse of each matrix, if it exists.

4. $\begin{bmatrix} 4 & -1 \\ -6 & 3 \end{bmatrix}$ $|A| = 12 - 6 = 6$

$A^{-1} = \frac{1}{6}\begin{bmatrix} 3 & 1 \\ 6 & 4 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{2} & \frac{1}{6} \\ 1 & \frac{2}{3} \end{bmatrix}$

5. $\begin{bmatrix} 8 & -4 \\ 2 & -1 \end{bmatrix}$ $|A| = -8 + 8 = 0$

No Inverse

6. $\begin{bmatrix} 8 & -10 \\ 8 & 5 \end{bmatrix}$

7. $\begin{bmatrix} 3 & -8 \\ 3 & -6 \end{bmatrix}$

8. $\begin{bmatrix} -6 & 12 \\ 1 & -2 \end{bmatrix}$

9. $\begin{bmatrix} -5 & -9 \\ 10 & -6 \end{bmatrix}$

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**** This is a 2-page document! ****

Directions: Find the determinant of each matrix.

1. $\begin{bmatrix} 7 & -12 \\ -1 & -7 \end{bmatrix}$

2. $\begin{bmatrix} 6 & -5 \\ 11 & -9 \end{bmatrix}$

3. $\begin{bmatrix} -3 & 4 \\ 8 & 10 \end{bmatrix}$

4. $\begin{bmatrix} 7 & -4 & 3 \\ -1 & 2 & 2 \\ -2 & 5 & -3 \end{bmatrix}$

5. Triangle DEF has vertices D(4, -3), E(-2, 1), and F(-9, -6). Find the area of the triangle.

6. Gila's house is eight miles south and two miles east of her workplace. Her fitness center is three miles north and one mile east of her workplace. Find the area of the land formed between these three locations.

Directions: Determine whether each pair of matrices are inverses.

7. $X = \begin{bmatrix} 3 & 4 \\ 0 & -2 \end{bmatrix}, Y = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ 0 & -\frac{1}{2} \end{bmatrix}$

8. $P = \begin{bmatrix} 8 & 6 \\ 7 & -5 \end{bmatrix}, Q = \begin{bmatrix} -\frac{5}{2} & -3 \\ \frac{7}{2} & 4 \end{bmatrix}$

9. $C = \begin{bmatrix} -5 & 0 & 2 \\ -1 & 2 & -1 \\ 6 & 5 & -6 \end{bmatrix}, D = \begin{bmatrix} -7 & 10 & -4 \\ -12 & 18 & -7 \\ -17 & 25 & -10 \end{bmatrix}$

Directions: Find the inverse of each matrix, if it exists.

10. $\begin{bmatrix} -4 & 6 \\ 4 & -7 \end{bmatrix}$

11. $\begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix}$

12. $\begin{bmatrix} -11 & -7 \\ 11 & 7 \end{bmatrix}$

13. $\begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}$

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COEFFICIENT MATRIX

Linear System	**Coefficient Matrix**
$\begin{cases} x-5y=9 \\ -7x+2y=-30 \end{cases}$	$\begin{bmatrix} 1 & -5 \\ -7 & 2 \end{bmatrix}$

Solving Systems with CRAMER'S RULE

(Two-Variables)

Cramer's Rule, named after the Swiss mathematician Gabriel Cramer, uses the coefficient matrix and determinants to solve a system of linear equations.

- Given $\begin{cases} ax+by=e \\ cx+dy=f \end{cases}$, let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (the coefficient matrix)
- If $|A| \neq 0$, then the system has a unique solution given by:

$$x=\frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{|A|} \text{ and } y=\frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{|A|}$$

Note: If the determinant of the coefficient matrix equals 0, then the system does not have a unique solution. It has either no solution or an infinite number of solutions.

Use Cramer's Rule to solve each system of equations.

EXAMPLE 1:

$\begin{cases} x-5y=9 \\ -7x+2y=-30 \end{cases}$

$A=\begin{bmatrix} 1 & -5 \\ -7 & 2 \end{bmatrix}$ $|A|=2-35=-33$

$$X=\frac{\begin{vmatrix} 9 & -5 \\ -30 & 2 \end{vmatrix}}{-33}=\frac{18-150}{-33}=\frac{-132}{-33}=4$$

$$Y=\frac{\begin{vmatrix} 1 & 9 \\ -7 & -30 \end{vmatrix}}{-33}=\frac{-30+63}{-33}=\frac{33}{-33}=-1$$

$$\boxed{(4,-1)}$$

EXAMPLE 2:

$\begin{cases} 2x+6y=38 \\ 5x+y=-3 \end{cases}$

EXAMPLE 3:

$\begin{cases} 6x+y=-22 \\ -3x-2y=17 \end{cases}$

Solving Systems with
CRAMER'S RULE
(Three-Variables)

- Given $\begin{cases} ax+by+cz=j \\ dx+ey+fz=k \\ gx+hy+iz=l \end{cases}$, let $A=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ (the coefficient matrix)
- If $|A| \neq 0$, then the system has a unique solution given by:

$$x=\frac{\begin{vmatrix} \boldsymbol{j} & b & c \\ \boldsymbol{k} & e & f \\ \boldsymbol{l} & h & i \end{vmatrix}}{|A|}, \quad y=\frac{\begin{vmatrix} a & \boldsymbol{j} & c \\ d & \boldsymbol{k} & f \\ g & \boldsymbol{l} & i \end{vmatrix}}{|A|}, \text{ and } z=\frac{\begin{vmatrix} a & b & \boldsymbol{j} \\ d & e & \boldsymbol{k} \\ g & h & \boldsymbol{l} \end{vmatrix}}{|A|}$$

Use Cramer's Rule to solve each system of equations.

EXAMPLE 4:

$$\begin{cases} -x+7y-2z=-14 \\ 2x+y+9z=43 \\ 3x-5y+2z=2 \end{cases}$$

EXAMPLE 5:

$$\begin{cases} 3x-y+z=-23 \\ x+2y-7z=63 \\ 5x-6y-2z=-14 \end{cases}$$

EXAMPLE 6:

$$\begin{cases} -x+4y+z=-26 \\ 8x+3y-2z=-19 \\ -2x-y+9z=32 \end{cases}$$

EXAMPLE 7:

$$\begin{cases} x-y-z=9 \\ 4x-11y+2z=12 \\ -x-7y+9z=-41 \end{cases}$$

EXAMPLE 8:

$$\begin{cases} -5x-2y+3z=-50 \\ 2x-y+6z=-34 \\ -x+4y-z=-6 \end{cases}$$

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**** This is a 2-page document! ****

Directions: Use Cramer's Rule to solve each system of equations.

1. $\begin{cases} 4x+6y=22 \\ -3x-8y=-41 \end{cases}$

2. $\begin{cases} -2x+5y=-51 \\ 7x-y=30 \end{cases}$

3. $\begin{cases} -9x+4y=14 \\ 8x+11y=-27 \end{cases}$

4. $\begin{cases} x+3y-z=-7 \\ 5x-2y+3z=48 \\ -2x+y-4z=-31 \end{cases}$

5. $\begin{cases} 3x - y + 2z = -27 \\ -4x - 2y + z = 23 \\ x + 5y - 6z = 11 \end{cases}$

6. $\begin{cases} 2x + 7y - 3z = -1 \\ 6x - y + 5z = 35 \\ -x + 4y - z = -17 \end{cases}$

7. At the beginning of baseball season, three coaches bought extra supplies for their seasons. The Braves' coach ordered 2 bats, 8 balls, and 5 gloves for $300. The Marlins' coach ordered 3 bats, 5 balls, and 4 gloves for $310. The Astros' coach bought 1 bat, 9 balls, and 7 gloves for $314. Use Cramer's Rule to determine the price of each item.

* Desmos has a matrix calculator + a help video

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MATRIX EQUATION

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

- A system of linear equations can be written as a **matrix equation**:

$$\begin{cases} ax+by=e \\ cx+dy=f \end{cases} \rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$$

- If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, and $B = \begin{bmatrix} e \\ f \end{bmatrix}$, then $AX = B$.
- Therefore, $X = A^{-1}B$.

Solving Systems with INVERSE MATRICES

Steps to solve a system using inverse matrices:

❶ Write the system as the matrix equation, $AX = B$.

❷ Find A^{-1}, the inverse of the coefficient matrix.

❸ Multiply A^{-1}, by the constant matrix, B. $(X = A^{-1}B)$

EXAMPLE 1:

$$\begin{cases} 7x+5y=-14 \\ -6x+y=-25 \end{cases}$$

Use an inverse matrix to solve the system of equations.

$$\begin{bmatrix} 7 & 5 \\ -6 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -14 \\ -25 \end{bmatrix}$$

$|A| = 7+30 = 37$

$$A^{-1} = \frac{1}{37}\begin{bmatrix} 1 & -5 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 1/37 & -5/37 \\ 6/37 & 7/37 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} \frac{1}{37} & \frac{-5}{37} \\ \frac{6}{37} & \frac{7}{37} \end{bmatrix} \cdot \begin{bmatrix} -14 \\ -25 \end{bmatrix} = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$$

$\boxed{(3,-7)}$

EXAMPLE 2:

$$\begin{cases} 12x+11y=-35 \\ -8x-5y=21 \end{cases}$$

EXAMPLE 3:

$$\begin{cases} 6x+8y=-16 \\ 5x+3y=-28 \end{cases}$$

EXAMPLE 4:

$$\begin{cases} 2x+y-z=-16 \\ 3x+4y-2z=-23 \\ x+6y-3z=-17 \end{cases}$$

EXAMPLE 5:

$$\begin{cases} -x+5y-3z=-40 \\ 7x-2y-z=27 \\ -2x-y+6z=-5 \end{cases}$$

EXAMPLE 6:

$$\begin{cases} -4x-3y+5z=8 \\ x-2y-13z=-29 \\ 5x-9y+2z=53 \end{cases}$$

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**** This is a 2-page document! ****

Directions: Use an inverse matrix to solve each system of equations.

