

1-4 Study Guide and Intervention**Identity and Equality Properties**

Identity and Equality Properties The identity and equality properties in the chart below can help you solve algebraic equations and evaluate mathematical expressions.

Additive Identity	For any number a , $a + 0 = a$.
Multiplicative Identity	For any number a , $a \cdot 1 = a$.
Multiplicative Property of 0	For any number a , $a \cdot 0 = 0$.
Multiplicative Inverse Property	For every number $\frac{a}{b}$, $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \cdot \frac{b}{a} = 1$.
Reflexive Property	For any number a , $a = a$.
Symmetric Property	For any numbers a and b , if $a = b$, then $b = a$.
Transitive Property	For any numbers a , b , and c , if $a = b$ and $b = c$, then $a = c$.
Substitution Property	If $a = b$, then a may be replaced by b in any expression.

Example 1 Name the property used in each equation. Then find the value of n .

a. $8n = 8$

Multiplicative Identity Property

$n = 1$, since $8 \cdot 1 = 8$

b. $n \cdot 3 = 1$

Multiplicative Inverse Property

$n = \frac{1}{3}$, since $\frac{1}{3} \cdot 3 = 1$

Example 2 Name the property used to justify each statement.

a. $5 + 4 = 5 + 4$

Reflexive Property

b. If $n = 12$, then $4n = 4 \cdot 12$.

Substitution Property

Exercises

Name the property used in each equation. Then find the value of n .

1. $6n = 6$

2. $n \cdot 1 = 8$

3. $6 \cdot n = 6 \cdot 9$

4. $9 = n + 9$

5. $n + 0 = \frac{3}{8}$

6. $\frac{3}{4} \cdot n = 1$

Name the property used in each equation.

7. If $4 + 5 = 9$, then $9 = 4 + 5$.

8. $0 + 21 = 21$

9. $0(15) = 0$

10. $(1)94 = 94$

11. If $3 + 3 = 6$ and $6 = 3 \cdot 2$, then $3 + 3 = 3 \cdot 2$.

12. $4 + 3 = 4 + 3$

13. $(14 - 6) + 3 = 8 + 3$

1-5 Study Guide and Intervention

The Distributive Property

Evaluate Expressions The Distributive Property can be used to help evaluate expressions.

Distributive Property	For any numbers a , b , and c , $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ and $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$.
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Example 1 Rewrite $6(8 + 10)$ using the Distributive Property. Then evaluate.

$$\begin{aligned} 6(8 + 10) &= 6 \cdot 8 + 6 \cdot 10 && \text{Distributive Property} \\ &= 48 + 60 && \text{Multiply.} \\ &= 108 && \text{Add.} \end{aligned}$$

Example 2 Rewrite $-2(3x^2 + 5x + 1)$ using the Distributive Property. Then simplify.

$$\begin{aligned} -2(3x^2 + 5x + 1) &= -2(3x^2) + (-2)(5x) + (-2)(1) && \text{Distributive Property} \\ &= -6x^2 + (-10x) + (-2) && \text{Multiply.} \\ &= -6x^2 - 10x - 2 && \text{Simplify.} \end{aligned}$$

Exercises

Rewrite each expression using the Distributive Property. Then simplify.

1. $2(10 - 5)$

2. $6(12 - t)$

3. $3(x - 1)$

4. $6(12 + 5)$

5. $(x - 4)3$

6. $-2(x + 3)$

7. $5(4x - 9)$

8. $3(8 - 2x)$

9. $12\left(6 - \frac{1}{2}x\right)$

10. $12\left(2 + \frac{1}{2}x\right)$

11. $\frac{1}{4}(12 - 4t)$

12. $3(2x - y)$

13. $2(3x + 2y - z)$

14. $(x - 2)y$

15. $2(3a - 2b + c)$

16. $\frac{1}{4}(16x - 12y + 4z)$

17. $(2 - 3x + x^2)3$

18. $-2(2x^2 + 3x + 1)$

1-8 Study Guide and Intervention**Number Systems**

Square Roots A square root is one of two equal factors of a number. For example, the square roots of 36 are 6 and -6 , since $6 \cdot 6$ or 6^2 is 36 and $(-6)(-6)$ or $(-6)^2$ is also 36. A rational number like 36, whose square root is a rational number, is called a **perfect square**.

