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1-1 Study Guide and Intervention

Variables and Expressions

Write Mathematical Expressions In the **algebraic expression**, ℓw, the letters ℓ and w are called **variables**. In algebra, a variable is used to represent unspecified numbers or values. Any letter can be used as a variable. The letters ℓ and w are used above because they are the first letters of the words *length* and *width*. In the expression ℓw, ℓ and w are called factors, and the result is called the **product**.

Example 1 **Write an algebraic expression for each verbal expression.**

a. four more than a number n

The words *more than* imply addition.

four more than a number n

$4 + n$

The algebraic expression is $4 + n$.

b. the difference of a number squared and 8

The expression *difference of* implies subtraction.

the difference of a number squared and 8

$n^2 - 8$

The algebraic expression is $n^2 - 8$.

Example 2 **Evaluate each expression.**

a. 3^4

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$ Use 3 as a factor 4 times.

$= 81$ Multiply.

b. five cubed

Cubed means raised to the third power.

$5^3 = 5 \cdot 5 \cdot 5$ Use 5 as a factor 3 times.

$= 125$ Multiply.

Exercises

Write an algebraic expression for each verbal expression.

1. a number decreased by 8 $b - 8$
2. a number divided by 8 $\frac{h}{8}$
3. a number squared n^2
4. four times a number $4n$
5. a number divided by 6 $\frac{n}{6}$
6. a number multiplied by 37 $37n$
7. the sum of 9 and a number $9 + n$
8. 3 less than 5 times a number $5n - 3$
9. twice the sum of 15 and a number $2(15 + n)$
10. one-half the square of b $\frac{1}{2}b^2$
11. 7 more than the product of 6 and a number $6n + 7$
12. 30 increased by 3 times the square of a number $30 + 3n^2$

Evaluate each expression.

13. 5^2 25
14. 3^3 27
15. 10^4 10,000
16. 12^2 144
17. 8^3 512
18. 2^8 256

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1-1 Study Guide and Intervention *(continued)*

Variables and Expressions

Write Verbal Expressions Translating algebraic expressions into verbal expressions is important in algebra.

Example **Write a verbal expression for each algebraic expression.**

a. $6n^2$

the product of 6 and n squared

b. $n^3 - 12m$

the difference of n cubed and twelve times m

Exercises

Write a verbal expression for each algebraic expression. 1–18. Sample answers are given.

1. $w - 1$
   one less than w
2. $\frac{1}{3}a^3$
   one third the cube of a
3. $81 + 2x$
   eighty-one increased by twice x
4. $12c$
   12 times c
5. 8^4
   eight to the fourth power
6. 6^2
   the square of 6
7. $2n^2 + 4$ the sum of 4 and twice the square of n
8. $a^3 \cdot b^3$
   a cubed times b cubed
9. $2x^3 - 3$ the difference of twice a number cubed and 3
10. $\frac{6k^3}{5}$
    6 times the cube of k divided by 5
11. $\frac{1}{4}b^2$
    one-fourth the square of b
12. $7n^5$
    seven times the fifth power of n
13. $3x + 4$ the sum of three times a number and 4
14. $\frac{2}{3}k^5$
    two-thirds the fifth power of k
15. $3b^2 + 2a^3$ 3 times b squared plus 2 times a cubed
16. $4(n^2 + 1)$ 4 times the sum of the square of n and 1
17. $3^2 + 2^3$
    3 squared plus 2 cubed
18. $6n^2 + 3$
    the sum of 6 times n squared and 3

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1-2 Lesson Reading Guide

Order of Operations

Get Ready for the Lesson

Read the introduction to Lesson 1-2 in your textbook.

In the expression 4.95 + 0.99(117 − 100), 4.95 represents the regular monthly cost of internet service, 0.99 represents the cost of each additional hour after 100 hours, and (117 − 100) represents the number of hours over 100 used by Nicole in a given month.

Read the Lesson

1. The first step in evaluating an expression is to evaluate inside grouping symbols. List four types of grouping symbols found in algebraic expressions.
   parentheses, brackets, braces, and fraction bars

2. What does *evaluate powers* mean? Use an example to explain.
   Sample answer: To evaluate a power means to find the value of the power. To evaluate 4^3, find the value of $4 \times 4 \times 4$.

3. Read the order of operations on page 11 in your textbook. For each of the following expressions, write *addition*, *subtraction*, *multiplication*, *division*, or *evaluate powers* to tell what operation to use first when evaluating the expression.

   a. $400 - 5[12 + 9]$ addition

   b. $26 - 8 + 14$ subtraction

   c. $17 + 3 \cdot 6$ multiplication

   d. $69 + 57 \div 3 + 16 \cdot 4$ division

   e. $\frac{19 + 3 \cdot 4}{6 \div 2}$ multiplication

   f. $\frac{51 \div 729}{9^2}$ evaluate powers

Remember What You Learned

4. The sentence *Please Excuse My Dear Aunt Sally* (PEMDAS) is often used to remember the order of operations. The letter P represents parentheses and other grouping symbols. Write what each of the other letters in PEMDAS means when using the order of operations.
   E—exponents (powers), M—multiply, D—divide, A—add, S—subtract

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1-2 Study Guide and Intervention

Order of Operations

Evaluate Rational Expressions Numerical expressions often contain more than one operation. To evaluate them, use the rules for order of operations shown below.

Order of Operations	
	Step 1 Evaluate expressions inside grouping symbols.
	Step 2 Evaluate all powers.
	Step 3 Do all multiplication and/or division from left to right.
	Step 4 Do all addition and/or subtraction from left to right.

Example 1 **Evaluate each expression.**

a. $7 + 2 \cdot 4 - 4$

$7 + 2 \cdot 4 - 4 = 7 + 8 - 4$ Multiply 2 and 4.
$= 15 - 4$ Add 7 and 8.
$= 11$ Subtract 4 from 15.

b. $3(2) + 4(2 + 6)$

$3(2) + 4(2 + 6) = 3(2) + 4(8)$ Add 2 and 6.
$= 6 + 32$ Multiply left to right.
$= 38$ Add 6 and 32.

Example 2 **Evaluate each expression.**

a. $3[2 + (12 \div 3)^2]$

$3[2 + (12 \div 3)^2] = 3(2 + 4^2)$ Divide 12 by 3.
$= 3(2 + 16)$ Find 4 squared.
$= 3(18)$ Add 2 and 16.
$= 54$ Multiply 3 and 18.

b. $\frac{3 + 2^3}{4^2 \cdot 3}$

$\frac{3 + 2^3}{4^2 \cdot 3} = \frac{3 + 8}{4^2 \cdot 3}$ Evaluate power in numerator.
$= \frac{11}{4^2 \cdot 3}$ Add 3 and 8 in the numerator.
$= \frac{11}{16 \cdot 3}$ Evaluate power in denominator.
$= \frac{11}{48}$ Multiply.

Exercises

Evaluate each expression.