1. $\begin{cases} 3x - 7y = -33 \\ -2x + 4y = 20 \end{cases}$

2. $\begin{cases} -x - 5y = -35 \\ 8x + 3y = 58 \end{cases}$

3. $\begin{cases} 6x + y = -21 \\ -11x + 9y = -59 \end{cases}$

4. $\begin{cases} 2x - 3y + z = 19 \\ -4x + 2y - 5z = -25 \\ 7x - y - 3z = 22 \end{cases}$

5. $\begin{cases} x+4y-8z=47 \\ -3x-2y+5z=-46 \\ 2x+5y-3z=29 \end{cases}$

6. $\begin{cases} 6x-3y-4z=29 \\ -2x+2y-z=4 \\ -x+4y+3z=22 \end{cases}$

7. The local theatre performed a Saturday matinee to a crowd of 215 people. Tickets cost $11 per child and $15 per adult, bringing in a total of $2877. Use an inverse matrix to determine the number of each type of ticket sold.

8. Myles had 103 coins in a jar - all nickels, dimes, and quarters. If the total value of the coins is $15 and he has three more quarters than dimes, use an inverse matrix to determine the number of each type of coin.

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AUGMENTED MATRIX

The **augmented matrix** of a system of linear equations uses the coefficients and constants of the equations written in standard form.

Linear System	**Augmented Matrix**
$\begin{cases} 2x-7y+z=-25 \\ x+5y-8z=81 \\ -x+4y+3z=-18 \end{cases}$	$\begin{bmatrix} 2 & -7 & 1 & -25 \\ 1 & 5 & -8 & 81 \\ -1 & 4 & 3 & -18 \end{bmatrix}$

ROW-ECHELON FORM

A system is in **row-echelon form** (or triangular form) when the last equation contains just one variable, each equation above contains the variable(s) below and an additional variable, and the leading coefficient of each equation is one. Give the corresponding augmented matrix of the row-echelon system below.

Linear System in Row-Echelon Form	**Augmented Matrix in Row-Echelon Form**
$\begin{cases} x-2y+5z=-29 \\ y-6z=26 \\ z=-3 \end{cases}$	$\begin{bmatrix} 1 & -2 & 5 & -29 \\ 0 & 1 & -6 & 26 \\ 0 & 0 & 1 & -3 \end{bmatrix}$

Row-echelon form is convienent because the remaining variables can be solved by **back-substitution.**

ELEMENTARY
Row Operations

The following operations, called **elementary row operations,** produce equivalent systems and can be applied to an augmented matrix to transform it to row-echelon form.

- Interchange any two rows.
- Multiply a row by a non-zero constant.
- Add a multiple of one row to another.

GAUSSIAN ELIMINATION

The following process of solving a system of equations is known as **Gaussian Elimination,** named after the German mathematician Carl Friedrich Gauss.

❶	Write the system as an augmented matrix.
❷	Transform the matrix to row-echelon form using elementary row operations.
❸	Use the row-echelon form of the augmented matrix to rewrite the system of equations.
❹	Use back-substitution to solve.

Use augmented matrices and Gaussian elimination to solve each system.

EXAMPLE 1:

$$\begin{cases} x+2y-9z=-59 \\ y+4z=12 \\ x-y+3z=25 \end{cases}$$

$\downarrow$

①

$$\begin{bmatrix} 1 & 2 & -9 & -59 \\ 0 & 1 & 4 & 12 \\ 1 & -1 & 3 & 25 \end{bmatrix}$$

② Ⓐ $R_3 - R_1 \rightarrow R_3$

$$\begin{bmatrix} 1 & 2 & -9 & -59 \\ 0 & 1 & 4 & 12 \\ 0 & -3 & 12 & 84 \end{bmatrix}$$

Ⓑ $R_3 + 3R_2 \rightarrow R_3$

$$\begin{bmatrix} 1 & 2 & -9 & -59 \\ 0 & 1 & 4 & 12 \\ 0 & 0 & 24 & 120 \end{bmatrix}$$

Ⓒ $\frac{1}{24}R_3 \rightarrow R_3$

$$\begin{bmatrix} 1 & 2 & -9 & -59 \\ 0 & 1 & 4 & 12 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

③

$x + 2y - 9z = -59$

$y + 4z = 12$

$z = 5$

$y + 4(5) = 12$

$y + 20 = 12$

$y = -8$

$x + 2(-8) - 9(5) = -59$

$x - 16 - 45 = -59$

$x - 61 = -59$

$x = 2$

$$\boxed{(2, -8, 5)}$$

EXAMPLE 2:

$$\begin{cases} x-y+4z=6 \\ x+y-2z=-14 \\ x+2y+5z=6 \end{cases}$$

EXAMPLE 3:

$$\begin{cases} 4x-4y+8z=12 \\ x-2y+z=11 \\ -x+y-3z=-1 \end{cases}$$

EXAMPLE 4:

$$\begin{cases} 3x+6y-12z=-42 \\ -x-y+2z=9 \\ 2x+y+4z=29 \end{cases}$$

EXAMPLE 5:

$$\begin{cases} 2x-8y-6z=10 \\ x-2y+z=-19 \\ x-y-4z=25 \end{cases}$$

EXAMPLE 6:

$$\begin{cases} -x-6y+2z=28 \\ x-4y+3z=32 \\ 2x+3y-z=-5 \end{cases}$$

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**** This is a 2-page document! ****

Directions: Use augmented matrices and Gaussian elimination to solve each system.

1. $\begin{cases} x + y - 3z = 11 \\ 2x + y + z = -4 \\ x + 5z = -19 \end{cases}$

2. $\begin{cases} -3x + 3y + 9z = -30 \\ -x - 2y + 3z = -31 \\ 2x - y - 4z = 19 \end{cases}$

3. $\begin{cases} 4x-4y+4z=-4 \\ 4x-y-2z=-1 \\ -3x-3y-4z=-16 \end{cases}$

4. $\begin{cases} 2x+4y-2z=38 \\ -x+2y+3z=1 \\ x-3y-5z=3 \end{cases}$

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GAUSS-JORDAN ELIMINATION

(reduced row-echelon form)

By continuing to apply elementary row operations to the row-echelon form of an augmented matrix, you can write the matrix in **reduced row-echelon form:**

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array}\right]$$

This is useful because back subsitution will not be needed. Once a matrix is in reduced row-echelon form, we know that $x = a$, $y = b$ and $z = c$. Transforming an augmented matrix to reduced row-echelon form is called Gauss-Jordan elimination, named after Carl Friedrich Gauss and Wilhelm Jordan.

EXAMPLE 1:

$$\begin{cases} x + y - z = -6 \\ 2x - y + z = -3 \\ -x - 5y - 3z = -14 \end{cases}$$

$\downarrow$

$$\begin{bmatrix} 1 & 1 & -1 & -6 \\ 2 & -1 & 1 & -3 \\ -1 & -5 & -3 & -14 \end{bmatrix}$$

$(-3, 1, 4)$

Solve each system by Gauss-Jordan elimination.

Ⓐ $R_2 - 2R_1 \rightarrow R_2$

$$\begin{bmatrix} 1 & 1 & -1 & -6 \\ 0 & -3 & 3 & 9 \\ -1 & -5 & -3 & -14 \end{bmatrix}$$

Ⓑ $-\frac{1}{3}R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & 1 & -1 & -6 \\ 0 & 1 & -1 & -3 \\ -1 & -5 & -3 & -14 \end{bmatrix}$$

Ⓒ $R_3 + R_1 \rightarrow R_3$

$$\begin{bmatrix} 1 & 1 & -1 & -6 \\ 0 & 1 & -1 & -3 \\ 0 & -4 & -4 & -20 \end{bmatrix}$$

Ⓓ $R_3 + 4R_2 \rightarrow R_3$

$$\begin{bmatrix} 1 & 1 & -1 & -6 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & -8 & -32 \end{bmatrix}$$

Ⓔ $-\frac{1}{8}R_3 \rightarrow R_3$

$$\begin{bmatrix} 1 & 1 & -1 & -6 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Ⓕ $R_2 + R_3 \rightarrow R_2$

$$\begin{bmatrix} 1 & 1 & -1 & -6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Ⓖ $R_1 + R_3 \rightarrow R_1$

$$\begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Ⓗ $R_1 - R_2 \rightarrow R_1$

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

EXAMPLE 2:

$$\begin{cases} 2x+2y-6z=0 \\ x-3y-z=30 \\ -x+2y+5z=-26 \end{cases}$$

EXAMPLE 3:

$$\begin{cases} 3x-12y-6z=-27 \\ -x-y+2z=-1 \\ x-5y-z=-15 \end{cases}$$

EXAMPLE 4:

$$\begin{cases} x+2y-3z=-14 \\ -2x+4y+8z=-32 \\ 3x-y+4z=-19 \end{cases}$$

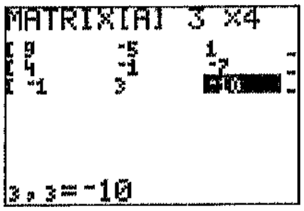
USING THE CALCULATOR

$$\begin{cases} 9x-5y+z=-7 \\ 4x-y-7z=-49 \\ -x+3y-10z=-61 \end{cases}$$

Try using Desmos matrix calculator

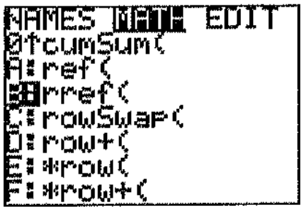
Use the steps below to solve the system to the left.

Step 1: Press **2ND** then x^{-1}, and arrow over to **EDIT.** Choose a matrix to edit. Input the augmented form of the system. Then, hit **2ND** then **MODE** to quit and return to the main screen.

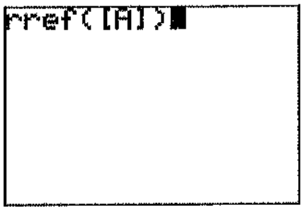


```
MATRIX[A] 3 ×4
[ 9      -5      1
[ 4      -1      -7
[ -1     3       -10
3,3=-10
```

Step 2: Press **2ND** then x^{-1}, and arrow over to **MATH.** Choose **B↓rref(** and press **ENTER.**

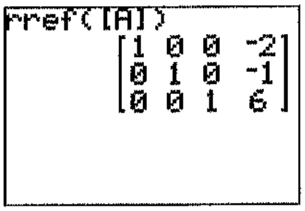


Step 3: Press **2ND** then x^{-1}, choose the matrix from the **NAME** menu, then press **ENTER.**



```
rref([A])
```

Step 4: The matrix is now in reduced row-echelon form. What is the solution?



```
rref([A])
        [1 0 0 -2]
        [0 1 0 -1]
        [0 0 1  6]
```

SPECIAL SOLUTIONS

From previous lessons, you know that not all linear systems have a unique solution. Linear systems can also have no solution or an infinite number of solutions. The row-echelon form of a linear system can reveal whether it has one solution, no solution, or an infinite number of solutions.