The symbol $\sqrt{\quad}$ is a **radical sign**. It indicates the nonnegative, or **principal**, square root of the number under the radical sign. So $\sqrt{36} = 6$ and $-\sqrt{36} = -6$. The symbol $\pm\sqrt{36}$ represents both square roots.

Example 1 Find $-\sqrt{\frac{25}{49}}$.

$-\sqrt{\frac{25}{49}}$ represents the negative square root of $\frac{25}{49}$.

$$\frac{25}{49} = \left(\frac{5}{7}\right)^2 \rightarrow -\sqrt{\frac{25}{49}} = -\frac{5}{7}$$

Exercises

Find each square root.

1. $\sqrt{64}$

2. $-\sqrt{81}$

3. $\sqrt{16.81}$

4. $\pm\sqrt{100}$

5. $-\sqrt{\frac{4}{25}}$

6. $-\sqrt{121}$

7. $\pm\sqrt{\frac{25}{144}}$

8. $-\sqrt{\frac{25}{16}}$

9. $\pm\sqrt{\frac{121}{100}}$

10. $-\sqrt{3600}$

11. $-\sqrt{6.25}$

12. $\pm\sqrt{0.0004}$

13. $\sqrt{\frac{144}{196}}$

14. $-\sqrt{\frac{36}{49}}$

15. $\pm\sqrt{1.21}$

Example 2 Find $\pm\sqrt{0.16}$.

$\pm\sqrt{0.16}$ represents the positive and negative square roots of 0.16.

$$0.16 = 0.4^2 \text{ and } 0.16 = (-0.4)^2$$

$$\pm\sqrt{0.16} = \pm 0.4$$

2-5 Study Guide and Intervention

Solving Equations with the Variable on Each Side

Variables on Each Side To solve an equation with the same variable on each side, first use the Addition or the Subtraction Property of Equality to write an equivalent equation that has the variable on just one side of the equation. Then solve the equation.

Example 1 Solve $5y - 8 = 3y + 12$.

$$\begin{aligned} 5y - 8 &= 3y + 12 \\ 5y - 8 - 3y &= 3y + 12 - 3y \\ 2y - 8 &= 12 \\ 2y - 8 + 8 &= 12 + 8 \\ 2y &= 20 \\ \frac{2y}{2} &= \frac{20}{2} \\ y &= 10 \end{aligned}$$

The solution is 10.

Example 2 Solve $-11 - 3y = 8y + 1$.

$$\begin{aligned} -11 - 3y &= 8y + 1 \\ -11 - 3y + 3y &= 8y + 1 + 3y \\ -11 &= 11y + 1 \\ -11 - 1 &= 11y + 1 - 1 \\ -12 &= 11y \\ \frac{-12}{11} &= \frac{11y}{11} \\ -1\frac{1}{11} &= y \end{aligned}$$

The solution is $-1\frac{1}{11}$.

Exercises

Solve each equation. Then check your solution.

1. $6 - b = 5b + 30$

2. $5y - 2y = 3y + 2$

3. $5x + 2 = 2x - 10$

4. $4n - 8 = 3n + 2$

5. $1.2x + 4.3 = 2.1 - x$

6. $4.4s + 6.2 = 8.8s - 1.8$

7. $\frac{1}{2}b + 4 = \frac{1}{8}b + 88$

8. $\frac{3}{4}k - 5 = \frac{1}{4}k - 1$

9. $8 - 5p = 4p - 1$

10. $4b - 8 = 10 - 2b$

11. $0.2x - 8 = -2 - x$

12. $3y - 1.8 = 3y - 1.8$

13. $-4 - 3x = 7x - 6$

14. $8 + 4k = -10 + k$

15. $20 - a = 10a - 2$

16. $\frac{2}{3}n + 8 = \frac{1}{2}n + 2$

17. $\frac{2}{5}y - 8 = 9 - \frac{3}{5}y$

18. $-4r + 5 = 5 - 4r$

19. $-4 - 3x = 6x - 6$

20. $18 - 4k = -10 - 4k$

21. $12 + 2y = 10y - 12$

2-6**Study Guide and Intervention****Ratios and Proportions**

Ratios and Proportions A **ratio** is a comparison of two numbers by division. The ratio of x to y can be expressed as x to y , $x:y$ or $\frac{x}{y}$. Ratios are usually expressed in simplest form.