1. $(8 - 4) \cdot 2$ 8
2. $(12 + 4) \cdot 6$ 96
3. $10 + 2 \cdot 3$ 16
4. $10 + 8 \cdot 1$ 18
5. $15 - 12 \div 4$ 12
6. $\frac{15 + 60}{30 - 5}$ 3
7. $12(20 - 17) - 3 \cdot 6$ 18
8. $24 \div 3 \cdot 2 - 3^2$ 7
9. $8^2 \div (2 \cdot 8) + 2$ 6
10. $3^2 \div 3 + 2^2 \cdot 7 - 20 \div 5$ 27
11. $\frac{4 + 3^2}{12 + 1}$ 1
12. $\frac{8(2) - 4}{8 \div 4}$ 6
13. $250 \div [5(3 \cdot 7 + 4)]$ 2
14. $\frac{2 \cdot 4^2 - 8 \div 2}{(5 + 2) \cdot 2}$ 2
15. $\frac{4 \cdot 3^2 - 3 \cdot 2}{3 \cdot 5}$ 2
16. $\frac{4(5^2) - 4 \cdot 3}{4(4 \cdot 5 + 2)}$ 1
17. $\frac{5^2 - 3}{20(3) + 2(3)}$ $\frac{1}{3}$
18. $\frac{8^2 - 2^2}{(2 \cdot 8) + 4}$ 3

Answers

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1-3 Lesson Reading Guide

Open Sentences

Get Ready for the Lesson

Read the introduction to Lesson 1-3 in your textbook.

How is the open sentence different from the expression $15.50 + 5n$?

The open sentence has two expressions joined by the $\geq$ symbol.

Read the Lesson

1. How can you tell whether a mathematical sentence is or is not an open sentence?
   An open sentence must contain one or more variables.

2. How would you read each inequality symbol in words?

Inequality Symbol	Words
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to

3. Consider the equation $3n + 6 = 15$ and the inequality $3n + 6 \leq 15$. Suppose the replacement set is $\{0, 1, 2, 3, 4, 5\}$.

   a. Describe how you would find the solutions of the equation.
   Replace n with each member of the replacement set. The members of the replacement set that make the equation true are the solutions.

   b. Describe how you would find the solutions of the inequality.
   Replace n with each member of the replacement set. The members of the replacement set that make the inequality true are the solutions.

   c. Explain how the solution set for the equation is different from the solution set for the inequality.
   The solution set for the equation contains only one number, 3. The solution set for the inequality contains the four numbers 0, 1, 2, and 3.

Remember What You Learned

4. Look up the word *solution* in a dictionary. What is one meaning that relates to the way we use the word in algebra?
   Sample answer: answer to a problem

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1-3 Study Guide and Intervention

Open Sentences

Solve Equations A mathematical sentence with one or more variables is called an **open sentence**. Open sentences are **solved** by finding replacements for the variables that result in true sentences. The set of numbers from which replacements for a variable may be chosen is called the **replacement set**. The set of all replacements for the variable that result in true statements is called the **solution set** for the variable. A sentence that contains an equal sign, $=$, is called an **equation**.

Example 1 **Find the solution set of $3a + 12 = 39$ if the replacement set is $\{6, 7, 8, 9, 10\}$.**

Replace a in $3a + 12 = 39$ with each value in the replacement set.

$3(6) + 12 \stackrel{?}{=} 39 \rightarrow 30 \neq 39$ false
$3(7) + 12 \stackrel{?}{=} 39 \rightarrow 33 \neq 39$ false
$3(8) + 12 \stackrel{?}{=} 39 \rightarrow 36 \neq 39$ false
$3(9) + 12 \stackrel{?}{=} 39 \rightarrow 39 = 39$ true
$3(10) + 12 \stackrel{?}{=} 39 \rightarrow 42 \neq 39$ false

Since $a = 9$ makes the equation $3a + 12 = 39$ true, the solution is 9.

The solution set is $\{9\}$.

Example 2 **Solve $\frac{2(3+1)}{3(7-4)} = b$.**

$\frac{2(3+1)}{3(7-4)} = b$ Original equation

$\frac{2(4)}{3(3)} = b$ Add in the numerator; subtract in the denominator.

$\frac{8}{9} = b$ Simplify.

The solution is $\frac{8}{9}$.

Exercises

Find the solution of each equation if the replacement sets are $X = \left\{\frac{1}{4}, \frac{1}{2}, 1, 2, 3\right\}$ and $Y = \{2, 4, 6, 8\}$.

1. $x + \frac{1}{2} = \frac{5}{2}$ $\{2\}$
2. $x + 8 = 11$ $\{3\}$
3. $y - 2 = 6$ $\{8\}$
4. $x^2 - 1 = 8$ $\{3\}$
5. $y^2 - 2 = 34$ $\{6\}$
6. $x^2 + 5 = 5\frac{1}{16}$ $\left\{\frac{1}{4}\right\}$
7. $2(x + 3) = 7$ $\left\{\frac{1}{2}\right\}$
8. $\frac{1}{4}(y + 1)^2 = \frac{9}{4}$ $\{2\}$
9. $y^2 + y = 20$ $\{4\}$

Solve each equation.

10. $a = 2^3 - 1$ 7
11. $n = 6^2 - 4^2$ 20
12. $w = 6^2 \cdot 3^2$ 324
13. $\frac{1}{4} + \frac{5}{8} = k$ $\frac{7}{8}$
14. $\frac{18-3}{2+3} = p$ 3
15. $s = \frac{15-6}{27-24}$ 3
16. $18.4 - 3.2 = m$ 15.2
17. $k = 9.8 + 5.7$ 15.5
18. $c = 3\frac{1}{2} + 2\frac{1}{4}$ $5\frac{3}{4}$

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1-4 Lesson Reading Guide

Identity and Equality Properties

Get Ready for the Lesson

Read the introduction to Lesson 1-4 in your textbook.

Write an open sentence to represent the change in rank r of Auburn from week 6 to week 7. Explain why the solution is the same as the solution in the introduction.

$2 + r = 2$; Sample answer: The rank did not change for either team from week 6 to week 7.

Read the Lesson

1. Write the Roman numeral of the sentence that best matches each term.

a. additive identity V

b. multiplicative identity III

c. Multiplicative Property of Zero VIII

d. Multiplicative Inverse Property I

e. Reflexive Property II

f. Symmetric Property IV

g. Transitive Property VI

h. Substitution Property VII

I. $\frac{5}{7} \cdot \frac{7}{5} = 1$

II. $18 = 18$

III. $3 \cdot 1 = 3$

IV. If $12 = 8 + 4$, then $8 + 4 = 12$.

V. $6 + 0 = 6$

VI. If $2 + 4 = 5 + 1$ and $5 + 1 = 6$, then $2 + 4 = 6$.

VII. If $n = 2$, then $5n = 5 \cdot 2$.

VIII. $4 \cdot 0 = 0$

Remember What You Learned

2. The prefix *trans-* means "across" or "through." Explain how this can help you remember the meaning of the Transitive Property of Equality.

Sample answer: The Transitive Property of Equality tells you that when $a = b$ and $b = c$, you can go from a through b to get to c.

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1-4 Study Guide and Intervention

Identity and Equality Properties

Identity and Equality Properties The identity and equality properties in the chart below can help you solve algebraic equations and evaluate mathematical expressions.