ONE SOLUTION	$\begin{bmatrix} \mathbf{1} & 0 & 0 & -7 \\ 0 & \mathbf{1} & 0 & 2 \\ 0 & 0 & \mathbf{1} & -3 \end{bmatrix}$	The matrix is written in reduced row-echelon form with 1's along the diagonals and 0's.
NO SOLUTION	$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -2 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{5} \end{bmatrix}$	A row contains three zeros, then a number.
INFINITELY MANY SOLUTIONS	$\begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 0 & 3 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$	A row contains all zeros.

EXAMPLES

Using your calculator, determine whether the system has one solution, no solution, or an infinite number of solutions. Give the solution if it has one.

5. $\begin{cases} -7x - 8y + 7z = 31 \\ 2x - 3y - 2z = -9 \\ -4x + 8y + 4z = 40 \end{cases}$

6. $\begin{cases} -3x + 3y - 2z = -12 \\ 4x + 5y - 5z = -33 \\ -2x - 4y + 5z = 21 \end{cases}$

7. $\begin{cases} -2x + 2y - 2z = -2 \\ -3x - 5y + 5z = -19 \\ 3x - 5y + 5z = -1 \end{cases}$

8. $\begin{cases} 6x + 4y - 5z = -31 \\ 5x + 4y + 5z = -14 \\ 2x - 2y - 5z = -27 \end{cases}$

9. $\begin{cases} 5x - 4y + 4z = -34 \\ 5x - y - 2z = 14 \\ 8x + 3y + 2z = 24 \end{cases}$

10. $\begin{cases} -2x - 5y + 2z = -3 \\ -3x - 5y - 7z = -37 \\ -x - 4y + 7z = 18 \end{cases}$

11. $\begin{cases} x + y - 4z = -11 \\ -x + y + 5z = -30 \\ -2x - 2y + 8z = 12 \end{cases}$

12. $\begin{cases} -8x + 5y - 3z = -2 \\ 5x - y - 5z = -3 \\ -3x + 4y + 5z = -5 \end{cases}$

Name: ______________________________ **Unit 10:** Systems and Matrices

Date: ____________________ **Per:** ________ **Homework 11:** Gauss-Jordan Elimination

**** This is a 2-page document! ****

Directions: Solve each system of equations with Gauss-Jordan elimination. Verify your answers using the calculator.

1. $$\begin{cases} 3x+4y+2z=37 \\ -2x+y+3z=-15 \\ 4x+5y+z=51 \end{cases}$$

2. $$\begin{cases} -x+3y+z=16 \\ 2x-2y+z=-13 \\ x+4y+3z=16 \end{cases}$$

3. $\begin{cases} -5x+10y+20z=-35 \\ 2x-3y-6z=5 \\ -3x+6y+15z=0 \end{cases}$

Directions: Using your calculator, determine whether the system has one solution, no solution, or an infinite number of solutions. Give the solution if it has one.

4. $\begin{cases} -4x+y-2z=16 \\ x-5y+6z=-27 \\ -5x-3y+4z=1 \end{cases}$

5. $\begin{cases} 2x-3z=4 \\ -2x+y+6z=-12 \\ -2x-3y-6z=-30 \end{cases}$

6. $\begin{cases} 5x+6y+z=27 \\ 4x-2y+4z=24 \\ 3x-2y-5z=-23 \end{cases}$

7. $\begin{cases} -x-5y+2z=-20 \\ -3x-5y+6z=-10 \\ -x+2y+2z=15 \end{cases}$

8. $\begin{cases} -4x+3y-3z=30 \\ 2x+3y-3z=24 \\ 3x-y+z=-10 \end{cases}$

9. $\begin{cases} 3x-2y-z=-26 \\ x-5y+7z=-10 \\ 5x+3y+5z=-24 \end{cases}$

Name:	**Date:**
Topic:	**Class:**

Main Ideas/Questions	Notes/Examples
PARTIAL FRACTIONS	A rational expression can often be written as the sum of two or more fractions, called the **partial fraction decomposition**. **Example:** $\frac{9x+12}{x^2+3x+2}=\frac{6}{x+2}+\frac{3}{x+1}$ The individual fractions are called partial fractions.
DECOMPOSING Proper Rational Expressions	A rational expression is **proper** when the degree of the numerator is less than the degree of the denominator. Follow the steps below to decompose the expression $\frac{3x+2}{x^2+6x+8}$:

❶	Factor the denominator.	$x^2+6x+8=(x+4)(x+2)$
❷	Write the expression as the sum of two fractions, using A and B as the numerators and the linear factors as the denominators.	$\frac{3x+2}{x^2+6x+8}=\frac{A}{x+4}+\frac{B}{x+2}$
❸	Eliminate the denominators by multiplying by the LCD.	$3x+2=A(x+2)+B(x+4)$
❹	Distribute and group like terms.	$3x+2=Ax+2A+Bx+4B$ $3x+2=Ax+Bx+2A+4B$ $3x+2=(A+B)x+(2A+4B)$
❺	Write a system of equations to find A and B.	$A+B=x$ $2A+4B=2$
❻	Solve the system using a method of your choice.	$\begin{bmatrix} 1 & 1 & 3 \\ 2 & 4 & 2 \end{bmatrix}$ $\rightarrow (5,-2)$
❼	Write the partial fraction decomposition of the expression.	$\frac{5}{x+4}-\frac{2}{x+2}$

	Find the partial fraction decomposition of each rational expression.
EXAMPLE 1: $\frac{7x-15}{x^2-9}$	
EXAMPLE 2: $\frac{-7x-9}{2x^2-7x+3}$	
EXAMPLE 3: $\frac{-5x^2+19x+12}{x^3-2x^2-3x}$	

EXAMPLE 4:

$$\frac{-8x^2-31x+75}{x^3+2x^2-25x-50}$$

EXAMPLE 5:

$$\frac{12x^2+67x+7}{16x^3-32x^2-x+2}$$

☐

Name: ______________________________ **Unit 10:** Systems and Matrices

Date: ________________________ **Per:** ________ **Homework 12:** Partial Fractions (Day 1 - Proper)

**** This is a 2-page document! ****

Directions: Write the partial fraction decomposition of each rational expression.

1. $\dfrac{11x+10}{x^2+4x-12}$

2. $\dfrac{-2x-4}{x^2-8x+7}$

3. $\dfrac{-2x+17}{2x^2+11x+12}$

4. $\dfrac{11x-1}{x^2-4x-5}$

5. $\dfrac{3x^2+47x-60}{x^3-x^2-30x}$

6. $\dfrac{26x^2+47x-21}{3x^3+20x^2-7x}$

7. $\dfrac{x^2-17x-6}{x^3+x^2-4x-4}$

Name:

Date:

Topic:

Class:

Main Ideas/Questions	**Notes/Examples**
DECOMPOSING Improper Rational Expressions	➢ A rational expression is **improper** when the degree of the numerator is greater than or equal to the degree of the denominator. ➢ If a rational expression is improper, you must use **long division** first to rewrite the expression as the sum of a polynomial and proper expression.
	Find the partial fraction decomposition of each rational expression.
EXAMPLE 1: $\frac{-x^2+8x+5}{x^2+x}$	-1 $x^2+x\,\overline{)\,-x^2+8x+5}$ $-(-x^2-x)$ $9x+5$ $\frac{-x^2+8x+5}{x^2+x} = -1 + \frac{9x+5}{x^2+x}$ $\frac{9x+5}{x^2+x} = \frac{A}{x} + \frac{B}{x+1}$ $9x+5 = A(x+1) + Bx$ $9x+5 = Ax + A + Bx$ $9x+5 = Ax + Bx + A$ $9x+5 = x(A+B) + A$ $A+B = 9$ $A = 5$ $A=5,\ B=4$ $-1 + \frac{5}{x} + \frac{4}{x+1}$
EXAMPLE 2: $\frac{7x^2+50x-108}{x^2+6x-16}$	
EXAMPLE 3: $\frac{3x^2+7x-20}{x^2-25}$	

EXAMPLE 4:

$$\frac{2x^3-30}{x^3+x^2-9x-9}$$

EXAMPLE 5:

$$\frac{x^3-8x^2+x+26}{x^2-4x-12}$$

EXAMPLE 6:

$$\frac{x^3+3x^2-27x-36}{x^2+x-12}$$

Name: ______________________________ **Unit 10:** Systems and Matrices

Date: ________________________ **Per:** ________ **Homework 13:** Partial Fractions (Day 2 - Improper)

**** This is a 2-page document! ****

Directions: Find the partial fraction decomposition of each rational expression.