An equation stating that two ratios are equal is called a **proportion**. To determine whether two ratios form a proportion, express both ratios in simplest form or check cross products.

Example 1 Determine whether the ratios $\frac{24}{36}$ and $\frac{12}{18}$ form a proportion.

$$\frac{24}{36} = \frac{2}{3} \text{ when expressed in simplest form.}$$

$$\frac{12}{18} = \frac{2}{3} \text{ when expressed in simplest form.}$$

The ratios $\frac{24}{36}$ and $\frac{12}{18}$ form a proportion because they are equal when expressed in simplest form.

Example 2 Use cross products to determine whether $\frac{10}{18}$ and $\frac{25}{45}$ form a proportion.

$$\frac{10}{18} \stackrel{?}{=} \frac{25}{45}$$

Write the proportion.

$$10(45) \stackrel{?}{=} 18(25)$$

Cross products

$$450 = 450$$

Simplify.

The cross products are equal, so $\frac{10}{18} = \frac{25}{45}$. Since the ratios are equal, they form a proportion.

Exercises

Use cross products to determine whether each pair of ratios forms a proportion.

1. $\frac{1}{2}, \frac{16}{32}$

2. $\frac{5}{8}, \frac{10}{15}$

3. $\frac{10}{20}, \frac{25}{49}$

4. $\frac{25}{36}, \frac{15}{20}$

5. $\frac{12}{32}, \frac{3}{16}$

6. $\frac{4}{9}, \frac{12}{27}$

7. $\frac{0.1}{2}, \frac{5}{100}$

8. $\frac{15}{20}, \frac{9}{12}$

9. $\frac{14}{21}, \frac{20}{30}$

10. 2:3, 20:30

11. 5 to 9, 25 to 45

12. $\frac{72}{64}, \frac{9}{8}$

13. 5:5, 30:20

14. 18 to 24, 50 to 75

15. 100:75, 44:33

16. $\frac{0.05}{1}, \frac{1}{20}$

17. $\frac{1.5}{2}, \frac{6}{8}$

18. $\frac{0.1}{0.2}, \frac{0.45}{0.9}$

2-7 Study Guide and Intervention**Percent of Change**

Percent of Change When an increase or decrease in an amount is expressed as a percent, the percent is called the **percent of change**. If the new number is greater than the original number, the percent of change is a **percent of increase**. If the new number is less than the original number, the percent of change is the **percent of decrease**.

Example 1

Find the percent of increase.

original: 48

new: 60

First, subtract to find the amount of increase. The amount of increase is $60 - 48 = 12$.

Then find the percent of increase by using the original number, 48, as the base.

$$\frac{12}{48} = \frac{r}{100} \quad \text{Percent proportion}$$

$$12(100) = 48(r) \quad \text{Cross products}$$

$$1200 = 48r \quad \text{Simplify.}$$

$$\frac{1200}{48} = \frac{48r}{48} \quad \text{Divide each side by 48.}$$

$$25 = r \quad \text{Simplify.}$$

The percent of increase is 25%.

Example 2

Find the percent of decrease.

original: 30

new: 22

First, subtract to find the amount of decrease. The amount of decrease is $30 - 22 = 8$.

Then find the percent of decrease by using the original number, 30, as the base.

$$\frac{8}{30} = \frac{r}{100} \quad \text{Percent proportion}$$

$$8(100) = 30(r) \quad \text{Cross products}$$

$$800 = 30r \quad \text{Simplify.}$$

$$\frac{800}{30} = \frac{30r}{30} \quad \text{Divide each side by 30.}$$

$$26\frac{2}{3} = r \quad \text{Simplify.}$$

The percent of decrease is $26\frac{2}{3}\%$, or about 27%.