Additive Identity	For any number a, $a + 0 = a$.
Multiplicative Identity	For any number a, $a \cdot 1 = a$.
Multiplicative Property of 0	For any number a, $a \cdot 0 = 0$.
Multiplicative Inverse Property	For every number $\frac{a}{b}$, $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \cdot \frac{b}{a} = 1$.
Reflexive Property	For any number a, $a = a$.
Symmetric Property	For any numbers a and b, if $a = b$, then $b = a$.
Transitive Property	For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
Substitution Property	If $a = b$, then a may be replaced by b in any expression.

Example 1 **Name the property used in each equation. Then find the value of n.**

a. $8n = 8$
Multiplicative Identity Property
$n = 1$, since $8 \cdot 1 = 8$

b. $n \cdot 3 = 1$
Multiplicative Inverse Property
$n = \frac{1}{3}$, since $\frac{1}{3} \cdot 3 = 1$

Example 2 **Name the property used to justify each statement.**

a. $5 + 4 = 5 + 4$
Reflexive Property

b. **If $n = 12$, then $4n = 4 \cdot 12$.**
Substitution Property

Exercises

Name the property used in each equation. Then find the value of n.

1. $6n = 6$
Mult. Identity; 1

2. $n \cdot 1 = 8$
Mult. Identity; 8

3. $6 \cdot n = 6 \cdot 9$
Substitution Property; 9

4. $9 = n + 9$
Add. Identity; 0

5. $n + 0 = \frac{3}{8}$
Add. Identity; $\frac{3}{8}$

6. $\frac{3}{4} \cdot n = 1$
Mult. Inverse; $\frac{4}{3}$

Name the property used in each equation.

7. If $4 + 5 = 9$, then $9 = 4 + 5$.
Symmetric Property

8. $0 + 21 = 21$
Add. Identity

9. $0(15) = 0$ Mult. Prop. of Zero

10. $(1)94 = 94$ Mult. Identity

11. If $3 + 3 = 6$ and $6 = 3 \cdot 2$, then $3 + 3 = 3 \cdot 2$. Transitive Property

12. $4 + 3 = 4 + 3$
Reflexive Property

13. $(14 - 6) + 3 = 8 + 3$
Substitution Property

Answers

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1-5 Study Guide and Intervention

The Distributive Property

Evaluate Expressions The Distributive Property can be used to help evaluate expressions.

Distributive Property	For any numbers a, b, and c, $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ and $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$.

Example 1 Rewrite $6(8 + 10)$ using the Distributive Property. Then evaluate.

$6(8 + 10) = 6 \cdot 8 + 6 \cdot 10$ Distributive Property
$= 48 + 60$ Multiply.
$= 108$ Add.

Example 2 Rewrite $-2(3x^2 + 5x + 1)$ using the Distributive Property. Then simplify.

$-2(3x^2 + 5x + 1) = -2(3x^2) + (-2)(5x) + (-2)(1)$ Distributive Property
$= -6x^2 + (-10x) + (-2)$ Multiply.
$= -6x^2 - 10x - 2$ Simplify.

Exercises

Rewrite each expression using the Distributive Property. Then simplify.

1. $2(10 - 5)$ **10**
2. $6(12 - t)$ **$72 - 6t$**
3. $3(x - 1)$ **$3x - 3$**
4. $6(12 + 5)$ **102**
5. $(x - 4)3$ **$3x - 12$**
6. $-2(x + 3)$ **$-2x - 6$**
7. $5(4x - 9)$ **$20x - 45$**
8. $3(8 - 2x)$ **$24 - 6x$**
9. $12\left(6 - \frac{1}{2}x\right)$ **$72 - 6x$**
10. $12\left(2 + \frac{1}{2}x\right)$ **$24 + 6x$**
11. $\frac{1}{4}(12 - 4t)$ **$3 - t$**
12. $3(2x - y)$ **$6x - 3y$**
13. $2(3x + 2y - z)$ **$6x + 4y - 2z$**
14. $(x - 2)y$ **$xy - 2y$**
15. $2(3a - 2b + c)$ **$6a - 4b + 2c$**
16. $\frac{1}{4}(16x - 12y + 4z)$ **$4x - 3y + z$**
17. $(2 - 3x + x^2)3$ **$6 - 9x + 3x^2$**
18. $-2(2x^2 + 3x + 1)$ **$-4x^2 - 6x - 2$**

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1-5 Study Guide and Intervention *(continued)*

The Distributive Property

Simplify Expressions A **term** is a number, a variable, or a product or quotient of numbers and variables. **Like terms** are terms that contain the same variables, with corresponding variables having the same powers. The Distributive Property and properties of equalities can be used to simplify expressions. An expression is in **simplest form** if it is replaced by an **equivalent** expression with no like terms or parentheses.

Example Simplify $4(a^2 + 3ab) - ab$.

$4(a^2 + 3ab) - ab = 4(a^2 + 3ab) - 1ab$ Multiplicative Identity
$= 4a^2 + 12ab - 1ab$ Distributive Property
$= 4a^2 + (12 - 1)ab$ Distributive Property
$= 4a^2 + 11ab$ Substitution

Exercises

Simplify each expression. If not possible, write *simplified*.

1. $12a - a$ **11a**
2. $3x + 6x$ **$9x$**
3. $3x - 1$ **simplified**
4. $12g - 10g + 1$ **$2g + 1$**
5. $-2x - 12$ **simplified**
6. $4x^2 + 3x + 7$ **simplified**
7. $20a + 12a - 8$ **$32a - 8$**
8. $3x^2 + 2x^2$ **$5x^2$**
9. $-6x + 3x^2 + 10x^2$ **$-6x + 13x^2$**
10. $2p + \frac{1}{2}q$ **simplified**
11. $10xy - 4(xy + xy)$ **$2xy$**
12. $21c + 18c + 31b - 3b$ **$39c + 28b$**
13. $3x - 2x - 2y + 2y$ **x**
14. $xy - 2xy$ **$-xy$**
15. $12a - 12b + 12c$ **simplified**
16. $4x + \frac{1}{4}(16x - 20y)$ **$8x - 5y$**
17. $2 - 1 - 6x + x^2$ **$1 - 6x + x^2$**
18. $4x^2 + 3x^2 + 2x$ **$7x^2 + 2x$**

Answers

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1-6 Lesson Reading Guide

Commutative and Associative Properties

Get Ready for the Lesson

Read the introduction to Lesson 1-6 in your textbook.

How are the expressions 0.4 + 1.5 and 1.5 + 0.4 alike? different?

The numbers and the operation are the same; the order of the numbers is different.

Read the Lesson

1. Write the Roman numeral of the term that best matches each equation.

a. $3 + 6 = 6 + 3$ **III**	I.	Associative Property of Addition
b. $2 + (3 + 4) = (2 + 3) + 4$ **I**	II.	Associative Property of Multiplication
c. $2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4$ **II**	III.	Commutative Property of Addition
d. $2 \cdot (3 \cdot 4) = 2 \cdot (4 \cdot 3)$ **IV**	IV.	Commutative Property of Multiplication

2. What property can you use to change the order of the terms in an expression?
Commutative Property of Addition

3. What property can you use to change the way three factors are grouped?
Associative Property of Multiplication

4. What property can you use to combine two like terms to get a single term?
Distributive Property

5. To use the Associative Property of Addition to rewrite the sum of a group of terms, what is the least number of terms you need? **three**

Remember What You Learned

6. Look up the word *commute* in a dictionary. Find an everyday meaning that is close to the mathematical meaning and explain how it can help you remember the mathematical meaning.
Sample answer: To travel back and forth, as between a suburb and a city; in the Commutative Property of Addition, $a + b = b + a$, the quantities a and b are switched back and forth.