1. $\dfrac{2x^2+9x-47}{x^2+3x-10}$

2. $\dfrac{-5x^2+3x+280}{x^2-49}$

3. $\dfrac{9x^2+14x-29}{x^2+x-2}$

4. $\dfrac{3x^2-43x+134}{x^2-10x+24}$

5. $\dfrac{4x^3+37x^2+29x+38}{x^3+8x^2-x-8}$

6. $\dfrac{x^3-3x^2-19x+43}{x^2-8x+15}$

7. $\dfrac{x^3+10x^2+13x-68}{x^2+11x+28}$

Name:	**Date:**
Topic:	**Class:**

Main Ideas/Questions	**Notes/Examples**
REPEATED Linear Factors	If the denominator of a rational expression has a linear factor that repeats n times, then the decomposition must include a partial fraction for each factor from the 1st power through the nth power. **Example:** $\frac{-6x+22}{x^2-8x+16}=\frac{-6x+22}{(x-4)^2}=\frac{A}{x-4}+\frac{B}{(x-4)^2}$
EXAMPLE 1: $\frac{-x-12}{x^2+14x+49}$	**Find the partial fraction decomposition of each rational expression.**
EXAMPLE 2: $\frac{-2x+16}{x^2-10x+25}$	
EXAMPLE 3: $\frac{12x-15}{9x^2-12x+4}$	

EXAMPLE 4:

$$\frac{5x^2+6x-1}{x^3+2x^2+x}$$

EXAMPLE 5:

$$\frac{40x^2-74x+27}{16x^3-24x^2+9x}$$

EXAMPLE 6: $\frac{-2x^2-7x-2}{x^2+4x+4}$	
PRIME Quadratic Factors	If the denominator of a rational expression contains a prime quadratic factor, then the decomposition must include a partial fraction with a linear factor of the form $Ax + B$ for each power of the factor. **Example:** $\frac{x^2+x-12}{x^4+3x^2+2}=\frac{x^2+x-12}{(x^2+1)(x^2+2)}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{x^2+2}$
EXAMPLE 7: $\frac{-3x^2+x-12}{x^3+4x}$	
EXAMPLE 8: $\frac{x^2-14}{x^4-7x^2+10}$	

EXAMPLE 9: $\frac{-x^2-4}{x^4-4x^2+4}$	
EXAMPLE 10: $\frac{5x^2+8x+31}{x^3+2x^2+3x+6}$	
EXAMPLE 11: $\frac{4x^3-9x^2-25x+57}{x^3-2x^2-7x+14}$	

Name: ______________________________

Date: ____________________ **Per:** ________

Unit 10: Systems and Matrices

Homework 14: Partial Fractions
(Day 3 – Special Case Factors)

**** This is a 2-page document! ****

Directions: Find the partial fraction decomposition of each rational expression.

1. $\frac{-4x+15}{x^2-4x+4}$

2. $\frac{4x+4}{16x^2-8x+1}$

3. $\frac{10x^2+30x+18}{x^3+6x^2+9x}$

4. $\frac{5x^2-2x+35}{x^3+7x}$

5. $\dfrac{-2x^2+20}{x^4-4}$

6. $\dfrac{7x^2+46}{x^4+10x^2+25}$

7. $\dfrac{7x^2-87x+265}{x^2-12x+36}$

8. $\dfrac{-3x^3-5x^2-30x-52}{x^3+2x^2+8x+16}$

Name:

Date:

Topic:

Class:

Main Ideas/Questions	**Notes/Examples**
SEQUENCE	A set of numbers with a particular order or pattern. Each number is called a term.
FINITE Sequence	A sequence with a limited number of terms **Example:** {1, 2, 3, 4, 5}
INFINITE Sequence	A sequence with an unlimited number of terms; no end. **Example:** {0, 1, 2, 3, 4, 5, 6, 7, ...} ←whole #'s

TERM NOTATION

- The first term in a sequence is denoted a_1.
- Each subsequent term is denoted a_{n+1}, where $n+1$ is the term number in the sequence.

17, 20, 23, 26, 29, 32, 35, 38, 41 . . . .

Example: Given {2, 5, 8, 11, 14, ...}, identify the following terms:

a_1: 2 a_5: 14 a_8: 23 a_{14}: 41

Sequences as **FUNCTIONS**

Since each term value is paired with exactly one term number, a sequence is a function with the following properties:

The **domain** is the set of term numbers.

n	1	2	3	4	5
a_n	2	5	8	11	14

The **range** is the set of term values.

In an infinite set, the domain is the the set of natural numbers

RECURSIVE Formulas

A rule in which one or more previous terms are used to generate the next term

The Fibonacci Sequence: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

In this sequence, $a_1 = 0$, $a_2 = 1$, then for each subsequent term,

$$a_n = a_{n-1} + a_{n-2}$$

EXAMPLES

Directions: Give the first 5 terms of each sequence.

1. $a_1 = 25;\ a_n = a_{n-1} - 7\ (n \geq 2)$

{25, 18, 11, 4, -3}

2. $a_1 = -2;\ a_n = 3a_{n-1} + 1\ (n \geq 2)$

3. $a_1 = 40;\ a_n = \frac{1}{2}a_{n-1} + a_{n-1}\ (n \ge 2)$

4. $a_1 = 8,\ a_2 = 10;\ a_n = \frac{a_{n-1}}{a_{n-2}}\ (n \ge 3)$

Directions: Write a recursive rule for each sequence. Then find a_8.

5. $\{9, 13, 17, 21, 25, \ldots\}$

6. $\{1, 3, 8, 22, 60, \ldots\}$

EXPLICIT
Formulas

A rule in which the n^{th} term is defined as a function of n

EXAMPLES

Directions: Give the first 5 terms of each sequence.

7. $a_n = 5n + 1$

8. $a_n = \left(\frac{3}{2}\right)^{n-1}$

9. $a_n = \frac{n-1}{n+2}$

10. $a_n = \sqrt{3n}$

11. $a_n = n^3 - (n-1)^2$

12. $a_n = n! - 4$

Directions: Write an explicit rule for each sequence. Then find a_{25}.

13. $\{1, 4, 9, 16, 25, \ldots\}$

14. $\{9, 11, 13, 15, 17, \ldots\}$

EXPLICIT VS. RECURSIVE
Rules

Directions: Write an explicit and recursive rule for each sequence.

15. $\{4, 7, 10, 13, 16, \ldots\}$

Explicit: $a_n = 3n + 1$

Recursive: $a_1 = 4;\ a_n = a_{n-1} + 3$ $(n \ge 2)$

16. $\left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots\right\}$

Explicit:

Recursive:

Name: ______________________________ **Unit 11:** Sequences and Series

Date: ____________________ **Per:** _______ **Homework 1:** Introduction to Sequences

**** This is a 2-page document! ****

Directions: Give the first 5 terms of each sequence.

1. $a_1 = 17;\ a_n = a_{n-1} + 13$

2. $a_1 = \frac{1}{2};\ a_n = -2a_{n-1} - 5$

3. $a_1 = -1,\ a_2 = 3;\ a_n = a_{n-1} \cdot a_{n-2}$

4. $a_1 = 3,\ a_2 = 7;\ a_n = a_{n-2} + 2 \cdot a_{n-1}$

Directions: Write a recursive rule for each sequence. Then find a_9.

5. $\{-7,\ 21,\ -63,\ 189,\ -567,\ \ldots\}$

6. $\left\{\frac{15}{2},\ 6,\ \frac{9}{2},\ 3,\ \frac{3}{2},\ \ldots\right\}$

7. $\{1,\ 2,\ 6,\ 24,\ 120,\ \ldots\}$

8. $\left\{4,\ 8,\ 6,\ 7,\ \frac{13}{2},\ \ldots\right\}$

Directions: Give the first 5 terms of each sequence.

9. $a_n = -3n + 7$

10. $a_n = \frac{1}{2}n - 4$

11. $a_n = \frac{4n}{n+5}$

12. $a_n = 6n^{\frac{1}{2}}$

13. $a_n = \frac{(n+1)^2}{n!}$

14. $a_n = \frac{3n^3 + 4}{(2n-1)^2}$

Directions: Write an explicit rule for each sequence. Then find a_{18}.

15. $\{4,\ 7,\ 12,\ 19,\ 28,\ \ldots\}$

16. $\{-23,\ -21,\ -19,\ -17,\ -15,\ \ldots\}$

17. $\left\{-\frac{4}{3}, -1, -\frac{4}{5}, -\frac{2}{3}, -\frac{4}{7}, \ldots\right\}$

18. $\left\{\frac{1}{3}, \frac{4}{5}, \frac{9}{7}, \frac{16}{9}, \frac{25}{11}, \ldots\right\}$

Directions: Write an explicit and recursive rule for each sequence.

19. $\{2,\ -6,\ 18,\ -54,\ 162,\ \ldots\}$

20. $\{-3,\ 2,\ 7,\ 12,\ 17,\ \ldots\}$

Explicit:	**Recursive:**	**Explicit:**	**Recursive:**

21. Elan makes time every day at the end of basketball practice to work on his free throws. During one week, he makes 11, 15, 19, 23, and 27 throws. Assuming this sequence continues, find the recursive and explicit rules.

22. A florist is selling roses in preparation for Valentine's Day. Their sales on the first five days are 3, 9, 27, 81, and 243 respectively. Assuming this sequence continues, find the recursive and explicit rules.

23. Rachel's friend was pushing her on a swing while another friend measured her path. On the first swing, she traveled 32 feet. On the second swing, she traveled 16 feet. On the next consecutive swings, she traveled 8 feet, 4 feet, and 2 feet. Assuming this sequence continues, find the recursive and explicit rules.

Name:	**Date:**
Topic:	**Class:**

Main Ideas/Questions	**Notes/Examples**
ARITHMETIC SEQUENCE	A sequence in which the difference between any two consecutive terms is constant
COMMON DIFFERENCE	The numerical difference between any two terms; variable = d
EXAMPLES	**Directions:** Determine the common difference, then find the next four terms of the sequence. **1.** $\{29, 37, 45, 53, 61, \ldots\}$ **2.** $\{-7, -10, -13, -16, -19, \ldots\}$ **3.** $\{25, 11, -3, -17, -31, \ldots\}$ **4.** $\{-12, 7, 26, 45, 64, \ldots\}$
ARITHMETIC SEQUENCE FORMULA	The n^{th} term of any arithmetic sequence can be found using the formula: $\boxed{a_n = d(n-1) + a_1}$ where a_1 is the first term and d is the common difference
EXAMPLES	**Directions:** Write a rule for each sequence, then find the indicated term. **5.** $\{7, 13, 19, 25, 31, \ldots\}$; a_{32} $a_1 = 7,\ d = 6$ $a_n = 6(n-1) + 7$ $a_n = 6n - 6 + 7$ $a_n = 6n + 1$ $a_{32} = 6(32) + 1 = \boxed{193}$ **6.** $\{4, -7, -18, -29, -40, \ldots\}$; a_{24} **7.** $\{-23, -19, -15, -11, -7, \ldots\}$; a_{45} **8.** $\{-3, -5.5, -8, -10.5, -13, \ldots\}$; a_{39}

Directions: Use the information given to find the indicated value.

9. If $a_1 = 29$ and $d = 7$, find a_{36}.

10. If $a_{22} = -156$ and $d = -9$, find a_1.

11. If $a_1 = 11$ and $a_{25} = 107$ find d.

12. If $a_1 = 5$ and $a_{18} = -199$ find a_{13}.

13. If $a_n = -76$, $a_1 = -21$, and $d = -5$, find n.

14. If $a_n = 310$, $a_1 = -15$, and $d = 13$, find n.

EXPLICIT VS. RECURSIVE

Formulas

Directions: Write an explicit and recursive formula for each sequence.