Exercises

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

1. original: 50
new: 80

2. original: 90
new: 100

3. original: 45
new: 20

4. original: 77.5
new: 62

5. original: 140
new: 150

6. original: 135
new: 90

7. original: 120
new: 180

8. original: 90
new: 270

9. original: 27.5
new: 25

10. original: 84
new: 98

11. original: 12.5
new: 10

12. original: 250
new: 500

Algebra

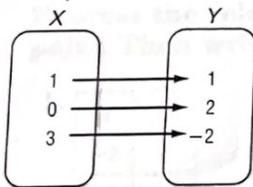
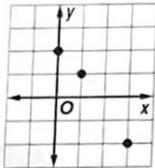
3-1 Study Guide and Intervention

Representing Relations

Represent Relations A relation is a set of ordered pairs. A relation can be represented by a set of ordered pairs, a table, a graph, or a **mapping**. A mapping illustrates how each element of the domain is paired with an element in the range.

Example 1 Express the relation $\{(1, 1), (0, 2), (3, -2)\}$ as a table, a graph, and a mapping. State the domain and range of the relation.

x	y
1	1
0	2
3	-2



The domain for this relation is $\{0, 1, 3\}$.
The range for this relation is $\{-2, 1, 2\}$.

Example 2 A person playing racquetball uses 4 calories per hour for every pound he or she weighs.

a. Make a table to show the relation between weight and calories burned in one hour for people weighing 100, 110, 120, and 130 pounds.

x	y
100	400
110	440
120	480
130	520

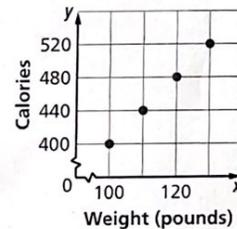
Source: *The Math Teacher's Book of Lists*

b. Give the domain and range.

domain: $\{100, 110, 120, 130\}$

range: $\{400, 440, 480, 520\}$

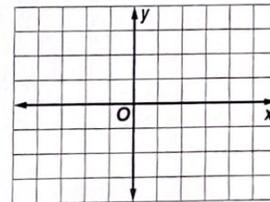
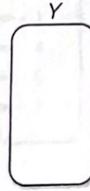
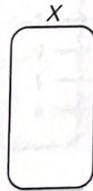
c. Graph the relation.



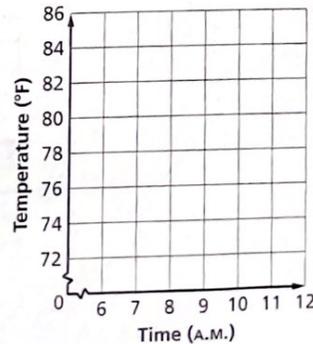
Exercises

1. Express the relation $\{(-2, -1), (3, 3), (4, 3)\}$ as a table, a graph, and a mapping. Then determine the domain and range.

x	y



2. The temperature in a house drops 2° for every hour the air conditioner is on between the hours of 6 A.M. and 11 A.M. Make a graph to show the relationship between time and temperature if the temperature at 6 A.M. was 82°F .

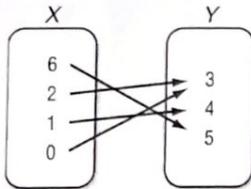


3-1 Study Guide and Intervention *(continued)*

Representing Relations

Inverse Relations The inverse of any relation is obtained by switching the coordinates in each ordered pair.

Example Express the relation shown in the mapping as a set of ordered pairs. Then write the inverse of the relation.



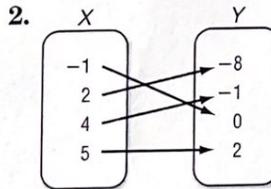
Relation: $\{(6, 5), (2, 3), (1, 4), (0, 3)\}$
 Inverse: $\{(5, 6), (3, 2), (4, 1), (3, 0)\}$

Exercises

Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of each relation.