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1-6 Study Guide and Intervention

Commutative and Associative Properties

Commutative and Associative Properties The Commutative and Associative Properties can be used to simplify expressions. The Commutative Properties state that the order in which you add or multiply numbers does not change their sum or product. The Associative Properties state that the way you group three or more numbers when adding or multiplying does not change their sum or product.

Commutative Properties	For any numbers a and b, $a + b = b + a$ and $a \cdot b = b \cdot a$.
Associative Properties	For any numbers a, b, and c, $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.

Example 1 **Evaluate $6 \cdot 2 \cdot 3 \cdot 5$.**

$6 \cdot 2 \cdot 3 \cdot 5 = 6 \cdot 3 \cdot 2 \cdot 5$ Commutative Property
$= (6 \cdot 3)(2 \cdot 5)$ Associative Property
$= 18 \cdot 10$ Multiply.
$= 180$ Multiply.

The product is 180.

Example 2 **Evaluate $8.2 + 2.5 + 2.5 + 1.8$.**

$8.2 + 2.5 + 2.5 + 1.8$
$= 8.2 + 1.8 + 2.5 + 2.5$ Commutative Prop.
$= (8.2 + 1.8) + (2.5 + 2.5)$ Associative Prop.
$= 10 + 5$ Add.
$= 15$ Add.

The sum is 15.

Exercises

Evaluate each expression.

1. $12 + 10 + 8 + 5$ **35**
2. $16 + 8 + 22 + 12$ **58**
3. $10 \cdot 7 \cdot 2.5$ **175**
4. $4 \cdot 8 \cdot 5 \cdot 3$ **480**
5. $12 + 20 + 10 + 5$ **47**
6. $26 + 8 + 4 + 22$ **60**
7. $3\frac{1}{2} + 4 + 2\frac{1}{2} + 3$ **13**
8. $\frac{3}{4} \cdot 12 \cdot 4 \cdot 2$ **72**
9. $3.5 + 2.4 + 3.6 + 4.2$ **13.7**
10. $4\frac{1}{2} + 5 + \frac{1}{2} + 3$ **13**
11. $0.5 \cdot 2.8 \cdot 4$ **5.6**
12. $2.5 + 2.4 + 2.5 + 3.6$ **11**
13. $\frac{4}{5} \cdot 18 \cdot 25 \cdot \frac{2}{9}$ **80**
14. $32 \cdot \frac{1}{5} \cdot \frac{1}{2} \cdot 10$ **32**
15. $\frac{1}{4} \cdot 7 \cdot 16 \cdot \frac{1}{7}$ **4**
16. $3.5 + 8 + 2.5 + 2$ **16**
17. $18 \cdot 8 \cdot \frac{1}{2} \cdot \frac{1}{9}$ **8**
18. $\frac{3}{4} \cdot 10 \cdot 16 \cdot \frac{1}{2}$ **60**

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1-8 Lesson Reading Guide

Number Systems

Get Ready for the Lesson

Read the introduction to Lesson 1-8 in your textbook.

The expression $\sqrt{3600}$ is read, "the square root of 3600." How would you read the expression $\sqrt{64}$?

the square root of 64

Read the Lesson

Complete each statement below.

1. The symbol $\sqrt{\ }$ is called a ___radical sign___ and is used to indicate a nonnegative or principal square root of the expression under the symbol.
2. A ___rational approximation___ of an irrational number is a rational number that is close to, but not equal to, the value of the irrational number.
3. The positive square root of a number is called the ___principal___ square root of the number.
4. A number whose positive square root is a rational number is a ___perfect square___.
5. Write each of the following as a mathematical expression that uses the $\sqrt{\ }$ symbol.
   a. the positive square root of 1600 $\sqrt{1600}$
   b. the negative square root of 729 $-\sqrt{729}$
   c. the principal square root of 3025 $\sqrt{3025}$
6. The irrational numbers and rational numbers together form the set of ___real___ numbers.

Remember What You Learned

7. Use a dictionary to look up several words that begin with "ir-". What does the prefix "ir-" mean? How can this help you remember the meaning of the word *irrational*?

   Sample answer: The prefix "ir-" means *not*. So an irrational number is a number that is not a rational number.

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1-8 Study Guide and Intervention

Number Systems

Square Roots A **square root** is one of two equal factors of a number. For example, the square roots of 36 are 6 and -6, since $6 \cdot 6$ or 6^2 is 36 and $(-6)(-6)$ or $(-6)^2$ is also 36. A rational number like 36, whose square root is a rational number, is called a **perfect square**.

The symbol $\sqrt{\ }$ is a **radical sign**. It indicates the nonnegative, or **principal**, square root of the number under the radical sign. So $\sqrt{36} = 6$ and $-\sqrt{36} = -6$. The symbol $\pm\sqrt{36}$ represents both square roots.

Example 1 Find $-\sqrt{\frac{25}{49}}$.

$-\sqrt{\frac{25}{49}}$ represents the negative square root of $\frac{25}{49}$.

$\frac{25}{49} = \left(\frac{5}{7}\right)^2 \rightarrow -\sqrt{\frac{25}{49}} = -\frac{5}{7}$

Example 2 Find $\pm\sqrt{0.16}$.

$\pm\sqrt{0.16}$ represents the positive and negative square roots of 0.16.

$0.16 = 0.4^2$ and $0.16 = (-0.4)^2$

$\pm\sqrt{0.16} = \pm 0.4$

Exercises

Find each square root.

1. $\sqrt{64}$ 8
2. $-\sqrt{81}$ -9
3. $\sqrt{16.81}$ 4.1
4. $\pm\sqrt{100}$ ± 10
5. $-\sqrt{\frac{4}{25}}$ $-\frac{2}{5}$
6. $-\sqrt{121}$ -11
7. $\pm\sqrt{\frac{25}{144}}$ $\pm\frac{5}{12}$
8. $-\sqrt{\frac{25}{16}}$ $-\frac{5}{4}$
9. $\pm\sqrt{\frac{121}{100}}$ $\pm\frac{11}{10}$
10. $-\sqrt{3600}$ -60
11. $-\sqrt{6.25}$ -2.5
12. $\pm\sqrt{0.0004}$ ± 0.02
13. $\sqrt{\frac{144}{196}}$ $\frac{6}{7}$
14. $-\sqrt{\frac{36}{49}}$ $-\frac{6}{7}$
15. $\pm\sqrt{1.21}$ ± 1.1

Answers

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2-1 Study Guide and Intervention

Writing Equations

Write Equations Writing equations is one strategy for solving problems. You can use a variable to represent an unspecified number or measure referred to in a problem. Then you can write a verbal expression as an algebraic expression.