15. $\{-19, -17, -15, -13, -11, \ldots\}$

Explicit:

Recursive:

16. $\{37, 31, 25, 19, 13, \ldots\}$

Explicit:

Recursive:

ARITHMETIC MEANS

The terms between any two nonconsecutive terms in an arithmetic sequence

For example: Given the sequence {2, 8,14, 20, 26, 32, ...},
the arithmetic means between 8 and 32, are 14, 20, 26.

EXAMPLES

Directions: Find the indicated arithmetic means for each set of nonconsecutive terms.

17. 50 and 118; 3 means

$\{50, \_, \_, \_, 118\}$

$a_1 = 50 \quad a_5 = 118$

$118 = d(5-1) + 50$

$118 = 4d + 50$

$d = 17 \quad \boxed{67, 84, 101}$

18. -16 and -58; 5 means

19. 17 and 5; 7 means

20. 2,265 and 3,570; 4 means

APPLICATIONS

21. A theater has 35 seats in the first row, 39 seats in the second, 43 seats in the third, etc. Write both an explicit and recursive formula to find the number of seats in the n^{th} row.

Week	Weight
1	256.3
2	254.6
3	252.9
4	251.2

22. Ryan weighed himself at the end of each week into his new diet. The table to the left shows his weight at the end of each week for the first three weeks of his diet. Write an explicit rule to find his weight n weeks into his diet, then find his weight after 16 weeks.

23. On Anna's first birthday, her grandma gave her a check to open a college savings account. Her grandma continued to send checks on her birthday, each at the same amount. At 7 years old, Anna's grandma had given her a total of $610. At 18 years old, Anna's grandma had given her a total of $1,270. How much money did Anna's grandma give her on her first birthday?

Name: ______________________________ **Unit 11:** Sequences and Series

Date: ____________________ **Per:** ________ **Homework 2:** Arithmetic Sequences

**** This is a 2-page document! ****

Directions: Determine the common difference, then find the next four terms of the sequence.

1. $\{31, 22, 13, 4, -5, \ldots\}$

2. $\{14, 27, 40, 53, 66, \ldots\}$

3. $\{3, 7.5, 12, 16.5, 21, \ldots\}$

Directions: Write a rule for each sequence, then find the indicated term.

4. $\{-2, 1, 4, 7, 10, \ldots\}; a_{19}$

5. $\{52, 44, 36, 28, 20, \ldots\}; a_{37}$

6. $\{-27, -16, -5, 6, 17, \ldots\}; a_{26}$

7. $\{19.2, 17.4, 15.6, 13.8, 12, \ldots\}; a_{51}$

Directions: Use the information given to find the indicated value.

8. If $a_1 = 83$ and $d = -7$, find a_{24}.

9. If $a_{38} = -54$ and $d = 2$, find a_1.

10. If $a_1 = 104$ and $a_{33} = 328$, find d.

11. If $a_1 = -12$ and $a_{21} = -42$, find a_{15}.

12. If $a_1 = 20$ and $a_{24} = -118$, find a_{41}.

13. If $a_n = 2988$, $a_1 = -12$, and $d = 100$, find n.

Directions: Write an explicit and recursive formula for each sequence.

14. $\{225, 218, 211, 204, 197, \ldots\}$

Explicit:

Recursive:

15. $\{-6.7, -5.4, -4.1, -2.8, -1.5, \ldots\}$

Explicit:

Recursive:

Directions: Find the indicated arithmetic means for each set of nonconsecutive terms.

16. -357 and -297; 4 means

17. 512 and 360; 7 means

18. 74 and 98; 3 means

19. 9 and -1; 5 means

20. Nathaniel counts his newly sprouting crops at the beginning of each day. He counts 3 new crops of the first day, 9 new crops on the second day, 15 new crops on the third day, etc. Write the explicit and recursive formulas to find the number of new crops on the n^{th} day.

21. Amy reached the peak of her marathon training and begins a taper, where she decreases mileage each week. On the 1st week of the taper, she runs 65 total miles. On the 5th week of the taper, she runs 31 miles. Determine her weekly mileage for weeks 2, 3, and 4.

Name:	**Date:**
Topic:	**Class:**

Main Ideas/Questions	**Notes/Examples**
GEOMETRIC SEQUENCE	A sequence in which the ratio between any 2 consecutive terms is constant
COMMON RATIO	the numerical ratio between any two terms; variable $= r$
EXAMPLES	**Directions:** Determine the common ratio, then find the next three terms of the sequence. **1.** $\{8, 24, 72, 216, \ldots\}$ **2.** $\{512, -128, 32, -8, \ldots\}$ **3.** $\{-6.4, -16, -40, -100, \ldots\}$ **4.** $\left\{-\frac{243}{32}, \frac{81}{16}, -\frac{27}{8}, \frac{9}{4}, \ldots\right\}$
GEOMETRIC SEQUENCE FORMULA	The n^{th} term of any geometric sequence can be found using the formula: $\boxed{a_n = a_1 \cdot r^{n-1}}$ where a_1 is the first term and r is the common ratio.
EXAMPLES	**Directions:** Write an explicit rule for each sequence, then find a_8. **5.** $\{3, 12, 48, 192, \ldots\}$ $a_n = 3 \cdot 4^{n-1}$ $a_8 = 3 \cdot 4^{8-1}$ $a_8 = 3 \cdot 4^7$ $\boxed{49,152}$ **6.** $\{-2400, -1200, -600, -300, \ldots\}$ **7.** $\left\{\frac{7}{36}, -\frac{7}{6}, 7, -42, \ldots\right\}$ **8.** $\{-100000, 20000, -4000, 800, \ldots\}$

9. $\{-6400, -4800, -3600, -2700, \ldots\}$

10. $\left\{\frac{1}{2}, -\frac{3}{2}, \frac{9}{2}, -\frac{27}{2}, \ldots\right\}$

Directions: Use the information given to find the indicated value.

11. If $a_1 = 240$ and $r = -2$, find a_6.

12. If $a_2 = -8$ and $r = 4$, find a_{10}.

13. If $a_5 = \frac{1}{4}$ and $r = \frac{1}{2}$, find a_{11}.

14. If $a_3 = \frac{4}{9}$ and $r = -\frac{1}{3}$, find a_9.

15. If $a_1 = 2$ and $a_6 = 486$ find a_{12}.

16. If $a_4 = \frac{75}{2}$ and $a_7 = -\frac{75}{128}$, find a_2.

EXPLICIT VS. RECURSIVE

Formulas

Directions: Write an explicit and recursive formula for each sequence.

17. $\{2, -10, 50, -250, \ldots\}$

Explicit:
$a_n = 2 \cdot (-5)^{n-1}$

Recursive: $a_1 = 2$
$a_n = -5 \cdot a_{n-1} \ (n \geq 2)$

18. $\left\{-\frac{4}{3}, -\frac{2}{3}, -\frac{1}{3}, -\frac{1}{6}, \ldots\right\}$

Explicit:

Recursive:

GEOMETRIC MEANS

The terms between any 2 nonconsecutive terms in a geometric sequence

For example: Given the sequence {1, 3, 9, 27, 81, 243, ...},
the geometric means between 3 and 243, are 9, 27, 81.

EXAMPLES

Directions: Find the indicated geometric means for each set of nonconsecutive terms.

19. -4 and -2500; 3 means

-4, _, _, _, -2500

$a_1 = -4 \quad a_5 = -2500$

$-2500 = -4r^4$

$625 = r^4$

$r = \pm 5$

$\pm 20, -100, \pm 500$

20. 2 and $\frac{9}{8}$; 1 mean

21. 243 and -72; 2 means

22. -960 and $\frac{15}{16}$; 4 means

APPLICATIONS

23. In 2009, Eva purchased a new truck for $45,000. Each year, the value of the truck depreciates at a rate of 18%. Use a geometric sequence formula to find the value of Eva's truck in 2017.

24. Garrett signed a new cell phone contract in 2014. The table to the left shows his average monthly bill in 2014, 2015, and 2016. If this pattern continues, use a geometric sequence formula to find his average monthly bill in 2025.

Year	Average Bill
2014	$75.00
2015	$76.50
2016	$78.03

Name: ______________________________ **Unit 11:** Sequences and Series

Date: ____________________ **Per:** ________ **Homework 3:** Geometric Sequences

**** This is a 2-page document! ****

Directions: Determine the common ratio, then find the next four terms of the sequence.

1. $\{18, -108, 648, -3888, \ldots\}$

2. $\{27, 36, 48, 64, \ldots\}$

3. $\left\{10, 4, \frac{8}{5}, \frac{16}{25}, \ldots\right\}$

Directions: Write a rule for each sequence, then find the indicated term.

4. $\{-3, -9, -27, -81, \ldots\}$; a_7

5. $\left\{-18, 27, -\frac{81}{2}, \frac{243}{4}, \ldots\right\}$; a_9

6. $\left\{\frac{1}{40}, -\frac{1}{10}, \frac{2}{5}, -\frac{8}{5}, \ldots\right\}$; a_{11}

7. $\left\{100, 60, 36, \frac{108}{5} \ldots\right\}$; a_8

Directions: Use the information given to find the indicated value.

8. If $a_1 = -2$ and $r = 5$, find a_8.

9. If $a_2 = -9$ and $r = -3$, find a_6.

10. If $a_7 = \frac{704}{81}$ and $r = -\frac{2}{3}$, find a_{11}.

11. If $a_4 = 1$ and $r = \frac{1}{4}$, find a_7.

12. If $a_1 = \frac{3}{8}$ and $a_8 = 48$, find a_9.

13. If $a_2 = 20000$ and $a_7 = 625$, find a_{10}.

Directions: Write an explicit and recursive formula for each sequence.

14. $\left\{\frac{1}{12}, -\frac{1}{2}, 3, -18, \ldots\right\}$

Explicit:

Recursive:

15. $\left\{-\frac{9}{2}, -3, -2, -\frac{4}{3}, \ldots\right\}$

Explicit:

Recursive:

Directions: Find the indicated geometric means for each set of nonconsecutive terms.