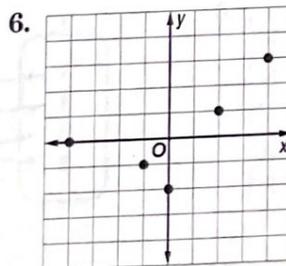
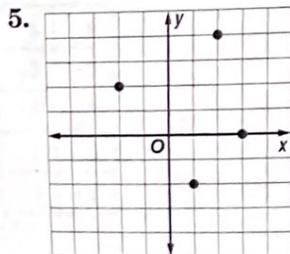
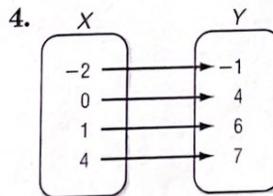
1.

x	y
-2	4
-1	3
2	1
4	5



3.

x	y
-3	5
-2	-1
1	0
2	4

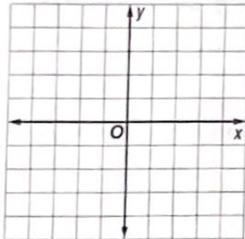


3-1 Skills Practice

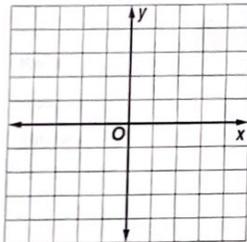
Representing Relations

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

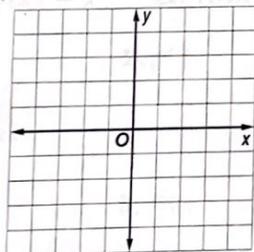
1. $\{(-1, -1), (1, 1), (2, 1), (3, 2)\}$



2. $\{(0, 4), (-4, -4), (-2, 3), (4, 0)\}$



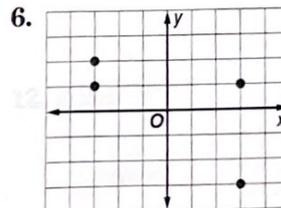
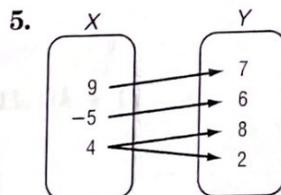
3. $\{(3, -2), (1, 0), (-2, 4), (3, 1)\}$



Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation.

4.

x	y
3	-5
-4	3
7	6
1	-2



3-2**Study Guide and Intervention** (continued)**Representing Functions**

Function Values Equations that are functions can be written in a form called **function notation**. For example, $y = 2x - 1$ can be written as $f(x) = 2x - 1$. In the function, x represents the elements of the domain, and $f(x)$ represents the elements of the range. Suppose you want to find the value in the range that corresponds to the element 2 in the domain. This is written $f(2)$ and is read "f of 2." The value of $f(2)$ is found by substituting 2 for x in the equation.

Example If $f(x) = 3x - 4$, find each value.

a. $f(3)$

$$\begin{aligned} f(3) &= 3(3) - 4 && \text{Replace } x \text{ with } 3. \\ &= 9 - 4 && \text{Multiply.} \\ &= 5 && \text{Simplify.} \end{aligned}$$

b. $f(-2)$

$$\begin{aligned} f(-2) &= 3(-2) - 4 && \text{Replace } x \text{ with } -2. \\ &= -6 - 4 && \text{Multiply.} \\ &= -10 && \text{Simplify.} \end{aligned}$$

Exercises

If $f(x) = 2x - 4$ and $g(x) = x^2 - 4x$, find each value.

1. $f(4)$

2. $g(2)$

3. $f(-5)$

4. $g(-3)$

5. $f(0)$

6. $g(0)$

7. $f(3) - 1$

8. $f\left(\frac{1}{4}\right)$

9. $g\left(\frac{1}{4}\right)$

10. $f(a^2)$

11. $f(k + 1)$

12. $g(2c)$

13. $f(3x)$

14. $f(2) + 3$

15. $g(-4)$

3-3 Study Guide and Intervention *(continued)*

Linear Functions

Graph Linear Equations The graph of a linear equation represents all the solutions of the equation. An x -coordinate of the point at which a graph of an equation crosses the x -axis is an **x -intercept**. A y -coordinate of the point at which a graph crosses the y -axis is called a **y -intercept**.