Example 1 **Translate each sentence into an equation or a formula.**

a. Ten times a number x is equal to 2.8 times the difference y minus z.

$10 \times x = 2.8 \times (y - z)$

The equation is $10x = 2.8(y - z)$.

b. A number m minus 8 is the same as a number n divided by 2.

$m - 8 = n \div 2$

The equation is $m - 8 = \frac{n}{2}$.

c. The area of a rectangle equals the length times the width. Translate this sentence into a formula.

Let A = area, ℓ = length, and w = width.

Formula: *Area equals length times width.*

$A = \ell \times w$

The formula for the area of a rectangle is $A = \ell w$.

Example 2 **Use the Four-Step Problem-Solving Plan.**

The population of the United States in 2005 was about 297,000,000, and the land area of the United States is about 3,500,000 square miles. Find the average number of people per square mile in the United States.

Source: www.census.gov

Step 1 *Explore* You know that there are 297,000,000 people. You want to know the number of people per square mile.

Step 2 *Plan* Write an equation to represent the situation. Let p represent the number of people per square mile.

$3{,}500{,}000 \times p = 297{,}000{,}000$

Step 3 *Solve* $3{,}500{,}000 \times p = 297{,}000{,}000$.

$3{,}500{,}000p = 297{,}000{,}000$ Divide each side by 3,500,000.

$p \approx 84.86$

There about 85 people per square mile.

Step 4 *Check* If there are 85 people per square mile and there are 3,500,000 square miles, $85 \times 3{,}500{,}000 = 297{,}500{,}000$, or about 297,000,000 people. The answer makes sense.

Exercises

Translate each sentence into an equation or formula.

1. Three times a number t minus twelve equals forty. $3t - 12 = 40$
2. One-half of the difference of a and b is 54. $\frac{1}{2}(a - b) = 54$
3. Three times the sum of d and 4 is 32. $3(d + 4) = 32$
4. The area A of a circle is the product of π and the radius r squared. $A = \pi r^2$

WEIGHT LOSS For Exercises 5–6, use the following information.

Lou wants to lose weight to audition for a part in a play. He weighs 160 pounds now. He wants to weigh 150 pounds.

5. If p represents the number of pounds he wants to lose, write an equation to represent this situation. $160 - p = 150$
6. How many pounds does he need to lose to reach his goal? 10 lb

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2-1 Study Guide and Intervention *(continued)*

Writing Equations

Write Verbal Sentences You can translate equations into verbal sentences.

Example **Translate each equation into a verbal sentence.**

a. $4n - 8 = 12$.

$4n \quad - \quad 8 \quad = \quad 12$

Four times n minus eight equals twelve.

b. $a^2 + b^2 = c^2$

$a^2 + b^2 \quad = \quad c^2$

The sum of the squares of a and b is equal to the square of c.

Exercises

Translate each equation into a verbal sentence.

1. $4a - 5 = 23$

   4 times a minus 5 is equal to 23.

2. $10 + k = 4k$

   The sum of 10 and k is equal to 4 times k.

3. $6xy = 24$

   6 times the product of x and y is equal to 24.

4. $x^2 + y^2 = 8$

   The sum of the squares of x and y is equal to 8.

5. $p + 3 = 2p$

   The sum of p and 3 is equal to 2 times p.

6. $b = \frac{1}{3}(h - 1)$

   b is $\frac{1}{3}$ of the difference of h and 1.

7. $100 - 2x = 80$

   100 minus 2 times x is equal to 80.

8. $3(g + h) = 12$

   3 times the sum of g and h is 12.

9. $p^2 - 2p = 9$

   The square of p minus 2 times p is equal to 9.

10. $C = \frac{5}{9}(F - 32)$

    C is equal to $\frac{5}{9}$ of the difference of F and 32.

11. $V = \frac{1}{3}Bh$

    V is equal to $\frac{1}{3}$ of the product of B and h.

12. $A = \frac{1}{2}hb$

    A is equal to $\frac{1}{2}$ of the product of h and b.

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2-2 Lesson Reading Guide

Solving Equations by Using Addition and Subtraction

Get Ready for the Lesson

Read the introduction to Lesson 2-2 in your textbook.

In the equation $66 = m - 50$, the number 5 represents the difference between the percent of growth for medical assistants and the percent of growth for travel agents, and the number 66 represents the rate of growth for travel agents.

Read the Lesson

1. To solve $x + 17 = 46$ using the Subtraction Property of Equality, you would subtract 17 from each side.

2. To solve $y - 9 = -30$ using the Addition Property of Equality, you would add 9 to each side.

3. Write an equation that you could solve by subtracting 32 from each side.
   Sample answer: $m + 32 = 50$

4. A student used the Subtraction Property of Equality to solve an equation. Explain why it would also be possible to use the Addition Property of Equality to solve the equation.
   Subtracting one number from another gives the same result as adding the opposite of the number that was subtracted.

Remember What You Learned

5. Explain how you decide whether to use the Addition Property or the Subtraction Property of Equality to solve an equation.
   Sample answer: If the given equation has a number added to the variable, then use the Subtraction Property of Equality. If the equation has a number subtracted from the variable, then use the Addition Property of Equality.

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2-2 Study Guide and Intervention

Solving Equations by Using Addition and Subtraction

Solve Using Addition If the same number is added to each side of an equation, the resulting equation is equivalent to the original one. In general if the original equation involves subtraction, this property will help you solve the equation.

Addition Property of Equality	For any numbers a, b, and c, if $a = b$, then $a + c = b + c$.

Example 1 Solve $m - 32 = 18$.

$m - 32 = 18$	Original equation
$m - 32 + 32 = 18 + 32$	Add 32 to each side.
$m = 50$	Simplify.

The solution is 50.

Example 2 Solve $-18 = p - 12$.

$-18 = p - 12$	Original equation
$-18 + 12 = p - 12 + 12$	Add 12 to each side.
$p = -6$	Simplify.

The solution is -6.

Exercises

Solve each equation. Then check your solution.

1. $h - 3 = -2$ 1
2. $m - 8 = -12$ -4
3. $p - 5 = 15$ 20
4. $20 = y - 8$ 28
5. $k - 0.5 = 2.3$ 2.8
6. $w - \frac{1}{2} = \frac{5}{8}$ $1\frac{1}{8}$
7. $h - 18 = -17$ 1
8. $-12 = -24 + k$ 12
9. $j - 0.2 = 1.8$ 2
10. $b - 40 = -40$ 0
11. $m - (-12) = 10$ -2
12. $w - \frac{3}{2} = \frac{1}{4}$ $\frac{7}{4}$

Write an equation for each problem. Then solve the equation and check the solution.

13. Twelve subtracted from a number equals 25. Find the number. $n - 12 = 25$; 37
14. What number decreased by 52 equals -12? $n - 52 = -12$; 40
15. Fifty subtracted from a number equals eighty. Find the number. $n - 50 = 80$; 130
16. What number minus one-half is equal to negative one-half? $n - \frac{1}{2} = -\frac{1}{2}$; 0
17. The difference of a number and eight is equal to 14. What is the number?
    $n - 8 = 14$; 22
18. A number decreased by fourteen is equal to eighteen. What is the number?
    $n - 14 = 18$; 32

Answers

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2-3 Study Guide and Intervention

Solving Equations by using Multiplication and Division

Solve Using Multiplication If each side of an equation is multiplied by the same number, the resulting equation is equivalent to the given one. You can use the property to solve equations involving multiplication and division.