16. 1 and 27; 2 means

17. -160 and $-\frac{5}{2}$; 5 means

18. -6 and -96; 3 means

19. 243 and 147; 1 mean

20. In 2003, Eric started a new job with a yearly salary of $52,000. If the company offers a 2.5% raise each year, use a geometric sequence formula to determine his salary in 2020.

21. A ball was dropped from a height of 125 feet. At each bounce, the ball reaches four-fifths the height of the previous bounce. If this pattern continues, use a geometric sequence formula to find the maximum height reached by the ball after its 5th bounce.

Name:

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Main Ideas/Questions | **Notes/Examples**

SERIES

The sum of the terms in a sequence

Sequence	$\{2, 4, 6, 8\}$	$\{4, 8, 12, 16, \ldots\}$	$\{\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \ldots\}$
Series	$\{2+4+6+8\}$	$\{4+8+12+16+\ldots\}$	$\{\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\frac{1}{24}+\ldots\}$

PARTIAL SUMS

$\boxed{S_n}$

the sum of a specified number of terms in a sequence

Directions: Find the partial sum for each given sequence.

1. $\{2, 6, 10, 14, 18, \ldots\}$; find S_7

22, 26

$2+6+10+14+18+22+26=\boxed{98}$

2. $\{1, -2, 4, -8, 16, \ldots\}$; find S_8

3. $\{1, 1, 2, 3, 5, 8, \ldots\}$; find S_{10}

4. $\left\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots\right\}$; find S_3

5. $a_n = n^3$; find S_5

6. $a_n = \frac{2}{n+1}$; find S_4

7. $a_n = -3n+7$; find S_8

8. $a_n = n^2\sqrt{5}$; find S_6

SUMMATION NOTATION

A way to represent a series using the greek letter Σ to denote the sum.

↖ sigma

$$\boxed{\text{last term \#}} \rightarrow 5 \qquad \sum_{n=1}^{5} 2n \leftarrow \boxed{\text{formula for the sequence}} \qquad \boxed{\text{first term \#}} \rightarrow n=1$$

Find the sum of the series above:

$2(1)+2(2)+2(3)+2(4)+2(5)=30$

EXAMPLES

Directions: Expand each series and evaluate.

9. $\sum_{n=1}^{14}(n+5)$

$6+7+8+9+10+11+12+13+14+15+16+17+18+19=$

$\boxed{175}$

10. $\sum_{n=1}^{11}(-12n)$

11. $\sum_{k=2}^{8}(k^2-k)$

12. $\sum_{c=2}^{6}\left(\frac{1}{c}-c\right)$

13. $\sum_{a=4}^{17}(5a+7)$

14. $\sum_{v=3}^{8}\left(\frac{v+1}{2v}\right)$

15. $-\frac{9}{4}\cdot\sum_{j=1}^{6}(2j^2+7)$

16. $2\cdot\sum_{m=3}^{11}(1-m)^3$

Name:

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Main Ideas/Questions	**Notes/Examples**

ARITHMETIC SERIES

The sum of an arithmetic sequence

To find the sum of an **arithmetic series**, use the following formula:

$$\boxed{S_n = n\left(\frac{a_1+a_n}{2}\right)}$$

where n is the number of terms,
a_1 is the first term, and a_n is the last term.

EXAMPLES

Directions: Find the indicated sum for each arithmetic series.

1. $\{7+10+13+16+\ldots\};\ S_{18}$

$a_n = 3(n-1)+7 = 3n+4$

$a_{18} = 3(18)+4 = 58$

$S_{18} = 18\left(\frac{7+58}{2}\right) = \boxed{585}$

2. $\{50+42+34+26+\ldots\};\ S_{35}$

3. $\{(-4)+(-1.5)+1+3.5+\ldots\};\ S_{21}$

4. $\left\{\frac{1}{10}+\left(-\frac{1}{15}\right)+\left(-\frac{7}{30}\right)+\left(-\frac{2}{5}\right)\ldots\right\};\ S_9$

5. $\sum_{k=1}^{10}(6k+1)$

$a_1 = 6(1)+1 = 7$

$a_{10} = 6(10)+1 = 61$

$S_{10} = 10\left(\frac{7+61}{2}\right) = \boxed{340}$

6. $\sum_{n=1}^{24}(-2n-67)$

7. $\sum_{v=1}^{16}\left(\frac{3}{4}v+\frac{1}{2}\right)$

8. $\sum_{y=2}^{20}(1-4y)$

9. $\sum_{j=4}^{17}(5j-27)$

10. $\sum_{m=3}^{38}\left(-8-\frac{3}{2}m\right)$

APPLICATIONS

11. Mikayla is training for a marathon. She ran 2.4 miles the first day, 2.55 miles the second day, 2.7 miles on the third day. If this pattern continues, find the total distance she ran after 60 days.

Minute	Tickets
1	35
2	43
3	51
4	59

12. Tickets for a concert opened up for sale online. The number of people that purchased tickets in each of the first four minutes is shown in the table to the left. If this pattern continues, and the concert venue can hold a maximum of 75,000 people, find the number of tickets left after the first two hours.

13. A brick retention wall was built such that the bottom row has 200 bricks and each row thereafter has 4 bricks fewer than the row beneath it. On which row was the 3,000th brick laid?

Name: ______________________________

Date: ____________________ **Per:** ________

Unit 11: Sequences and Series

Homework 4: Series & Summations;
Arithmetic Series

**** This is a 2-page document! ****

Directions: Find the partial sum of the given sequence.

1. $\{-9, -2, 5, 12, 19, ...\}$; find S_9

2. $\left\{27, \frac{103}{4}, \frac{49}{2}, \frac{93}{4}, ...\right\}$; find S_7

3. $a_n = \frac{4n}{n+2}$; S_6

Directions: Expand each series and evaluate.

4. $\sum_{p=1}^{9}(2-3p)$

5. $\sum_{j=1}^{15}(j^2-8)$

6. $\frac{7}{5}\cdot\sum_{m=2}^{12}(3m-m^2)$

7. $-3\cdot\sum_{y=6}^{11}\left(\frac{5}{y}+y\right)$

Directions: Find the indicated sum for each arithmetic series.

8. $\{29+25+21+17+...\}$; S_{15}

9. $\{13.1+18.7+24.3+29.9+...\}$; S_{20}

10. $\{(-8)+(-2)+4+10+...\}$; S_{17}

11. $\left\{3+\frac{11}{6}+\frac{2}{3}+\left(-\frac{1}{2}\right)+...\right\}$; S_{11}

12. $\sum_{k=1}^{21}(5k+7)$

13. $\sum_{c=1}^{14}(3-8c)$

14. $\sum_{f=1}^{39}\left(\frac{3}{5}f-9\right)$

15. $\sum_{x=5}^{26}\left(-7-\frac{4}{7}x\right)$

16. $\sum_{y=3}^{32}\left(-\frac{7}{6}y+1\right)$

17. $\sum_{h=7}^{19}(11h-6)$

18. Erin is making friendship bracelets for a fundraiser. She made 7 bracelets on the first day, 11 bracelets on the second day, and 15 bracelets on the third day. If this pattern continues, find the total amount of bracelets made after 4 weeks.

19. Lance created a tower with playing cards. He used 63 cards on the bottom row, then each row thereafter had 3 fewer cards than the row below it. If he used 6 cards on the top row, how many cards did he use to build the tower?

20. The Jefferson Theatre contains 2508 seats. If the first row has 12 seats and each successive row has 4 additional seats than the previous, determine the number of rows in the theatre.

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Main Ideas/Questions | **Notes/Examples**

GEOMETRIC SERIES

The sum of a geometric sequence

To find the sum of a **geometric series**, use the following formula:

$$\boxed{S_n = \frac{a_1(1-r^n)}{1-r}}$$

where n is the number of terms,
a_1 is the first term, and r is the common ratio.

EXAMPLES

Find the indicated sum for each geometric series.

1. $\{2+10+50+250+\ldots\};\ S_9$

$a_1=2 \quad r=5 \quad n=9$

$$S_9 = \frac{2(1-5^9)}{1-5}$$

$$\boxed{976{,}562}$$

2. $\{72+(-36)+18+(-9)+\ldots\};\ S_8$

3. $\{(-4)+(-8)+(-16)+\ldots+(-16{,}384)\}$

4. $\{2500+2000+1600+\ldots+655.36\}$

5. $\sum_{n=1}^{7} 3\cdot(-4)^{n-1}$

6. $\sum_{i=1}^{18} -5\cdot 2^{i-1}$

7. $\sum_{p=1}^{9} -96 \cdot \left(\frac{1}{2}\right)^{p-1}$

8. $\sum_{w=1}^{20} -64 \cdot \left(-\frac{1}{4}\right)^{w-1}$

9. $\sum_{c=2}^{13} 7 \cdot 3^{c-1}$

10. $\sum_{x=5}^{12} \frac{1}{20} \cdot (-5)^{x-1}$

APPLICATIONS

11. The grocery store started raising money for local schools. In their first month, they raised $4,000. Each month after, they raised 1.5% more than the previous month. Find the total money raised in 4 years.

Year	Interest
1	$800
2	$840
3	$882

12. Amari invested $16,000 in a retirement account. The amount of interest earned in each of the first three years is given to the left. If she made no other withdrawals or deposits and the trend in interest continues, what is the balance in her account after 30 years?

13. Carol wants to be a millionaire. If she saves one cent on the first day, two cents on the second day, four cents the third day, and so on, how many days will it take her to save a million dollars?

INFINITE GEOMETRIC SERIES

Find the partial sums for each infinite series below:

$\left\{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots\right\}$	
S_1	.5
S_2	.75
S_3	.875
S_4	.9375
S_5	.96875
S_6	.984375

$S_n \to 1$

A series that approaches a certain sum is called a **CONVERGENT SERIES.**

$\left\{\frac{1}{2}+1+2+4+8+\dots\right\}$	
S_1	.5
S_2	1.5
S_3	3.5
S_4	7.5
S_5	15.5
S_6	31.5

$S_n \to \infty$

A series that does not have a certain sum is called a **DIVERGENT SERIES.**

- If $|r|<1$, then the series is convergent.
- If $|r|>1$, then the series is divergent.