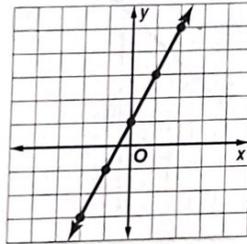
Example 1 Graph the equation $y - 2x = 1$ by making a table.

Solve the equation for y .

$$\begin{aligned} y - 2x &= 1 && \text{Original equation.} \\ y - 2x + 2x &= 1 + 2x && \text{Add } 2x \text{ to each side.} \\ y &= 2x + 1 && \text{Simplify.} \end{aligned}$$

Select five values for the domain and make a table. Then graph the ordered pairs and draw a line through the points.

x	$2x + 1$	y	(x, y)
-2	$2(-2) + 1$	-3	$(-2, -3)$
-1	$2(-1) + 1$	-1	$(-1, -1)$
0	$2(0) + 1$	1	$(0, 1)$
1	$2(1) + 1$	3	$(1, 3)$
2	$2(2) + 1$	5	$(2, 5)$



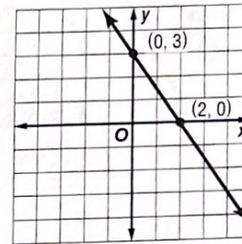
Example 2 Graph the equation $3x + 2y = 6$ by using the x -intercept and y -intercept.

To find the x -intercept, let $y = 0$ and solve for x . The x -intercept is 2. The graph intersects the x -axis at $(2, 0)$.

To find the y -intercept, let $x = 0$ and solve for y .

The y -intercept is 3. The graph intersects the y -axis at $(0, 3)$.

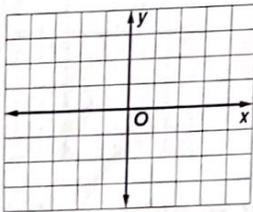
Plot the points $(2, 0)$ and $(0, 3)$ and draw the line through them.



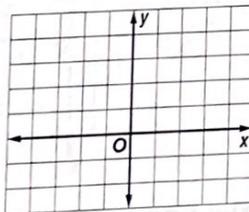
Exercises

Graph each equation by making a table.

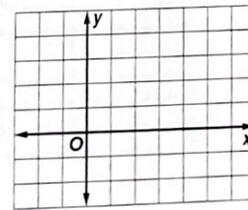
1. $y = 2x$



2. $x - y = -1$

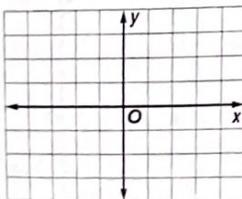


3. $x + 2y = 4$

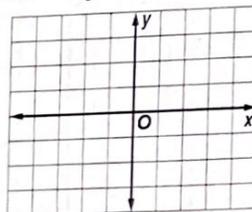


Graph each equation by using the x -intercept and y -intercept.

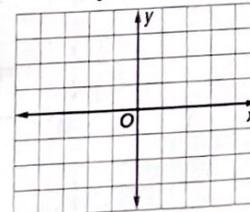
4. $2x + y = -2$



5. $3x - 6y = -3$



6. $-2x + y = -2$



Only complete

#10-18

NAME _____

DATE _____

PERIOD _____

3-3

Skills Practice

Linear Functions

Determine whether each equation is a linear equation. If so, write the equation in standard form.

1. $xy = 6$

2. $y = 2 - 3x$

3. $5x = y - 4$

4. $y = 2x + 5$

5. $y = -7 + 6x$

6. $y = 3x^2 + 1$

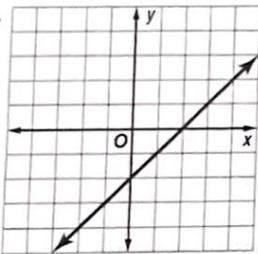
7. $y - 4 = 0$

8. $5x + 6y = 3x + 2$

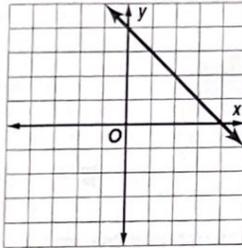
9. $\frac{1}{2}y = 1$

Determine the x -intercept and y -intercept of each graph.