Multiplication Property of Equality	For any numbers a, b, and c, if $a = b$, then $ac = bc$.

Example 1 Solve $3\frac{1}{2}p = 1\frac{1}{2}$.

$3\frac{1}{2}p = 1\frac{1}{2}$	Original equation
$\frac{7}{2}p = \frac{3}{2}$	Rewrite each mixed number as an improper fraction.
$\frac{2}{7}\left(\frac{7}{2}p\right) = \frac{2}{7}\left(\frac{3}{2}\right)$	Multiply each side by $\frac{2}{7}$.
$p = \frac{3}{7}$	Simplify.

The solution is $\frac{3}{7}$.

Example 2 Solve $-\frac{1}{4}n = 16$.

$-\frac{1}{4}n = 16$	Original equation
$-4\left(-\frac{1}{4}n\right) = -4(16)$	Multiply each side by -4.
$n = -64$	Simplify.

The solution is -64.

Exercises

Solve each equation. Then check your solution.

1. $\frac{h}{3} = -2$ -6
2. $\frac{1}{8}m = 6$ 48
3. $\frac{1}{5}p = \frac{3}{5}$ 3
4. $5 = \frac{y}{12}$ 60
5. $-\frac{1}{4}k = -2.5$ 10
6. $-\frac{m}{8} = \frac{5}{8}$ -5
7. $-1\frac{1}{2}h = 4$ $-\frac{8}{3}$
8. $-12 = -\frac{3}{2}k$ 8
9. $\frac{j}{3} = \frac{2}{5}$ $1\frac{1}{5}$
10. $-3\frac{1}{3}b = 5$ $-1\frac{1}{2}$
11. $\frac{7}{10}m = 10$ $14\frac{2}{7}$
12. $\frac{p}{5} = -\frac{1}{4}$ $-1\frac{1}{4}$

Write an equation for each problem. Then solve the equation.

13. One-fifth of a number equals 25. Find the number. $\frac{1}{5}n = 25$; 125
14. What number divided by 2 equals -18? $\frac{n}{2} = -18$; -36
15. A number divided by eight equals 3. Find the number. $\frac{n}{8} = 3$; 24
16. One and a half times a number equals 6. Find the number. $1\frac{1}{2}n = 6$; 4

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2-3 Study Guide and Intervention *(continued)*

Solving Equations by Using Multiplication and Division

Solve Using Division To solve equations with multiplication and division, you can also use the Division Property of Equality. If each side of an equation is divided by the same number, the resulting equation is true.

Division Property of Equality	For any numbers a, b, and c, with $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

Example 1 Solve $8n = 64$.

$8n = 64$	Original equation
$\frac{8n}{8} = \frac{64}{8}$	Divide each side by 8.
$n = 8$	Simplify.

The solution is 8.

Example 2 Solve $-5n = 60$.

$-5n = 60$	Original equation
$\frac{-5n}{-5} = \frac{60}{-5}$	Divide each side by -5.
$n = -12$	Simplify.

The solution is -12.

Exercises

Solve each equation. Then check your solution.

1. $3h = -42$ -14
2. $8m = 16$ 2
3. $-3t = 51$ -17
4. $-3r = -24$ 8
5. $8k = -64$ -8
6. $-2m = 16$ -8
7. $12h = 4$ $\frac{1}{3}$
8. $-2.4p = 7.2$ -3
9. $0.5j = 5$ 10
10. $-25 = 5m$ -5
11. $6m = 15$ $2\frac{1}{2}$
12. $-1.5p = -75$ 50

Write an equation for each problem. Then solve the equation.

13. Four times a number equals 64. Find the number. $4n = 64$; 16
14. What number multiplied by -4 equals -16? $-4n = -16$; 4
15. A number times eight equals -36. Find the number. $8n = -36$; $-4\frac{1}{2}$

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2-4 Study Guide and Intervention *(continued)*

Solving Multi-Step Equations

Solve Multi-Step Equations To solve equations with more than one operation, often called **multi-step equations**, undo operations by working backward. Reverse the usual order of operations as you work.

Example **Solve $5x + 3 = 23$.**

$5x + 3 = 23$	Original equation.
$5x + 3 - 3 = 23 - 3$	Subtract 3 from each side.
$5x = 20$	Simplify.
$\frac{5x}{5} = \frac{20}{5}$	Divide each side by 5.
$x = 4$	Simplify.

Exercises

Solve each equation. Then check your solution.

1. $5x + 2 = 27$ **5**
2. $6x + 9 = 27$ **3**
3. $5x + 16 = 51$ **7**
4. $14n - 8 = 34$ **3**
5. $0.6x - 1.5 = 1.8$ **5.5**
6. $\frac{7}{8}p - 4 = 10$ **16**
7. $16 = \frac{d - 12}{14}$ **236**
8. $8 + \frac{3n}{12} = 13$ **20**
9. $\frac{g}{-5} + 3 = -13$ **80**
10. $\frac{4b + 8}{-2} = 10$ **−7**
11. $0.2x - 8 = -2$ **30**
12. $3.2y - 1.8 = 3$ **1.5**
13. $-4 = \frac{7x - (-1)}{-8}$ **$4\frac{3}{7}$**
14. $8 = -12 + \frac{k}{-4}$ **−80**
15. $0 = 10y - 40$ **4**

Write an equation and solve each problem.

16. Find three consecutive integers whose sum is 96.
$n + (n + 1) + (n + 2) = 96$; 31, 32, 33

17. Find two consecutive odd integers whose sum is 176.
$n + (n + 2) = 176$; 87, 89

18. Find three consecutive integers whose sum is −93.
$n + (n + 1) + (n + 2) = -93$; −32, −31, −30

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2-4 Skills Practice

Solving Multi-Step Equations

Solve each problem by working backward.

1. A number is divided by 2, and then the quotient is added to 8. The result is 33. Find the number. **50**

2. Two is subtracted from a number, and then the difference is divided by 3. The result is 30. Find the number. **92**

3. A number is multiplied by 2, and then the product is added to 9. The result is 49. What is the number? **20**

4. **ALLOWANCE** After Ricardo received his allowance for the week, he went to the mall with some friends. He spent half of his allowance on a new paperback book. Then he bought himself a snack for $1.25. When he arrived home, he had $5.00 left. How much was his allowance? **$12.50**

Solve each equation. Then check your solution.