Convergent Series FORMULA

To find the sum of a **convergent infinite geometric series**, use the formula:

$$\boxed{S_n = a_1\left(\frac{1}{1-r}\right)}$$

EXAMPLES

Determine if the series is converent or divergent. If convergent, find the sum.

14. $\{2+(-12)+72+(-432)+\dots\}$

$r = -6$; divergent

15. $\left\{72+24+8+\frac{8}{3}+\dots\right\}$ $r=\frac{1}{3}$; convergent

$S_n = 72\left(\frac{1}{1-\frac{1}{3}}\right) = 108$

16. $\{(-180)+90+(-45)+22.5+\dots\}$

17. $\left\{1+\frac{5}{4}+\frac{25}{16}+\frac{125}{64}+\dots\right\}$

18. $\sum_{k=1}^{\infty} 224\cdot\left(\frac{3}{4}\right)^{k-1}$

19. $\sum_{m=1}^{\infty} 45\cdot(-0.2)^{m-1}$

20. $\sum_{p=1}^{\infty} \frac{1}{3}\cdot\left(-\frac{7}{6}\right)^{p-1}$

21. $\sum_{y=1}^{\infty} -\frac{8}{5}\cdot\left(\frac{1}{2}\right)^{y-1}$

Name: ______________________________ **Unit 11:** Sequences and Series

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**** This is a 2-page document! ****

Directions: Find the indicated sum for each geometric series.

1. $\{7+(-14)+28+(-56)+\ldots\};\ S_8$

2. $\{12.8+16+20+25+\ldots\};\ S_7$

3. $\{5+25+125+\ldots+78{,}125\}$

4. $\left\{6400+1600+400+\ldots+\frac{25}{16}\right\}$

5. $\sum_{n=1}^{8} 5(-3)^{n-1}$

6. $\sum_{p=1}^{14} -80\left(-\frac{1}{2}\right)^{p-1}$

7. $\sum_{i=1}^{11} -3(4)^{i-1}$

8. $\sum_{c=1}^{7} 512\left(\frac{3}{4}\right)^{c-1}$

9. $\sum_{m=4}^{12} -\frac{3}{8}(2)^{m-1}$

10. $\sum_{k=3}^{10} 2700\left(\frac{1}{3}\right)^{k-1}$

Directions: Determine if the series is convergent or divergent. If convergent, find the sum.

11. $\left\{(-32)+(-24)+(-18)+\left(-\frac{27}{2}\right)+\ldots\right\}$

12. $\{2+(-6)+18+(-54)+\ldots\}$

13. $\{200+40+8+1.6+\ldots\}$

14. $\left\{\left(-\frac{512}{3}\right)+64+(-24)+9+\ldots\right\}$

15. $\sum_{d=1}^{\infty}-15(2)^{d-1}$

16. $\sum_{m=1}^{\infty}9\left(-\frac{2}{7}\right)^{m-1}$

17. $\sum_{k=2}^{\infty}\frac{5}{2}\left(\frac{1}{3}\right)^{k-1}$

18. $\sum_{i=7}^{\infty}\frac{1}{8}(-4)^{i-1}$

19. Analeigh earned interest from her investment account with the interest earned since opening the account shown in the table below. Find the total amount of interest Analeigh will earn after 22 years.

Year	Interest
1	\$60
2	\$64.80
3	\$69.98

20. Jonas was collecting donations during a football game. He collected \$32 in the first minute, another \$28.80 in the second minute, and \$25.92 in the third minute. If this pattern continues, what is the total amount of money he will collect after a half of an hour?

21. A loan payment is made such that the first payment is \$5 and each subsequent payment is 10% more than the previous. If the sum of payments made is \$3,263, determine the number of loan payments made.

22. On its first swing, a pendulum travels 5 meters backwards then returns 3.75 meters forward. On the next swing, the pendulum swings 3.75 meters backwards, then returns 2.8125 meters forward. Approximate the total distance the pendulum travels, until it stops moving.

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Main Ideas/Questions	Notes/Examples
Arithmetic vs. Geometric **SEQUENCES**	**Directions:** Determine whether the sequence is arithmetic, geometric, or neither. If arithmetic or geometric, write an explicit formula to find the n^{th} term, then find a_7. **1.** $\{14, -42, 126, -378, \ldots\}$ **2.** $\{1, -1, 2, -2, \ldots\}$ **3.** $\{53, 44, 35, 26, \ldots\}$ **4.** $\left\{-\frac{15}{4}, -\frac{13}{4}, -\frac{11}{4}, -\frac{9}{4}, \ldots\right\}$ **5.** $\left\{\frac{1}{10}, \frac{2}{9}, \frac{3}{8}, \frac{4}{7}, \ldots\right\}$ **6.** $\{320, 80, 20, 5, \ldots\}$ **7.** $\{-145, -119, -93, -67, \ldots\}$ **8.** $\left\{-\frac{18}{25}, \frac{6}{5}, -2, \frac{10}{3}, \ldots\right\}$
Arithmetic vs. Geometric **SERIES**	**Directions:** Determine whether the series is arithmetic or geometric. Then find the indicated sum. **9.** $\{18+25+32+39+\ldots\}$; S_{19} **10.** $\{3-15+75-375+\ldots\}$; S_7

11. $\{-9+(-14)+(-19)+(-24)+\ldots\};\ S_{11}$

12. $\left\{-\frac{1}{2}-\frac{1}{4}-\frac{1}{8}-\frac{1}{16}\ldots\right\};\ S_8$

13. $\{13.5+9+6+4+\ldots\};\ S_{10}$

14. $\left\{\frac{12}{5}+\frac{56}{15}+\frac{76}{15}+\frac{32}{5}+\ldots\right\};\ S_{13}$

Directions: Determine whether the series is arithmetic or geometric, then find the sum, if possible.

15. $\sum_{m=1}^{28}(23-6m)$

16. $\sum_{y=1}^{13}-5\cdot 2^{y-1}$

17. $\sum_{c=5}^{34}\left(\frac{1}{3}c-\frac{25}{12}\right)$

18. $\sum_{n=3}^{7}\frac{20}{3}\cdot\left(-\frac{1}{4}\right)^{n-1}$

19. $\sum_{i=1}^{\infty}-\frac{7}{4}\cdot\left(-\frac{1}{5}\right)^{i-1}$

20. $\sum_{p=1}^{\infty}-4\cdot\left(-\frac{3}{2}\right)^{p-1}$

Name:	**Date:**
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Main Ideas/Questions	Notes/Examples
SEQUENCE Applications	**1.** A library book that is one day late is charged a $1.95 fee. Each day thereafter, it is charged an extra $0.20. Find the fee for a book that is 35 days late.
	2. Tucker took an 800-milligram dose of medicine for his headache. The table to the left shows the amount of medicine remaining in his bloodstream after each of the first three hours. After how many hours will the amount of medicine reach 50 milligrams?
SERIES Applications	**3.** Stocks at a company were initially issued at $9.80 per share. The value of the shares has increased by 25% each year. If Ari bought 20 shares each year since they were issued, find the total amount she has spent in buying shares after 15 years.
	4. Evan got a job with a starting salary of $36,000, with a $1,500 raise each subsequent year. How many years will it take for his total earnings to reach $1,000,000?
MIXED Applications	**5.** A ball is dropped from a tower. The table below shows the height of the ball after each of the first three bounces. Find the height of the ball after the 12th bounce.

Hour	Milligrams
1	800
2	680
3	578

Bounce	Height (ft)
1	50
2	45
3	40.5

6. Logs are stacked so that they are 40 logs on the bottom row and each row thereafter has 2 logs fewer than the row below it. If the top row has 8 logs, find the total number of logs in the stack.

7. When Michelle brought her newborn son John home, he slept just three hours the first night. Each night thereafter, he slept an extra 5 minutes than the previous night. How many nights will it take John to sleep an 8-hour stretch?

8. Elijah started a new Instagram account and gained 8 new followers in his first week. Each subsequent week, he gained twice as many new followers than he did the previous week. How many total followers does Elijah have after 16 weeks?

9. There are 20 seats in the first row of a concert hall. Each row thereafter has 3 seats more than the previous row. If 600 students are coming to the hall for a field trip, how many rows will be needed, assuming they are seated starting with the first row?

10. The table to the left shows the value of a car that was manufactured in 2012, along with its value for three subsequent years. In what year will the value of the car reach $4,000?

Year	Value
2012	$37,500
2013	$31,500
2014	$26,460
2015	$22,226.40

Name: ______________________________ **Unit 11:** Sequences and Series

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**** This is a 2-page document! ****

Directions: Determine whether the sequence is arithmetic, geometric, or neither. If arithmetic or geometric, write an explicit formula to find the n^{th} term, then find a_9.

1. $\{72, 64, 56, 48, \ldots\}$

2. $\left\{\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \ldots\right\}$

3. $\{(-16), (-24), (-36), (-54), \ldots\}$

4. $\{39, -117, 351, -1053, \ldots\}$

5. $\left\{-10, -\frac{5}{2}, 5, \frac{25}{2}, \ldots\right\}$

6. $\{1, 8, 27, 64, \ldots\}$

Directions: Determine whether the series is arithmetic or geometric. Then find the indicated sum.

7. $\{-3+12-48+192+\ldots\}$; S_8

8. $\{23+32+41+50+\ldots\}$; S_{19}

9. $\{-4-8-12-16-\ldots\}$; S_{22}

10. $\left\{5+\frac{5}{2}+\frac{5}{4}+\frac{5}{8}+\ldots\right\}$; S_8

11. $\left\{\frac{13}{6}+\frac{8}{3}+\frac{19}{6}+\frac{11}{3}+\ldots\right\}$; S_9

12. $\{135-45+15-5+\ldots\}$; S_6

Directions: Determine whether the series is arithmetic or geometric, then find the sum, if possible.

13. $\sum_{c=1}^{10} 3 \cdot 2^{c-1}$

14. $\sum_{k=1}^{21} (7k-6)$

15. $\sum_{i=3}^{19} \left(\frac{1}{3}i - \frac{19}{30}\right)$

16. $\sum_{n=6}^{13} \left(-\frac{1}{2}\right)^{n-1}$

17. $\sum_{j=1}^{\infty} 14 \cdot \left(-\frac{8}{7}\right)^{j-1}$

18. $\sum_{p=1}^{\infty} -5.6 \cdot (0.8)^{p-1}$

19. A brick wall was built such that the bottom row had 51 bricks and each row above contained three less than the previous. Determine which row contained 12 bricks.