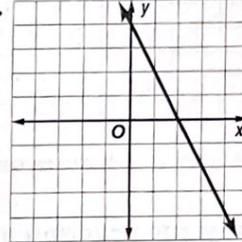
10.



11.

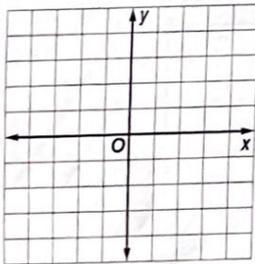


12.

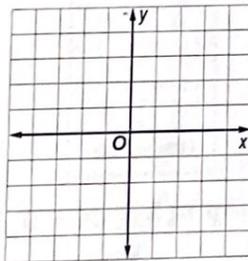


Graph each equation by making a table.

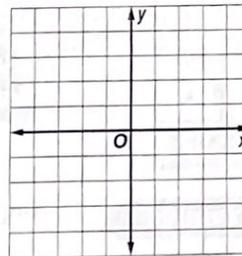
13. $y = 4$



14. $y = 3x$

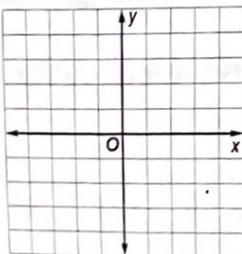


15. $y = x + 4$

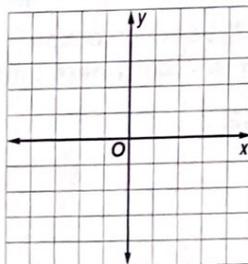


Graph each equation by using the x -intercept and y -intercept.

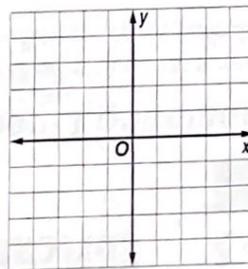
16. $x - y = 3$



17. $10x = -5y$



18. $4x = 2y + 6$



3-3 Enrichment

Translating Linear Graphs

Linear graphs can be **translated** on the coordinate plane. This means that the graph moves up, down, right, or left without changing its direction.

Translating the graphs up or down affects the y -coordinate for a given x value. Translating the graph right or left affects the x -coordinate for a given y value.

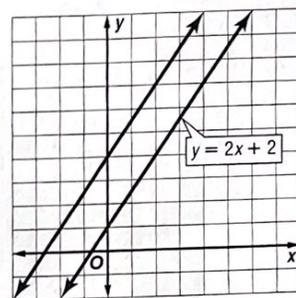
Example

Translate the graph of $y = 2x + 3$, 3 units up.

$y = 2x + 2$	
x	y
-1	0
0	2
1	4
2	6

Add 3 to each y value.

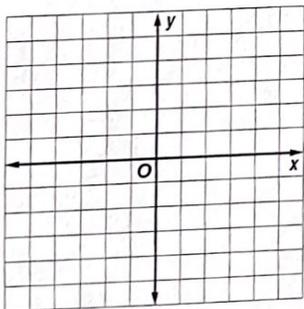
Translation	
x	y
-1	3
0	5
1	7
2	9



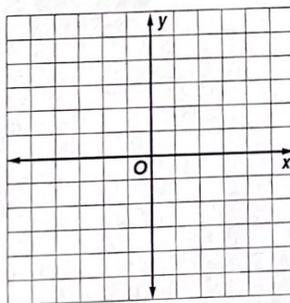
Exercises

Graph the function and the translation on the same coordinate plane.

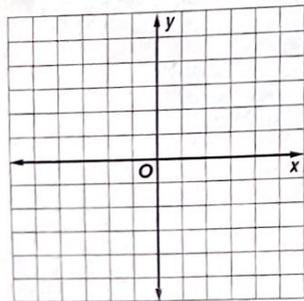
1. $y = x + 4$, 3 units down



2. $y = 2x - 2$, 2 units left



3. $y = -2x + 1$, 1 unit right



4. $y = -x - 3$, 2 units up