5. $5x + 3 = 23$ **4**
6. $4 = 3a - 14$ **6**
7. $2y + 5 = 19$ **7**
8. $6 + 5c = -29$ **−7**
9. $8 - 5w = -37$ **9**
10. $18 - 4v = 42$ **−6**
11. $\frac{n}{3} - 8 = -2$ **18**
12. $5 + \frac{x}{4} = 1$ **−16**
13. $-\frac{h}{3} - 4 = 13$ **−51**
14. $-\frac{d}{6} + 12 = -7$ **114**
15. $\frac{a}{5} - 2 = 9$ **55**
16. $\frac{w}{7} + 3 = -1$ **−28**
17. $\frac{3}{4}q - 7 = 8$ **20**
18. $\frac{2}{3}g + 6 = -12$ **−27**
19. $\frac{5}{2}z - 8 = -3$ **2**
20. $\frac{4}{5}m + 2 = 6$ **5**
21. $\frac{c - 5}{4} = 3$ **17**
22. $\frac{b + 1}{3} = 2$ **5**

Write an equation and solve each problem.

23. Twice a number plus four equals 6. What is the number? **$2n + 4 = 6$; 1**

24. Sixteen is seven plus three times a number. Find the number. **$16 = 7 + 3n$; 3**

25. Find two consecutive integers whose sum is 35. **$n + (n + 1) = 35$; 17, 18**

26. Find three consecutive integers whose sum is 36. **$n + (n + 1) + (n + 2) = 36$; 11, 12, 13**

Answers

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2-5 Study Guide and Intervention

Solving Equations with the Variable on Each Side

Variables on Each Side To solve an equation with the same variable on each side, first use the Addition or the Subtraction Property of Equality to write an equivalent equation that has the variable on just one side of the equation. Then solve the equation.

Example 1 Solve $5y - 8 = 3y + 12$.

$$\begin{aligned} 5y - 8 &= 3y + 12 \\ 5y - 8 - 3y &= 3y + 12 - 3y \\ 2y - 8 &= 12 \\ 2y - 8 + 8 &= 12 + 8 \\ 2y &= 20 \\ \frac{2y}{2} &= \frac{20}{2} \\ y &= 10 \end{aligned}$$

The solution is 10.

Example 2 Solve $-11 - 3y = 8y + 1$.

$$\begin{aligned} -11 - 3y &= 8y + 1 \\ -11 - 3y + 3y &= 8y + 1 + 3y \\ -11 &= 11y + 1 \\ -11 - 1 &= 11y + 1 - 1 \\ -12 &= 11y \\ \frac{-12}{11} &= \frac{11y}{11} \\ -1\frac{1}{11} &= y \end{aligned}$$

The solution is $-1\frac{1}{11}$.

Exercises

Solve each equation. Then check your solution.

1. $6 - b = 5b + 30$ — -4
2. $5y - 2y = 3y + 2$ — no solution
3. $5x + 2 = 2x - 10$ — -4
4. $4n - 8 = 3n + 2$ — 10
5. $1.2x + 4.3 = 2.1 - x$ — -1
6. $4.4s + 6.2 = 8.8s - 1.8$ — $\frac{20}{11}$
7. $\frac{1}{2}b + 4 = \frac{1}{8}b + 88$ — 224
8. $\frac{3}{4}k - 5 = \frac{1}{4}k - 1$ — 8
9. $8 - 5p = 4p - 1$ — 1
10. $4b - 8 = 10 - 2b$ — 3
11. $0.2x - 8 = -2 - x$ — 5
12. $3y - 1.8 = 3y - 1.8$ — all numbers
13. $-4 - 3x = 7x - 6$ — $\frac{1}{5}$
14. $8 + 4k = -10 + k$ — -6
15. $20 - a = 10a - 2$ — 2
16. $\frac{2}{3}n + 8 = \frac{1}{2}n + 2$ — -36
17. $\frac{2}{5}y - 8 = 9 - \frac{3}{5}y$ — 17
18. $-4r + 5 = 5 - 4r$ — all numbers
19. $-4 - 3x = 6x - 6$ — $\frac{2}{9}$
20. $18 - 4k = -10 - 4k$ — no solution
21. $12 + 2y = 10y - 12$ — 3

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2-5 Study Guide and Intervention *(continued)*

Solving Equations with the Variable on Each Side

Grouping Symbols When solving equations that contain grouping symbols, first use the Distributive Property to eliminate grouping symbols. Then solve.

Example Solve $4(2a - 1) = -10(a - 5)$.

$4(2a - 1) = -10(a - 5)$	Original equation
$8a - 4 = -10a + 50$	Distributive Property
$8a - 4 + 10a = -10a + 50 + 10a$	Add $10a$ to each side.
$18a - 4 = 50$	Simplify.
$18a - 4 + 4 = 50 + 4$	Add 4 to each side.
$18a = 54$	Simplify.
$\frac{18a}{18} = \frac{54}{18}$	Divide each side by 18.
$a = 3$	Simplify.

The solution is 3.

Exercises

Solve each equation. Then check your solution.

1. $-3(x + 5) = 3(x - 1)$ — -2
2. $2(7 + 3t) = -t$ — -2
3. $3(a + 1) - 5 = 3a - 2$ — all numbers
4. $75 - 9g = 5(-4 + 2g)$ — 5
5. $5(f + 2) = 2(3 - f)$ — $-\frac{4}{7}$
6. $4(p + 3) = 36$ — 6
7. $18 = 3(2c + 2)$ — 2
8. $3(d - 8) = 3d$ — no solution
9. $5(p + 3) + 9 = 3(p - 2) + 6$ — -12
10. $4(b - 2) = 2(5 - b)$ — 3
11. $1.2(x - 2) = 2 - x$ — 2
12. $\frac{3 + y}{4} = \frac{-y}{8}$ — -2
13. $\frac{a - 8}{12} = \frac{2a + 5}{3}$ — -4
14. $2(4 + 2k) + 10 = k$ — -6
15. $2(w - 1) + 4 = 4(w + 1)$ — -1
16. $6(n - 1) = 2(2n + 4)$ — 7
17. $2[2 + 3(y - 1)] = 22$ — 4
18. $-4(r + 2) = 4(2 - 4r)$ — $1\frac{1}{3}$
19. $-3(x - 8) = 24$ — 0
20. $4(4 - 4k) = -10 - 16k$ — no solution
21. $6(2 - 2y) = 5(2y - 2)$ — 1

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2-6 Study Guide and Intervention

Ratios and Proportions

Ratios and Proportions A **ratio** is a comparison of two numbers by division. The ratio of x to y can be expressed as x to y, $x:y$ or $\frac{x}{y}$. Ratios are usually expressed in simplest form.

An equation stating that two ratios are equal is called a **proportion**. To determine whether two ratios form a proportion, express both ratios in simplest form or check cross products.

Example 1 **Determine whether the ratios $\frac{24}{36}$ and $\frac{12}{18}$ form a proportion.**

$\frac{24}{36} = \frac{2}{3}$ when expressed in simplest form.

$\frac{12}{18} = \frac{2}{3}$ when expressed in simplest form.

The ratios $\frac{24}{36}$ and $\frac{12}{18}$ form a proportion because they are equal when expressed in simplest form.

Example 2 **Use cross products to determine whether $\frac{10}{18}$ and $\frac{25}{45}$ form a proportion.**

$\frac{10}{18} \stackrel{?}{=} \frac{25}{45}$ Write the proportion.

$10(45) \stackrel{?}{=} 18(25)$ Cross products

$450 = 450$ Simplify.

The cross products are equal, so $\frac{10}{18} = \frac{25}{45}$. Since the ratios are equal, they form a proportion.