20. Alan deposits $5,000 into a retirement account, earning 6% annual interest. Find the balance after 26 years.

21. A credit card payment of $225 is due on the first of the month. Each day the payment is late, an additional 2.5% is charged. If Erick paid a total of $288, how late was his payment?

22. A mechanic charges $17 for the first hour, $15.75 for the second hour, and $14.50 for the third hour. If a certain job takes him 7 hours, determine the total labor cost.

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Main Ideas/Questions	**Notes/Examples**

Patterns & CONJECTURES

Discovery and proof are two aspects of mathematics. Math facts are generally established by discovering a pattern, making a conjecture, and finally proving the conjecture. Follow the example below:

$1 =$ 1

$1 + 3 =$ 4

$1 + 3 + 5 =$ 9

$1 + 3 + 5 + 7 =$ 16

$1 + 3 + 5 + 7 + 9 =$ 25

Find a pattern: Look for a pattern in the sum of the first 5 positive odd integers to the left.

Make a conjecture: If the n^{th} odd number can be written as $2n - 1$, write a conjecture about the sum of the first n positive odd integers:

$1 + 3 + 5 + 7 + 9 + \ldots + (2n - 1) =$ n^2.

PROOF BY Mathematical INDUCTION

A **proof** is a clear argument that demonstrates the truth of a conjecture. Proving a method false simply requires a counterexample. Proving a conjecture true requires a more formal method. One such method of proof is called **mathematical induction.**

The Principle of Mathematical Induction:

Let P_n be a statement depending on a positive integer n.
P_n is true for all positive integers n if and only if:

- P_1 is true.
- If P_k is true for any positive integer k, then P_{k+1} is true.

The main steps to prove a conjecture by induction are outlined below.

❶ **Base Case:** Prove P_1 is true.

❷ **Inductive Hypothesis:** For any positive integer k, assume P_k is true.

❸ **Inductive Step:** Prove P_{k+1} is true.

EXAMPLES

Use induction to prove each conjecture is true for all positive n integers.

1. $1+3+5+7+\ldots+(2n-1)=n^2$

① $P_1 : 1 = 1^2$

$1 = 1$ ✓

② If $n=k$, assume P_k is true

$P_k = 1+3+5+7+\ldots+(2k-1) = k^2$

③ Prove P_{k+1} is true

$$P_{k+1} = \underbrace{1+3+5+7+\ldots+(2k-1)}_{=k^2} + (2(k+1)-1) = (k+1)^2$$

$$= k^2 + (2(k+1)-1) = (k+1)^2$$

$$k^2 + 2k + 2 - 1 = (k+1)^2$$

$$k^2 + 2k + 1 = (k+1)^2$$

$$(k+1)^2 = (k+1)^2 \checkmark$$

	2. $2+4+6+\ldots+2n=n(n+1)$
	3. $1^2+2^2+3^2+\ldots+n^2=\dfrac{n(n+1)(2n+1)}{6}$
	4. $1+2+4+8+\ldots+2^{n-1}=2^n-1$

SIGMA NOTATION

5. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

① $P_1 : 1 = \frac{1(1+1)}{2}$

$1 = 1$ ✓

② $\sum_{i=1}^{n} i = \frac{k(k+1)}{2}$ if n=k is true

③ Prove $\sum_{i=1}^{k+1} i = \frac{(k+1)((k+1)+1)}{2}$

$= \frac{(k+1)(k+2)}{2}$

$= \frac{k^2+3k+2}{2}$

$\sum_{i=1}^{k+1} i = \sum_{i=1}^{k} i + (k+1)$

$= \frac{k(k+1)}{2} + k+1$

$= \frac{k^2+k}{2} + \frac{2(k+1)}{2}$

$\frac{k^2+3k+2}{2}$

✓

6. $\sum_{i=1}^{n} (4i-2) = 2n^2$

PROVING DIVISIBILITY RULES

Prove that each conjecture is true for all positive integers *n*.

7. $n^2 + n$ is divisible by 2

$P_n = n^2 + n$ is divisible by 2

① $P_1 = 1^2 + 1 = 2$

2 is divisible by 2 ✓

② For $n = k$ assume P_k is true, that $k^2 + k$ ~~is~~ divisible by 2 or $k^2 + k = 2r$ for some integer r.

③ Prove P_{k+1} is divisible by 2

$$P_{k+1} = (k+1)^2 + (k+1)$$
$$= k^2 + 2k + 1 + k + 1$$
$$= \underbrace{k^2 + k} + 2k + 2$$
$$= 2r + 2k + 2$$
$$= 2(r + k + 1)$$

since r and k are integers, $(r+k+1)$ is an integer

∴ P_{k+1} is divisible by 2.

8. $11^n - 6$ is divisible by 5

Name: ______________________________ **Unit 11:** Sequences and Series

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**** This is a 2-page document! ****

Directions: Use induction to prove each conjecture is true for all positive n integers.

1. $8+14+20+26+\ldots+(6n+2)=n(3n+5)$

2. $2+8+20+40+\ldots+(n^2+n)=\dfrac{n(n+1)(n+2)}{3}$

3. $\displaystyle\sum_{i=1}^{n}(2i-1)^2=\frac{n(2n-1)(2n+1)}{3}$

4. $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$

5. $n^3 + 2n$ is divisible by 3

6. $17n^2 - 13n$ is divisible by -2

Name:

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Main Ideas/Questions | **Notes/Examples**

PASCAL'S triangle

Pascal's triangle is a pattern of numbers that was discovered in the 17th century by French mathematician Blaise Pascal. Each number in Pascal's triangle is the sum of the two numbers diagonally above it. All outside numbers are 1.

Complete rows 4 and 5 of Pascal's triangle below:

Row 0 → 1
Row 1 → 1 1
Row 2 → 1 2 1
Row 3 → 1 3 3 1
Row 4 → $\boxed{1}$ $\boxed{4}$ $\boxed{6}$ $\boxed{4}$ $\boxed{1}$
Row 5 → $\boxed{1}$ $\boxed{5}$ $\boxed{10}$ $\boxed{10}$ $\boxed{5}$ $\boxed{1}$

BINOMIAL EXPANSION
Patterns

Consider the binomial expansion of $(a+b)^n$:

$(a+b)^0 = 1$
$(a+b)^1 = a+b$
$(a+b)^2 = a^2+2ab+b^2$
$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$
$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$
$(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$

What do you notice about the coefficients?

They are symmetrical and form the n^{th} row of Pascal's triangle

Other important patterns in the binomial expansion of $(a+b)^n$:

- The total number of terms is always $n+1$.
- The first term is 1 and the last term is 1.
- The exponent of a decreases by 1 from left to right.
- The exponent of b increases by 1 from left to right.
- The sum of the exponents in each term is n.

EXPANDING A BINOMIAL
Using Pascal's Triangle

Use Pascal's triangle to expand each binomial.

1. $(a+b)^6$ row 6: 1 6 15 20 15 6 1

$(a+b)^6 = 1a^6b^0 + 6a^5b^1 + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6a^1b^5 + 1a^0b^6$

$\boxed{= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6}$

2. $(x+2)^7$

3. $(3m-1)^4$

4. $(4c-3d)^5$

5. $\left(1+\frac{2}{x}\right)^3$

6. $\left(a-\frac{1}{a}\right)^6$

Name:

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Main Ideas/Questions	**Notes/Examples**

FIND ANY ELEMENT
in Pascal's Triangle

- Each element of Pascal's triangle is assigned a row n and element r number. **(Note both rows and elements begin with 0.)**
- The n^{th} row, r^{th} element of Pascal's triangle is denoted as $\binom{n}{r}$ and defined as:

$$\binom{n}{r} = {}_nC_r = \frac{n!}{r!(n-r)!}$$

rows

0   1
1   1 1
2   1 2 1
3   1 3 3 1
4   1 4 6 4 1
5   1 5 10 10 5 1

0 1 2 3 4 5
elements

Find each element in Pascal's triangle.

1. 6th row, 3rd element

2. 13th row, 5th element

3. 18th row, 2nd element

4. 25th row, 14th element

Finding BINOMIAL TERMS

The $(r+1)^{th}$ term in the binomial expansion of $(a+b)^n$ is defined as $\binom{n}{r}a^{n-r}b^r$.

Find the indicated term of each binomial expansion.

5. $(a+b)^{10}$; 8th term

6. $(x-7)^{12}$; 3rd term

7. $(p-q)^{8}$; 5th term

8. $(2k+1)^{10}$; 6th term

9. $(v+3)^{9}$; 7th term

10. $(3a-2)^{7}$; 4th term

THE BINOMIAL THEOREM

If $\boldsymbol{n}$ is a natural number, then $(a+b)^n =$

$${}_nC_0 \cdot a^n b^0 + {}_nC_1 \cdot a^{n-1} b^1 + {}_nC_2 \cdot a^{n-2} b^2 + \dots + {}_nC_n \cdot a^0 b^n = \boxed{\sum_{r=0}^{n} {}_nC_k \cdot a^{n-r} b^r}$$

Use the Binomial Theorem to expand each binomial.

11. $(a+2)^6$

12. $(x-3y)^4$

13. $(1-2k)^7$

14. $(3n+4)^5$

15. $\left(\frac{5}{x}+y\right)^4$

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**** This is a 2-page document! ****

Directions: Use Pascal's triangle to expand each binomial.

1. $(x-y)^8$

2. $(2k-3)^5$

3. $\left(4m+\frac{1}{m}\right)^4$

Directions: Find each element in Pascal's triangle.

4. 9th row, 6th element

5. 14th row, 8th element

6. 23rd row, 19th element

Directions: Find the indicated term of each binomial expansion.

7. $(x+5)^6$; 5th term

8. $(j-k)^{11}$; 8th term

9. $(2p+3)^9$; 4th term

10. $(4r-s)^7$; 7th term	**11.** $(5m-3n)^8$; 3rd term	**12.** $(3c-2d)^{10}$; 6th term

Directions: Use Binomial Theorem to expand each binomial.

13. $(p+7)^5$

14. $(5x-4)^4$

15. $(m+3n)^7$

16. $\left(2a-\frac{1}{b}\right)^5$