Exercises

Use cross products to determine whether each pair of ratios forms a proportion.

1. $\frac{1}{2}, \frac{16}{32}$ yes
2. $\frac{5}{8}, \frac{10}{15}$ no
3. $\frac{10}{20}, \frac{25}{49}$ no
4. $\frac{25}{36}, \frac{15}{20}$ no
5. $\frac{12}{32}, \frac{3}{16}$ no
6. $\frac{4}{9}, \frac{12}{27}$ yes
7. $\frac{0.1}{2}, \frac{5}{100}$ yes
8. $\frac{15}{20}, \frac{9}{12}$ yes
9. $\frac{14}{21}, \frac{20}{30}$ yes
10. 2:3, 20:30 yes
11. 5 to 9, 25 to 45 yes
12. $\frac{72}{64}, \frac{9}{8}$ yes
13. 5:5, 30:20 no
14. 18 to 24, 50 to 75 no
15. 100:75, 44:33 yes
16. $\frac{0.05}{1}, \frac{1}{20}$ yes
17. $\frac{1.5}{2}, \frac{6}{8}$ yes
18. $\frac{0.1}{0.2}, \frac{0.45}{0.9}$ yes

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2-6 Study Guide and Intervention

Ratios and Proportions

Solve Proportions If a proportion involves a variable, you can use cross products to solve the proportion. In the proportion $\frac{x}{5} = \frac{10}{13}$, x and 13 are called **extremes**. They are the first and last terms of the proportion. 5 and 10 are called **means**. They are the middle terms of the proportion. In a proportion, the product of the extremes is equal to the product of the means.

Means-Extremes Property of Proportions	For any numbers a, b, c, and d, if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Example **Solve $\frac{x}{5} = \frac{10}{13}$.**

$\frac{x}{5} = \frac{10}{13}$ Original proportion

$13(x) = 5(10)$ Cross products

$13x = 50$ Simplify.

$\frac{13x}{13} = \frac{50}{13}$ Divide each side by 13.

$x = 3\frac{11}{13}$ Simplify.

Exercises

Solve each proportion.

1. $\frac{-3}{x} = \frac{2}{8}$ −12
2. $\frac{1}{t} = \frac{5}{3}$ $\frac{3}{5}$
3. $\frac{0.1}{2} = \frac{0.5}{x}$ 10
4. $\frac{x+1}{4} = \frac{3}{4}$ 2
5. $\frac{4}{6} = \frac{8}{x}$ 12
6. $\frac{x}{21} = \frac{3}{63}$ 1
7. $\frac{9}{y+1} = \frac{18}{54}$ 26
8. $\frac{3}{d} = \frac{18}{3}$ $\frac{1}{2}$
9. $\frac{5}{8} = \frac{p}{24}$ 15
10. $\frac{4}{b-2} = \frac{4}{12}$ 14
11. $\frac{1.5}{x} = \frac{12}{x}$ no solution
12. $\frac{3+y}{4} = \frac{-y}{8}$ −2
13. $\frac{a-8}{12} = \frac{15}{3}$ 68
14. $\frac{12}{k} = \frac{24}{k}$ no solution
15. $\frac{2+w}{6} = \frac{12}{9}$ 6

Use a proportion to solve each problem.

16. **MODELS** To make a model of the Guadeloupe River bed, Hermie used 1 inch of clay for 5 miles of the river's actual length. His model river was 50 inches long. How long is the Guadeloupe River? 250 mi

17. **EDUCATION** Josh finished 24 math problems in one hour. At that rate, how many hours will it take him to complete 72 problems? 3 h

NAME _____ DATE _____ PERIOD ____

2-7 Lesson Reading Guide

Percent of Change

Get Ready for the Lesson

Read the introduction to Lesson 2-7 in your textbook.

- How many area codes were in use in 1947? 84 area codes
- How many *more* area codes were in use in 1999? 285 area codes

Read the Lesson

1. If you use (original amount) — (new amount) to find the change for a percent of change problem, then the problem involves a percent of ____decrease____ (increase/decrease).

2. If you use (new amount) — (original amount) to find the change for a percent of change problem, then the problem involves a percent of ____increase____ (increase/decrease).

Complete the chart.

	Original Amount	New Amount	Percent Proportion	Percent Increase or Percent Decrease?
3.	10	13	$\frac{\text{change}}{\text{original}} \rightarrow \frac{3}{10} = \frac{r}{100}$	increase
4.	10	7	$\frac{\text{change}}{\text{original}} \rightarrow \frac{3}{10} = \frac{r}{100}$	decrease
5.	50	42	$\frac{\text{change}}{\text{original}} \rightarrow \frac{8}{50} = \frac{r}{100}$	decrease
6.	50	58	$\frac{\text{change}}{\text{original}} \rightarrow \frac{8}{50} = \frac{r}{100}$	increase

7. When you find a discount price, do you add to or subtract from the original price? **subtract**

Remember What You Learned

8. If you remember only two things about the ratio used for finding percent of change, what should they be? **Subtract the prices, then divide by the original number.**

NAME _____ DATE _____ PERIOD ____

2-7 Study Guide and Intervention

Percent of Change

Percent of Change When an increase or decrease in an amount is expressed as a percent, the percent is called the **percent of change**. If the new number is greater than the original number, the percent of change is a **percent of increase**. If the new number is less than the original number, the percent of change is the **percent of decrease**.

Example 1

Find the percent of increase.
original: 48
new: 60

First, subtract to find the amount of increase. The amount of increase is $60 - 48 = 12$.

Then find the percent of increase by using the original number, 48, as the base.

$\frac{12}{48} = \frac{r}{100}$ Percent proportion
$12(100) = 48(r)$ Cross products
$1200 = 48r$ Simplify.
$\frac{1200}{48} = \frac{48r}{48}$ Divide each side by 48.
$25 = r$ Simplify.

The percent of increase is 25%.

Example 2

Find the percent of decrease.
original: 30
new: 22

First, subtract to find the amount of decrease. The amount of decrease is $30 - 22 = 8$.

Then find the percent of decrease by using the original number, 30, as the base.

$\frac{8}{30} = \frac{r}{100}$ Percent proportion
$8(100) = 30(r)$ Cross products
$800 = 30r$ Simplify.
$\frac{800}{30} = \frac{30r}{30}$ Divide each side by 30.
$26\frac{2}{3} = r$ Simplify.

The percent of decrease is $26\frac{2}{3}\%$, or about 27%.

Exercises

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

1. original: 50
   new: 80
   increase; 60%
2. original: 90
   new: 100
   increase; 11%
3. original: 45
   new: 20
   decrease; 56%
4. original: 77.5
   new: 62
   decrease; 20%
5. original: 140
   new: 150
   increase; 7%
6. original: 135
   new: 90
   decrease; 33%
7. original: 120
   new: 180
   increase; 50%
8. original: 90
   new: 270
   increase; 200%
9. original: 27.5
   new: 25
   decrease; 9%
10. original: 84
    new: 98
    increase; 17%
11. original: 12.5
    new: 10
    decrease; 20%
12. original: 250
    new: 500
    increase; 100%